Background Paper and Brief for the development of a new Primary Mathematics Curriculum

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Background paper for the development of a new primary mathematics curriculum

The current Primary School Mathematics Curriculum (PSMC) (Department of Education and Science [DES], 1999) was introduced in 1999, with in-service for mathematics provided in 2001–02, and implementation beginning in 2002–03 (DES, 2005). Much has changed and happened since then. In meeting the demands of unprecedented societal and educational change, it is important to review and update the curriculum to ensure children are afforded a high-quality, coherent, and more relevant mathematics education that will contribute towards their personal and academic learning and development. This background paper is not exhaustive but will, it is hoped, provoke rich discussion and provide emergent signposts towards the development of a new Primary Mathematics Curriculum (PMC[1]). The paper begins by setting out the context for change and posing the question, *What is mathematics?* before offering a brief synopsis and critique of the PSMC.

This background paper draws on an extensive suite of evidence which includes relevant national and international data and research. In particular, it utilises the National Council for Curriculum and Assessment's (NCCA) curriculum reviews (2005, 2008) and evaluations by the Department of Education and Skills (2005, 2010), the two recent mathematics research reports (NCCA Reports 17 and 18, 2014), and the international audit of mathematics curricula (Burke, 2014) commissioned by the NCCA. Findings from focus groups carried out to elicit teachers' and principals' views, beliefs and values regarding mathematics learning and pedagogy, and their ideas regarding the development of a new mathematics curriculum, are also included.

[1] For the purposes of distinguishing between the current and new primary math curricula, the 1999 curriculum will be abbreviated as PSMC whereas the new primary curriculum will be abbreviated as PMC.

The following online research reports, summary and audit are recommended in support of this background paper:

- Dunphy, E., Dooley, T., and Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education*. Research Report 17, National Council for Curriculum and Assessment, Dublin. Available at http://ncca.ie/en/Publications/Reports/NCCA_Research_Report_17.pdf

- Dooley, T., Dunphy, E., and Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education*. Research Report 18, National Council for Curriculum and Assessment, Report, Dublin. Available at http://ncca.ie/en/Publications/Reports/NCCA_Research_Report_18.pdf

- Burke, D. (2014). *Audit of Mathematics Curriculum Policy across 12 Jurisdictions*. National Council for Curriculum and Assessment, Dublin. Available at http://ncca.ie/en/Publications/Reports/Audit-mathematics-curriculum-policy.pdf

Context for change

Primary classrooms have changed a great deal since 1999. While the current mathematics curriculum is sometimes still referred to as 'new', Ireland has one of the oldest primary mathematics curricula in Europe and so it's important that we explore its suitability for the current context. Curriculum reviews and evaluations and feedback from teachers over the past decade have resulted in a call for a less 'crowded' primary curriculum that promotes collaborative learning, problem-solving approaches and supports teachers to cater for increasingly diverse needs in the classroom. Teachers have expressed concerns about meeting the challenging demands of wide-ranging and systemic factors that impact implementation such as textbooks, class size and standardised testing.

The 1999 mathematics curriculum has many strengths. With firm theoretical roots in Piagetian and radical constructivism, the curriculum promotes the development of children's meaning making, mathematical language, skills and concepts as well as fostering positive attitudes to maths. There remains, however, scope for improvement. Contemporary thinking and research offers fresh insights into 'how children learn' and 'why they learn in particular circumstances'. This thinking, which has strong Vygotskian influences promotes learning as a social and collaborative process where children's learning is enhanced through active participation, engaging in 'mathematization'[2], working collaboratively with others as well as children building positive identities of themselves as mathematicians. This shift in theoretical perspective demonstrates the need for revisiting the aims of the PMSC and identifying where improvements can be made building on the many strengths of the current curriculum.

The context for change and the development of a new primary mathematics curriculum is grounded in learning from recent research, literature, international studies, audits and national and international assessments available. The background paper aims to exemplify this learning and lay the foundations for change towards the development of the new mathematics curriculum.

The new primary mathematics curriculum will be presented using broad learning outcomes. These outcomes will replace the existing content objectives. Informed by research, the learning outcomes will describe the learning that children will be able to demonstrate at the end of a two-year period. It is intended that learning outcomes will give teachers more flexibility and opportunity to plan for, and provide rich learning experiences for children in the classroom. Progression continua, along with examples of children's mathematical learning, will support teachers to interpret and differentiate learning outcomes supporting children to learn at a level and pace appropriate to them. Furthermore, support material will help to bring to life practical ideas on effective approaches to teaching mathematics as evidenced in research.

What is mathematics?

Terms such as mathematics, numeracy, and mathematical or quantitative literacy have different meanings in different contexts, resulting *in difficulties in the debate about critical aspects of mathematical education* (Turner, 2012, p.1). Frequently there is ambiguity

[2] Mathematization involves children interpreting and expressing their everyday experiences in mathematical form and analysing real world problems in a mathematical way through engaging in key processes such as connecting, communicating, reasoning, argumentation, justifying, representing, problem-solving and generalising (Ginsburg, 2009; Treffers and Beishuizen, 1999).

between the way people commonly use these terms and their intended meaning. Some view numeracy as more practically oriented and a part of mathematics (Dunphy *et al*., 2014), while others consider mathematics as part of numeracy, or mathematical or quantitative literacy in general (Turner, 2012). Discourse regarding terminological issues is ongoing and precise meanings continue to be debated (INTO, 2013). Of late, the Department of Education and Skills appears to favour the term 'numeracy' in various publications regarding mathematics stating that *numeracy is not limited to the ability to use numbers, to add, subtract, multiply and divide* but *encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings* (DES, 2011, p.8). Similarly, the authors of the NCCA-commissioned research reports on mathematics (Reports 17 and 18, 2014), adopt Hersh's (1997) *view of mathematics as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context* (p.xi); and, in keeping with others (e.g. Dweck, 2000; Boaler, 2009), consider that everyone is able to solve problems, communicate their mathematical thinking, and make sense of the world through mathematics. Understanding the nature of mathematics and clarifying what it means for children to engage in doing mathematics is fundamental to the development of a new PMC, and would make a good starting point for discussion. NCCA Report 17 (Dunphy *et al*., 2014, pp.33-36) provides a more detailed account regarding contemporary definitions of mathematics education.

The Primary School Mathematics Curriculum (1999)

The 1999 Primary School Mathematics Curriculum (PSMC)[3] which replaced the 1971 mathematics curriculum, views mathematics as:

> *…the science of magnitude, number, shape, space, and their relationships and also as a universal language based on symbols and diagrams. It involves the handling (arrangement, analysis, manipulation and communication) of information, the making of predictions and the solving of problems through the use of language that is both concise and accurate.* (DES, p.2)

[3] Available online at http://curriculumonline.ie/Primary

The PSMC (1999) is based on constructivist principles and comprises the following five strands for children from junior infants to sixth class: Number, Algebra, Shape and Space, Measures, and Data; with Early Mathematical Activities an additional strand for junior infants only. These strands are considered interrelated and are subdivided into various strand units. The content of the PSMC is divided into four levels or stages (infants, first and second classes, third and fourth classes, and fifth and sixth classes), delineated by year and accompanied by Teacher Guidelines[4]. The curriculum identifies the following six mathematical skills which children need to develop: Applying and Problem-Solving, Communicating and Expressing, Integrating and Connecting, Reasoning, Implementing, and Understanding and Recall; and encourages each child *to be confident and to communicate effectively through the medium of mathematics* (p.2). The PSMC promotes a wide range of teaching methodologies with cross-curricular linkage and integration. Guided-discovery learning and less reliance on textbooks and/or workbooks are encouraged. Collaborative and active learning in a mathematics-rich environment is promoted along with the use of concrete learning resources and digital technology for all classes. Discussion and the development of mathematical language are highlighted as central to children's learning of mathematics and the importance of developing estimation skills is also emphasised. Real-life problem-solving is viewed as a key element of the curriculum since it helps develop higher-order thinking skills, and highlights how mathematics can be used in everyday life. The PSMC outlines what should be assessed and offers a range of assessment practices to elicit information regarding children's progress.

The suggested time allocation for mathematics (DES, 1999) was originally 2 hours 15 minutes per week in infant classes, and 3 hours per week in all other classes, but was subsequently changed to 3 hours and 25 minutes, and 4 hours and 10 minutes respectively (Circular 0056/2011).

Since 1999, NCCA has published materials to provide additional help with the implementation of the PSMC. These materials include planning resources for teachers, a glossary of mathematical terms and bridging materials for 5th/6th classes to help children prepare for post-primary school. The NCCA also developed a suite of materials to support parents in

[4] Available online at http://curriculumonline.ie/Primary

helping their children to learn mathematics. These materials include tip sheets and videos of children and parents learning together[5].

Critique of the PSMC

In general, the PSMC was well-received by teachers and schools. The PSMC has many strengths. A recent desktop audit by Burke (2014) affirmed the comparative strength of the PSMC to the mathematics curricula in 13 other jurisdictions. Content, structure, and banding arrangements of the current curriculum were considered to be typical of international mathematics curricula. Additionally, the succinct articulation of content objectives for each of the five strands, at each of the eight class levels, were identified as a strength. Indeed, a recent report (Eivers and Clerkin, 2013) found the PSMC, while outdated, to be reasonably well aligned with the TIMSS mathematics assessment framework and items.

In a review by the NCCA (2005), Number was identified by teachers at all classes as the most useful strand, with Data (new to the PSMC) identified as least useful. Table 1 illustrates the main strengths and challenges/weaknesses with the curriculum as reported by teachers in that review.

Table 1: Strengths and challenges/weaknesses of the PSMC reported by teachers (NCCA, 2005)

Strengths	**Challenges / Weaknesses**
Children's enjoyment of mathematics Child-centred Emphasis on practical work Children's success in specific content areas	Time Appropriate use of assessment tools Catering for the range of children's abilities

[5] Available online at http://www.ncca.ie/en/Curriculum_and_Assessment/Parents/Primary/

One of the main criticisms levelled at the wider primary curriculum (1999) was the apparent disconnect between curriculum and assessment, with assessment ostensibly treated as an 'add-on' activity, and a lacuna regarding Assessment for Learning (AfL) evident in official curriculum documentation (Sugrue, 2004; 2011). While the NCCA published *Assessment in the Primary School Curriculum: Guidelines for Schools* in 2007, few teachers received CPD regarding these guidelines, and while teachers are willing to embrace assessment in their classrooms, the guidelines may remain underused (INTO, 2010).

Teachers, while acknowledging that the curriculum was flexible and had many strengths, have highlighted that it created unrealistic expectations and resulted in excessive paperwork (INTO, 2015). Additionally, they have identified the issue of curriculum overload, believing there is too much content, coupled with too many subjects, making it impossible to teach all subjects to a high standard. Furthermore, teachers believe that the curriculum can only be implemented effectively when schools are properly resourced and in receipt of high quality, practical, whole-school focussed CPD (INTO, 2015). Consequently, while these issues refer to the 1999 curriculum as a whole, they also need to be considered when developing the new PMC, perhaps through increased integration or teacher autonomy (INTO, 2015).

By international standards, Ireland's range of curriculum supports in mathematics is limited and the articulation of attainment expectations and the provision of exemplars, lags behind other countries (Burke, 2014). Furthermore, mathematics curricula in many jurisdictions have recently undergone significant redevelopment and improvement, and have incorporated relevant research, literature and contemporary thinking in mathematics and assessment (Burke, 2014), further highlighting the need for review and redevelopment.

Implementation of the PSMC

Discrepancies can exist between the intended curriculum and how it is implemented. Looney (2014) discussed the belief often held by policy-makers that problems with curriculum implementation result from teachers failing to follow the instructions they have been given, rather postulating that *curriculum aims are rarely a good guide to curriculum experiences* (p.8). Accordingly, it is widely recognised that the problem of curriculum implementation is difficult to solve (e.g. Sahlberg, 2007).

Insights from the classroom

A review conducted by the NCCA (2005) of the PSMC found teachers had prioritised focusing on specific curriculum content, increasing their use of practical work, and giving more attention to the use of mathematical language (NCCA, 2005). An evaluation of curriculum implementation (DES, 2005) highlighted challenges with methodologies and differentiation strategies employed, problem-solving, and assessment practices evident in classrooms. More recently, findings from incidental inspections (DES, 2010) have provided a snapshot of mathematics curriculum implementation in Irish primary classrooms. A total of 527 mathematics lessons were observed by the Inspectorate between October 2009 and October 2010. While findings of the overall implementation of the PSMC were mixed, learning outcomes were satisfactory in 85.4% of the lessons inspected. Many strengths in the provision of mathematics education were identified. However, the report highlighted particular challenges in teacher preparation, teaching approaches and methodologies, as well as in assessment *in an unacceptably high proportion of the mathematics lessons observed.* Moreover, only half of the children observed were enabled to work collaboratively, while ICT was used in only 30% of the lessons.

Other important insights into implementation of the PSMC are provided by *The Primary Classroom: Insights from the 'Growing up in Ireland' Study* (McCoy, Smyth and Banks, 2012). This report highlighted that 40% of children were found to spend three hours or less per week on mathematics, while 25% spend five or more hours, deducing that *some students have over 18 full days less instruction than others* (p.iii). As with other studies, teachers reported difficulties in catering for the range of children's abilities in mathematics, but despite this, generally high levels of children's engagement were reported. Another recent report, *National Schools, International Contexts* (Eivers and Clerkin, 2013), looked beyond the test scores achieved by Irish students in TIMSS (2011) and explored Irish classrooms with a mission to explain children's performances. Classroom practice was found to place a heavy emphasis on the Number strand, arguably to the detriment of core mathematical skills. Furthermore, relative to other countries, it was found that insufficient time was spent developing more complex problem-solving skills or learning key skills (Close, 2013b). The report found that Irish fourth-class children were

> *more likely than their peers internationally to work out problems with their class under their teacher's guidance,…and somewhat less likely to relate what they learned in a mathematics lesson to their everyday lives, or to take a written mathematics test* (p.93).

Teachers' experiences and views

The NCCA recently organised a series of focus groups around the country to obtain more up-to-date views regarding mathematics teaching and learning. Almost 100 teachers attended the focus group sessions at nine education centres. While a convenience, non-probability sample was utilised to locate focus group participants, member-checking by Education Centre personnel was used to ensure participants were practicing teachers.

Teaching mathematics

Focus group participants expressed the view that mathematics is an important life skill and that children need to be able to use mathematics outside the classroom in the real world and be comfortable with mathematics. They also highlighted that developing mathematical skills impacts on other areas of learning, thinking and problem-solving. Children talking about mathematics and explaining their approaches to problem-solving was highlighted as being an important classroom activity.

> *Make links to maths in the real world, it has to be relevant and purposeful.*

The importance of teachers understanding how children learn and starting teaching at a child's level, was raised. It was also emphasised that a child's ability level needs to be recognised and that within a class the ability range can be broad and may broaden as children get older.

> *[We need] emphasis on how children learn, massive upskilling required.*

The usefulness of teaching programmes such as Maths Recovery[6] was highlighted by participants. Teachers who had received Maths Recovery training felt they had a better understanding of mathematics and of how the concepts develop which gave them the pedagogical content knowledge they needed to teach effectively. It was felt that similar training should be extended to all teachers and not just to teachers working in DEIS settings.

> *Maths Recovery moves away from rote method, the method is 'understand', we show children a method that makes sense to us not to them. [A child] can do a sum mechanically but don't know how it's done conceptually. We want them to be able to do it as problem-solving not just process.*

Critical factors at play

The prevalence of workbooks and textbooks for teaching mathematics was noted and their usefulness was called into question. Participants highlighted that there is a need to make mathematics real to children and to use other learning methods such as group work, talking about mathematics and concrete objects. The usefulness of concrete materials was noted, in particular how they help children engage more with their learning. The importance of teachers being skilled in how to use concrete materials was also highlighted.

> *Maths language is very important, children can't develop this from books.*
>
> *Children learn maths better in peer groups, far more engaging and productive than text books.*
>
> *Getting kids to talk about maths is more important than filling in workbooks.*

Participants expressed some concerned views about the influence of standardised tests, the results of which are seen as having high value by parents. Concerns were expressed about whether teachers would teach to the test and the point was made that there is a conflict between the teachers' desire for the child to perform well on the test but recognition that this will impact negatively on the allocation of resources.

[6] Maths Recovery is an intensive individualised teaching programme for low-attaining children in first class in primary school. The programme involves specialist teachers using a unique instructional approach, in addition to distinctive instructional activities and assessment procedures.

We plan around that test and Measure only gets one question so it's always left to end of the year. If it's not in the test much, it's not taught much, simple as.

The importance of parental influence was highlighted by teachers with the view that parents' own past experiences and understanding of mathematics can have a detrimental impact. Moreover, an exploration of perceptions of parental values suggested that many parents value traditional methods and believe that children should be taught as they were taught. The importance of engaging with parents was emphasised.

Parental expectations can value traditional learning of maths- workbooks, homework etc. This can be detrimental, they can be quite forceful that you are teaching them [their children] wrong.

In the main, participants highlighted that they valued the broad range of resources available including digital and online resources that can be used to engage children in mathematics.

Core maths hasn't changed, concepts and thinking etc but how we approach it has, we have fantastic opportunities to use other resources to help kids learn.

Provide us with online resources, suggested websites are fine but resources specially built for the curriculum have a huge benefit.

Points were made that a lack of confidence in teachers' own mathematical ability can impact upon how they approach teaching mathematics and that CPD and upskilling are needed to support teachers.

Teachers' attitudes to maths influence how children learn. Teachers can be fearful of maths and lack confidence, particularly substitute teachers coming in.

Sometimes teachers are afraid (those who are not confident in their maths ability) to try open-ended tasks.

More CPD or resources for self-improvement are needed.

Further challenges

Other perceptions of challenges impacting on the teaching and learning of mathematics shared by teachers in the focus groups, included the following.

- Class size was identified as a problem affecting how they teach mathematics and how children in their classrooms learn mathematics, as was the time available for teaching mathematics.
- Lack of classroom control was identified as impacting negatively on mathematics learning and teaching. It was noted that providing children with motivating and relevant learning experiences would help classroom management.
- Frustration was expressed at what teachers considered ‘fads and initiatives’.
- Problems related to content strands in the curriculum were raised, such as the relative importance given to Number and Measurement.
- Others highlighted some problems children have with understanding number and moving from concrete to abstract concepts.
- A small number of comments were made emphasising the usefulness of traditional teaching methods compared to more recent, active learning approaches.

Contexts for learning

While the PSMC encouraged less reliance on textbooks, evidence suggests that mathematics planning and instruction in Irish primary classrooms is still regularly based around textbooks rather than the curriculum, with most children using textbooks on a daily basis, even in infant classes (Dunphy, 2009; Eivers *et al*. 2010). The following views offered by teachers provide some insight into this.

> *Planning with textbook is helpful for timing and managing to cover the curriculum in the time provided.*
>
> *Planning is dictated to by the book because it gives you structure. Without the book planning would take more work, you could dip in and out of books and photocopy pages for assessment but this is time consuming.*
>
> *Time is an important factor, to be innovative you need more time. It is difficult to balance exploratory, hands-on approach with constraints of curriculum overload, finishing book, getting ready for Sigma-T, must be finished by mid-May.*
>
> *…Maths book good for teacher to build confidence.*

Primary school teachers – NCCA focus groups (Autumn 2015)

Mathematics textbooks in Ireland have been criticised for including volumes of repeated practice with little difference in difficulty levels (Dooley *et al.* 2014). Moreover, the 'worked examples' which tend to predominate textbooks in the Irish context, have been criticised for being set almost exclusively in mathematical contexts rather than in real-life contexts (e.g. Delaney, 2010). Notably, two thirds of current TIMSS assessment items are embedded in applied contexts. However, Close (2013b) found that these 'real-life' questions proved difficult for fourth class children in Ireland since they have limited exposure to such questions at school. Additionally, the way problems in textbooks are primarily located in dedicated sections, and are predominantly word problems, has also been criticised.

> *Abstract word problems always left to end. Students who struggle with abstract never get to word problems or where you see it in real life.*
>
> *The textbook doesn't motivate kids, you need to [make] maths real to them … Context is everything for kids and choosing a good context can integrate with other subjects such as visual arts.*
>
> Primary school teachers – NCCA focus groups (Autumn 2015)

As part of a recent study (Eivers, Delaney and Close, 2014), three commercially available mathematics textbooks at third class level, were analysed to find out how well they aligned with the PSMC (Table 2). As textbooks have been found to often be the medium through which children experience the PSMC, it is interesting to view these results.

Table 2: Percentages of pages in three Irish pupil textbooks that cover each PSMC strand (adapted from Eivers *et al*., 2014)

% of PSMC objectives (N=70)		% of pages		
		Textbook A (N=174)	Textbook B (N=172)	Textbook C (N=156)
Number and Algebra	42.8	65.2	61.2	52.9
Shape and Space	24.3	8.3	13.5	16.5
Measures	24.3	20.1	20.5	23.4
Data	8.6	6.3	4.8	7.2

Close (2013a) suggests that teachers should be supported to move away from over-dependence on textbook activities and recommends that a repository of *good tasks* aligned with high quality professional development should be provided.

Professional development for teachers

A comprehensive programme of ongoing continuous professional development (CPD) was provided to help teachers implement the PSMC effectively (Harford, 2010; Sugrue, 2011). Opinions from research evaluating the impact of this large-scale, centralised CPD programme are somewhat mixed but significant (Murchan *et al*. 2009; Harford, 2010; Sugrue, 2011). Harford (2010) highlights that CPD in the Irish context has primarily been focused on equipping teachers to respond to curriculum change instead of the development of pedagogical approaches and reflective practice. A review by Murchan *et al*. (2009) revealed improved teacher knowledge but modest and varied implementation of the 1999 curriculum, and suggested that better identification of teacher needs prior to the CPD would have focused resources where they were most needed. They highlighted *an over-emphasis on planning rather than on creating local communities of practice per se* (p.466), and voiced concern that this model of CPD could lead to *a culture whereby teachers feel incapable of embracing reforms and adjusting professional practice without first receiving externally provided PD* (p.468). Sugrue (2011) concurred and suggested that there is *a need for more school-based CPD, and schools need to take more responsibility for the professional learning of staff* (p.803). In sharing their views of professional development (NCCA focus groups, Autumn, 2015), teachers echoed this preference for school-based CPD.

> *Two days training 'in-school' would be very useful because it would give teachers the confidence to go out and teach the new curriculum.*
>
> *Someone coming to school to help teachers be familiar with strands not with skills.*

Teachers expressed a need for support and professional development in the following areas particularly.

> *To ensure that teachers use the resources appropriately.*

Huge investment needed in CPD in maths. Useless allocating worthless resources. Caution against spending on resources without training.

Need for workshops, also videos of implementing curriculum with all the pitfalls.

Access to the CPD that DEIS schools get should be open to all teachers.

Many teachers felt that CPD should help teachers understand mathematical progression**.**

CPD is needed to show people what maths really is.

CPD needs to help teachers understand mathematical progression so that they can help students where they need it and fill the gaps.

Everything is taught in isolation, no connections are made.

Teachers are also looking for assessment tools and tools that will help them to diagnose why there is difficulty in learning and identify appropriate actions/steps to help children overcome their difficulty.

Good assessment tools are needed.

Need help in diagnosis- why a child is having difficulty and what to do to help them.

In acknowledging the potential for digital resources to enhance children's learning experiences, teachers also cautioned on the importance of CPD for purposeful use of these resources.

Digital resources have a huge benefit when teaching and learning maths but like we said, you have to know how and why to use them.

Indeed, current best practice suggests that CPD for teachers is job-embedded, sustained, collaborative, and linked to practice (Darling-Hammond and Richardson, 2009; Desmione, 2009; Guskey, 2000; O'Sullivan, 2011; Teaching Council, 2015). Therefore, the type, quality and effectiveness of CPD offered will undoubtedly impact the implementation of the new PMC, and so, the provision of CPD should be factored into the new mathematics curriculum discussions and consultations.

Context for curriculum development

Myriad factors can impact on the development of curricula, for example, political, economic, technological, and social, to name but a few; while international, national and local determinants also come into play. This section investigates the current context which will influence the development of the PMC. It explores policy developments and constraints at national and local levels and discusses how these have influenced the teaching and learning of mathematics in recent years. Furthermore, it explores the recent request by the Minister for Education and Skills, Mr Richard Bruton, TD, for the new primary mathematics curriculum to ensure that every child has an opportunity to develop the computational, and flexible and creative thinking skills that are the basis of computer science and coding.

We have witnessed substantial change in the Irish primary education system since the publication of the PSMC in 1999. The past 16 years have seen huge societal changes such as changes in the patterns of community and family life as well as rapid and unprecedented change in how children use and engage with digital and other media. These changes, among others, have and continue to have significant implications for schools. More recently, there has been an increase in the number of children with English as an Additional Language (EAL) also children learning mathematics through Irish. For many children, Irish is a second language and for some, possible a third or fourth language. These changes can present challenges for teachers during mathematics lessons, particularly regarding children's understanding and use of mathematical language. Additionally, the policy focus on inclusion means that many more children with special educational needs (SEN) are now attending mainstream schools in comparison to when the PSMC was launched. Responding to increased diversity in classrooms and supporting an extending range of children's learning needs poses challenges for teachers, the pedagogical challenge made more acute by the absence or low levels of additional support.

Meanwhile, class size in Irish primary schools remains the second highest in Europe, after England, with an average of 25 children per class in comparison to 20 children on average in other EU21 countries (OECD, 2015). This can result in difficulty for teachers when trying to engage in group work or talk and discussion or when using concrete materials during mathematics class. Additionally, it presents challenges when catering for the range of abilities present in most primary classes in the Irish context or when trying to support the learning of individual children. Notwithstanding these arguments, the recent OECD report (2015) also suggests that while smaller class size can lessen behavioural problems, there is little evidence that children's achievement is increased.

Universal Design for Learning (UDL), a research-based set of principles for curriculum development have also been devised since the publication of the PSCM. These principles promote equity of opportunity for all children and as such present a new lens for the development of curricula that addresses the challenges faced by schools in meeting the needs of an increasingly diverse school population (Meyer, Rose, Gordon, 2012). The development of the new PMC will be cognisant of the myriad factors impacting schools in Ireland currently as well as new theoretical perspectives offered in the literature. Of note and concurrent with developments in primary mathematics, there will be ongoing work in redeveloping the wider primary curriculum.

National Literacy and Numeracy Strategy, 2011-2020

Data from national and international assessments which suggested that Irish students were underperforming in mathematics were instrumental in the development of *Literacy and Numeracy for Learning and Life*, *the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (DES, 2011). This is a key policy document in the Irish context and has had significant influence on mathematics education in recent years. The *National Literacy and Numeracy Strategy* (2011) acknowledges the importance of mathematics education for all young people and presents a shared vision for numeracy for all stakeholders. It adopts a broad focus and emphasises the need to support numeracy in all curriculum areas and subjects. The strategy sets out a comprehensive set of targets and outlines actions that need to be taken in order to improve the teaching of literacy and numeracy in Irish schools, including robust self-evaluation. Regarding mathematics at primary level, some of the key targets in the strategy are:

- To promote better attitudes to mathematics among young people;
- To enable children's ability to understand, appreciate and enjoy mathematics;
- To improve mathematical language and ideas at early childhood level;

- To increase the percentage of children performing at the highest levels and decrease the percentage of children performing at the lowest levels in national assessments of mathematics by at least 5 percentage points and;
- To improve the way assessment information is used.

Since the introduction of the *National Literacy and Numeracy Strategy*, there has been an increase in the amount of time allocated to the teaching of numeracy (1 hour and 10 mins per week). Other relevant changes at national level include changing the B.Ed. programme from three to four years to allow extra time for the development of teachers' knowledge and pedagogical skills, especially in the area of numeracy. It is also hoped this will help produce reflective practitioners capable of applying current knowledge, methodologies and strategies in the teaching and learning of numeracy, as well enabling them to use ICT to support the teaching of numeracy.

Previous to the introduction of the *National Literacy and Numeracy Strategy*, schools were required to administer standardised testing at only two mandatory points with flexibility as to when they tested; at the end of first class or the beginning of second class, and at the end of fourth class or the beginning of fifth class, along with a requirement that the results of these tests be reported to parents (DES Circular 138/2006). Since the introduction of the Strategy, standardised assessments are now compulsory at three mandatory points—at the end of second, fourth and sixth classes in primary schools—and results are sent to the DES at the end of each year, reported to the Board of Management, as well as to parents (Circular 0056, 2011).

The *National Literacy and Numeracy Strategy* also highlights the importance of digital literacy. During recent focus group interviews (INTO, 2015), teachers acknowledged the benefits of using ICT as a pedagogical tool but highlighted that it should not dominate practice. However, they criticised the lack of ICT resources in classrooms, the inadequate broadband connectivity, the lack of technical support, and insufficient teacher professional development. Following a period of little investment in ICT in schools, the *Digital Strategy for Schools 2015-2020* outlines the Government's vision for the integration of ICT into schools to:

Realise the potential of digital technologies to enhance teaching, learning and assessment so that Ireland's young people become engaged thinkers, active learners, knowledge constructors and global citizens to participate fully in society and the economy (p.5).

The strategy focuses on the following key themes:

- Teaching, Learning and Assessment Using ICT
- Teacher Professional Learning
- Leadership, Research and Policy
- ICT Infrastructure.

A core aim of the Strategy is to support and enable children to move beyond being passive users of technology to actively fostering creativity and ambition through technology. Some key objectives of the Strategy are that digital learning objectives should be embedded within future education policy and curriculum initiatives and that technology-assisted assessment should be promoted.

In the Strategy, the Minister for Education and Skills states that the NCCA will ensure that future curriculum specifications will incorporate clear statements of learning that focus on developing digital learning skills and the use of ICT in achieving learning outcomes at all levels of education (p.4). This is similar to what Shiel *et al.* suggested in 2014 when they highlighted the importance of paying adequate attention to the effective use of ICTs in mathematics lessons when developing and implementing a new PMC. Internationally, many countries provide interactive websites which offer myriad resources and lesson-enriching activities for teachers (Burke, 2014). Some countries, for example, Scotland, provide websites and applications that build on a gaming concept. Nevertheless, no country has yet organised its digital resources in line with grade or strand structures, thus making it time- consuming for teachers to access suitable resources.

Teachers' response to mandatory reporting of standardised test results

In the focus group sessions conducted by the NCCA (Autumn 2015), strong views were shared regarding the mandatory reporting of results to the DES, parents, and Boards of Management, with many teachers expressing concern about increased pressure, particularly those teaching second, fourth and sixth classes, to ensure their children performed well in standardised tests. Congruent with the findings of the INTO discussion papers (2013, 2015), Teachers in the NCCA focus groups believed standardised tests should reflect what they are teaching, and they considered that current tests do not take account of children's collaborative work or the needs of children with EAL. Teachers also questioned if current standardised tests are able to assess the range of problem-solving skills promoted in the PSMC.

> *Teaching to a standardised test means you neglect the development of reasoning, communication and problem solving skills.*
>
> *Test is limited and doesn't test skills. Some students guess and this is not valuable information.*

Teachers believed standardised tests have become 'high stakes' and query their usefulness. Participants felt that they don't test skills, they are not seen as diagnostic or a true reflection of children's ability or attainment. Moreover, teachers felt that children can guess and have a 'bad day', with some teachers teaching to the test.

> *Standardised tests are not useful because they are not designed to be diagnostic. Now they are used as high stakes and are not a true reflection.*
>
> *Sigma can influence planning, in fact it is the single most factor that influences our maths planning, we are definitely teaching to the sigma test.*

Teachers expressed that results of the tests, which may not be necessarily accurate, can have a detrimental knock-on impact on decisions about learning support and resource allocation.

> *Results inform learning support and affect resource allocation…Children who may be a 7 are not really a 7 and don't get resources and are not coping in class.*

Teachers felt that self-perceptions of children may be negatively impacted upon by the use of standardised testing and offered cautions in labelling children with STen scores.

> *Children say they are good or bad at maths, no grey area. Labelling themselves early on.*
>
> *I would much prefer to be able to tell a parent where their child is having difficulty and have a conversation around what can be done to help rather*

> *than a number ...there your child has a STEN of 6 I know you haven't a clue what it means but there you go.*

Diagnostic testing was seen by teachers as something that is needed but is currently missing. Sigma-T was not seen as diagnostic and issues were identified with it and with the Drumcondra test.

> *There is a huge lack of diagnostic tests…would love to have a diagnostic test that you could give to children before you start.*
>
> *Diagnostic is very important; this will actually have a positive impact on teaching learning planning etc.*

Anxiety and stress for teachers, parents and children has been attributed to the mandatory reporting of standardised test results. Discussion findings suggest that schools engage in different practices regarding standardised testing, and that frequently the test manual is not being utilised. Moreover, teachers feel it can be difficult to explain the test to parents who don't always understand what the tests are telling them. While it would also seem that parents want their child to get a high score and are uncomfortable with their child sitting tests they are unprepared for (INTO, 2015).

> *When standardised testing became mandatory there was concern that the scores would be used to compare kids with each other, instead of what they were designed for. Writing S.T. score on the report has caused anxiety and trauma on the child of going down [STEN score].*
>
> *There is too much emphasis on score and misinterpretation by parents.*
>
> *Kids are too young to be doing these long tasks and they get very anxious. Often teachers forewarn kids and put a lot of pressure on them.*

Critical to curriculum developments will be concurrent development and support in standardised testing and assessment. In this endeavour, the following quote from Shiel *et al.* (2014) is noteworthy:

> *The relatively large increase in performance observed in NA '14 suggest that the norms for existing school-based standardised tests may overestimate pupil performance, and hence may not be very useful for the purposes for which they are being used, such as setting school-level targets and identifying students with learning difficulties. This points to a need to benchmark performance on standardised tests used in schools against*

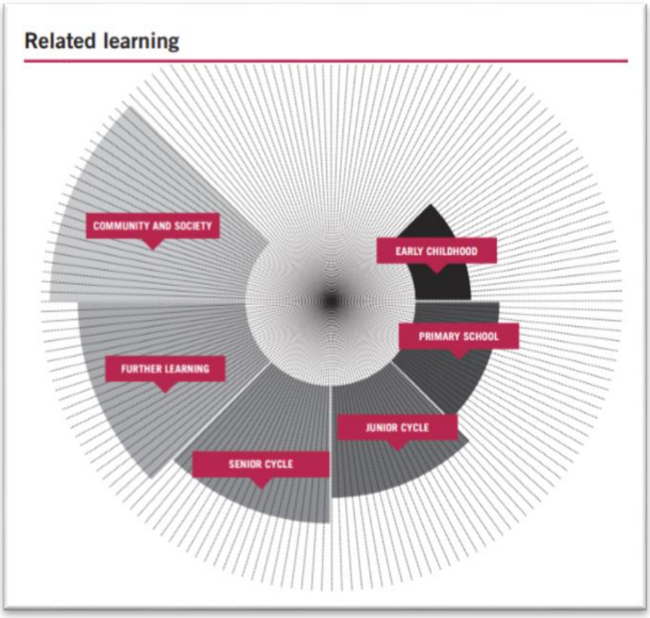
performance in NA '14, with a view to revising and renorming tests, perhaps in parallel with the implementation of revised curricula in English and Mathematics (p.xv).

Transitions: The mathematics continuum

Mathematics learning and development at primary level is part of a continuum which, in the context of state-provided education, begins in the pre-school years, through primary and includes post-primary and even tertiary mathematical learning. Account must therefore be taken of the various transitions involved in children's mathematical education. The importance of children's early mathematical learning and its significance for later mathematical learning and development is now generally recognised and so a new primary mathematics curriculum will need to ensure consistency with *Aistear: the Early Childhood Curriculum Framework* (2009), thereby facilitating and supporting progression in children's learning. *Aistear* emphasises the importance of play, relationships and language for children's learning from birth to six years, and lays important foundations for children's mathematical learning in primary school. In particular, *Aistear*'s *Exploring and Thinking*, and *Communicating* themes and its integration of play as a central teaching and learning approach, help foster children's mathematical learning in the early years, and should therefore feed into the new mathematics curriculum. Further, the *National Literacy and Numeracy Strategy* (2011) recommends that teaching and learning principles and approaches in infant classes should align with those advocated in *Aistear* and acknowledges that lower adult-child ratios would be required to implement these approaches in primary classrooms.

A new PMC will also have to be cognisant of children's subsequent learning at second level and so links with the mathematics syllabus developed through Project Maths, are also important. Like the PSMC, the revised mathematics syllabus at Junior and Leaving Certificate, recognises that mathematical learning is cumulative and that each level builds on previous learning. Consequently, it should encourage learners to utilise the numeracy and problem-solving skills developed in early childhood and primary education, thus attempting to ensure connected and integrated mathematical learning and understanding across the education continuum.

Figure 1: ***Extracted from Mathematics Syllabus Leaving Certificate*** **(2012)**



The syllabus developed through Project Maths emphasises greater understanding of mathematical concepts, and the application of mathematical knowledge and skills. It encourages students to relate mathematics to everyday life and requires sense-making, problem-solving, logical reasoning, higher-order thinking skills, and engagement in rich learning activities than heretofore. To help ease students' transition from primary to post-primary, the NCCA developed a bridging framework which illustrates how the objectives of the PSMC are continued and progressed at second-level, thus ensuring continuity and progression in children's mathematical learning. The framework shows the connections between topics studied in primary and post-primary mathematics and how learning is extended.

Student achievement

Mindful that national and international test results are only one proxy for judging the effectiveness of the PSMC, it must be acknowledged that increasingly these results appear to have assumed increased importance for the government in a globalised economy. Analysis of the results of national and international comparative assessments such as the Trends in International Mathematics and Science Study (TIMSS) and the National Assessments of Mathematics and English Reading (NA) provide an objective overview of the mathematics standards of Irish primary school children, highlighting strengths and weaknesses, and also changes that take place between assessments. Periodic assessments at primary level have revealed that Irish children are underachieving in mathematics (e.g. NA'2009; TIMSS, 2011), especially in important areas of the mathematics curriculum such as problem-solving and Measures.

The 2009 National Assessments of Mathematics Achievement (NA) revealed that traditional methods of instruction still predominated Irish classrooms, with whole class teaching, children working individually rather than in pairs or groups, and the use of textbooks and workbooks very much in evidence. Measures and children's ability to apply and problem-solve proved the most difficult items at both levels while no gender differences were discovered, apart from girls' and boys' performance on Measures in sixth class. Key recommendations suggest the adoption of a stronger social constructivist perspective in mathematics teaching and learning, as well as mandatory participation in CPD. The need for more discussion, collaborative problem-solving, use of AfL in every classroom, and increased sharing of good practice at school level are also advocated. Findings from whole school evaluations (WSE) and incidental inspections (DES, 2010; Ó Donnchadha and Keating, 2013) echo many of these points (NA, 2009), once again highlighting the need for attention to assessment practices, collaborative problem-solving and opportunities to learn through talk and discussion during lessons, while also recommending greater use of differentiation and resources.

Results from the NA '14 reflect a time where there was an increased emphasis on numeracy in schools and reveal the first statistically significant improvement in children's overall mathematics since 1980 and considerably higher than in NA'09 (Shiel *at al.*, 2014). These results are important since they provide data on how the *National Literacy and Numeracy* Strategy (DES, 2011) has impacted mathematics achievement and reveal if targets set out in the strategy have been achieved. Overall performance on mathematics in second and sixth classes was significantly higher in NA ’14 than in NA ‘09, with large effect sizes. There were reductions in the proportions of lower-achieving students (from 10% to 5-6%) and a small increase in the number of students performing at the higher-level. However, there is scope for students at both class levels to improve further on higher-level mathematical processes, including applying and problem-solving (Shiel, Kavanagh and Millar, 2014). The fact that there is considerable scope for improvement in mathematics in DEIS schools was also highlighted in NA ’14 by Shiel *et al*. (2014) as well as the fact that Irish children’s performance in mathematics lags behind that of literacy.

In 2011, Ireland (fourth class children only) participated in TIMSS for the first time since 1995. TIMSS provides overall achievement-related data outcomes for participating countries, thus facilitating both national and international comparisons. Additionally, TIMSS gathers other data related to the life and learning experiences of the participants such as attitudes towards school, generally and more specifically, and of particular relevance to the current paper, participants’ attitudes towards mathematics. In TIMSS 2011, Ireland was ranked 17th of 63 participating countries with a mean score of 527, above the TIMSS mathematics centre-point of 500, but significantly lower than the mean scores achieved by children in 13 other countries, including Northern Ireland and England (Eivers and Clerkin, 2012). Irish children performed strongly on Number and there were no significant gender differences in mean scores. Once again, Irish children displayed r*elative weaknesses on data display and on geometric shapes and measures* (p.27) and in the ability to reason. In comparison to TIMSS 1995, the strengths and weaknesses of Irish students remained roughly the same; there was no improvement in Ireland’s overall mean score for mathematics, but, low-achieving pupils did perform better (p.29).

Close (2013) argues that while Irish performance in TIMSS was generally satisfactory, many of the same weaknesses, highlighted in previous international studies (PISA and TIMSS) and in our own national assessments of mathematics remain and need to be addressed. Ireland has again participated in TIMSS 2015 with findings due in late 2016. TIMSS 2015 will show if Irish children's results are sustained or transferable to other contexts. An analysis of findings from the various national and international assessments discussed above highlighted that Irish children still need to improve in the area of applying and problem-solving in particular. Interestingly, Eivers and Clerkin (2013) argue that the poor performance of Irish second-level students in mathematics can be traced back to primary school mathematics, highlighting the need for any new primary mathematics curriculum to take cognisance of the new post-primary syllabus. Similarly, Close (2013b) also argues that results from TIMSS 2011 suggest *what primary pupils are taught may be at the root of the problem of Ireland's below-average standing in mathematics internationally when they move on to second level* (p.1).

Primary mathematics and its relationship with computer science and coding

The last ten years have brought unprecedented technological advances changing the way we communicate with each other, the way we access, process and manage information, and the way we ultimately think and view the world around us. Technology is now so permeated within children's everyday lives they are often referred to as 'digital natives'. What does this digital world mean though for children's learning in primary school and in particular, for their experiences with mathematics? Mathematics, like other subjects in the primary curriculum, can make an important contribution to developing children's computational, flexible and creative thinking. Such thinking is the foundation to computer science. Mathematics and computer science are complementary in so far as children's learning in mathematics will help them to develop computational thinking while computing is increasingly used in mathematics for problem-solving.

So what is computational thinking? It's a powerful thought process used to solve complex problems in schools and in the real world. Computational thinking involves taking a complex problem, understanding what the problem is and developing possible solutions which can be presented in a way that a computer or a human can understand. According to Wing (2006), computational thinking builds on the power and limits of computing processes, whether they are executed by a human or by a machine (p.33). Computational thinking involves children developing and using a number of concepts and processes including:

- logical reasoning (predicting and analysing)
- algorithms (devising steps and rules)
- decomposition (breaking down a problem into parts)
- patterns and generalisations (identifying and using simulations)
- abstractions (removing unnecessary detail)
- evaluation (making judgements).

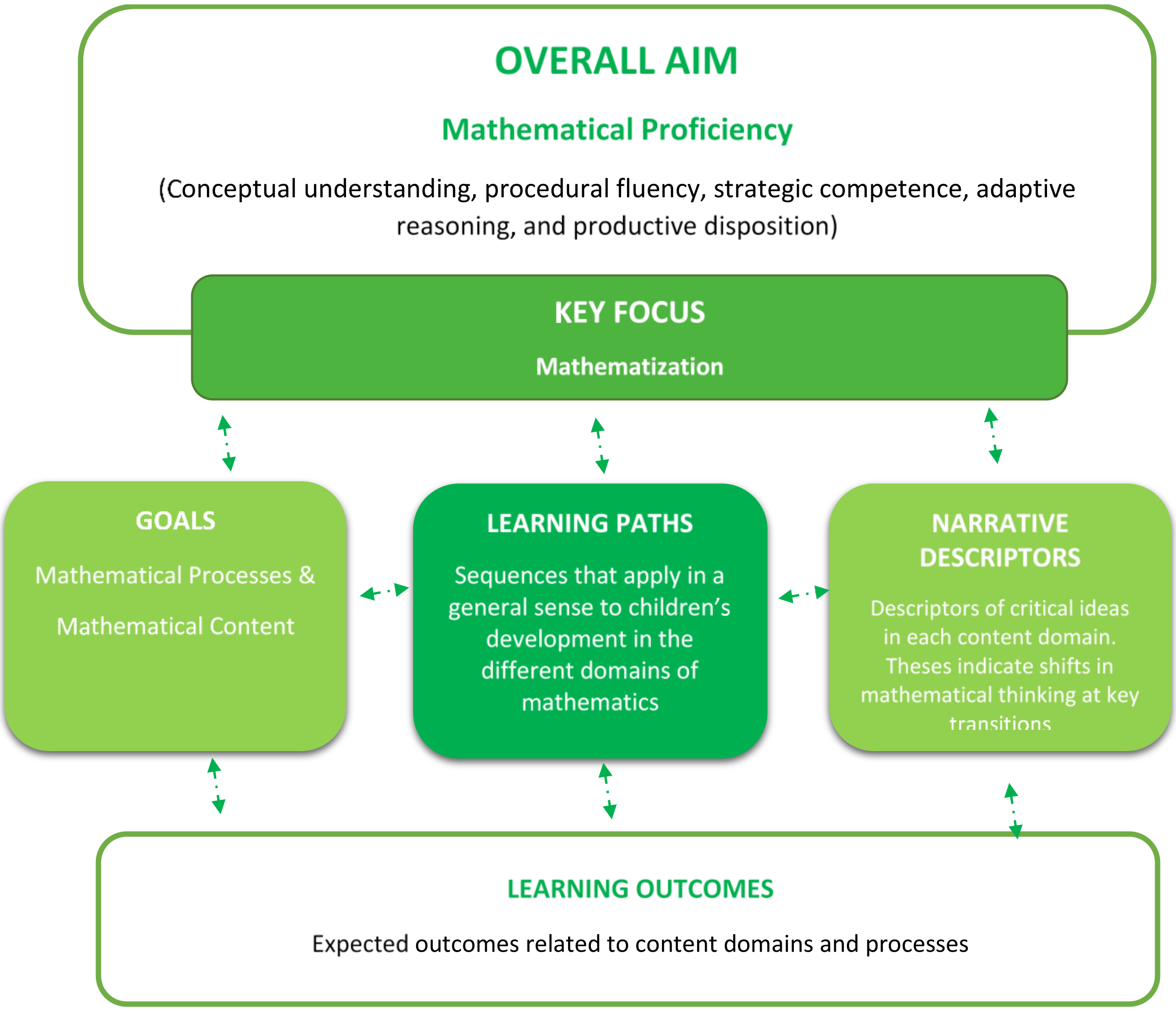
Computational thinking can be developed through playful and engaging learning experiences across the primary curriculum, for example, when writing stories children are encouraged to first plan, to think about main events and identify settings, characters, plot, etc. Or when using fair test investigations in science, children are encouraged to break the investigation down into steps, recognize patterns of what must be kept the same for each test, draw on existing understandings to reason their ideas, analyse results and draw conclusions.

Building on these foundations, computational thinking can then be further developed through rigorous and creative computer science applications such as coding. Such applications offer practical experience to children in using and extending their computational thinking as well as building the knowledge and understanding of the principles of information and computing. that leads to IT fluency. The place or significance of computer science and the extent to which its concepts, processes and applications can or should form part of the new PMC will be an important consideration in the development of the new PMC and the wider work in redeveloping the primary curriculum.

Theoretical underpinnings of a new mathematics curriculum

Research Reports 17 and 18 (Dunphy et al, 2014; Dooley et al, 2014) form a significant part of the suite of evidence used to support this background paper. Both reports are underpinned by the view that mathematics is for all and worthy of pursuit in its own right. Report 17 provides the theoretical underpinnings for the development of mathematics education in young people, and discusses current thinking and views on mathematics, specifically regarding definitions, theories, development and progression. The authors (Dunphy, Dooley and Shiel, 2014) recommend a combination of cognitive and sociocultural perspectives when envisaging a new primary mathematics curriculum (PMC). Report 18, meanwhile, deals with current thinking on the teaching and learning of mathematics. It investigates what constitutes good mathematics pedagogy and looks at appropriate structures for the development of mathematical knowledge for pre- and in-service teachers. It explores mathematical learning and development, in particular the process of mathematization. The report also discusses contemporary curricular issues and developments. Both reports suggest that the overall aim of the new mathematics curriculum should be mathematical proficiency. Mathematical proficiency consists of the five intertwined and interrelated strands of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (NRC, 2001). The reports recommend that since mathematization plays a pivotal role in the development of such proficiency it should permeate all mathematical teaching and learning. Additionally, the reports highlight how learning paths might be used effectively when formulating the new mathematics curriculum. Figure 2 succinctly illustrates the authors' conception of an emerging mathematics curriculum model. It includes content and process goals, learning paths and narrative descriptors, all leading to expected learning outcomes.

Figure 2*:* Emerging Curriculum Model (NCCA Report 18, 2014)



Since it is only possible to offer a brief synopsis of these two reports here, it is recommended that both reports or the executive summaries are read in full[7]. The following sections elaborate on three mathematical areas which are spotlighted in these reports, and are also emphasised in mathematics literature elsewhere. These are:

- Mathematization
- Mathematical knowledge for teaching
- Problem-solving.

[7] Available online at http://www.ncca.ie/en/Curriculum_and_Assessment/Early_Childhood_and_Primary_Education/Primary-Education/Primary_Developments/Maths/Review-and-Research/

Mathematization

The Organisation of Economic Co-operation and Development (OECD, 2002) claims that teaching students to 'mathematize' should be a primary goal of mathematics education. The term 'mathematization' was not used in the PSMC, although a number of its processes, for example, communicating, were implicit in that document. Notwithstanding, the authors of Research Reports 17 and 18 (Dunphy *et al.*, 2014; Dooley *et al.*, 2014) argue that mathematization should be central to the mathematical experience of all children.

Mathematization involves children interpreting and expressing their everyday experiences in mathematical form and comprehending the relations between abstract mathematics and real situations in the world around them (Ginsburg, 2009). This requires children to abstract, represent and elaborate on informal experiences and create models of their everyday activities. Teachers can play a critical role in facilitating children to mathematize by making meaningful connections between the mathematical strands, the real world and other areas of learning. Teachers can also assist children to mathematize by giving language to informal mathematics which children first understand on an intuitive and informal level (Clements and Sarama, 2009, p.244). For example, as a child naturally creates and extends a pattern while making a necklace with links, the teacher can effectively pose questions to encourage the child not only to use appropriate mathematical language to describe the pattern, but also to make predictions and generalisations.

Put simply, Rosales (2015) defines mathematization as *the process of understanding maths within the contexts of children's daily lives* (p.1). Enabling children to talk about their mathematical thinking (math-talk) and to engage in mathematization makes their mathematical thinking visible and helps develop their mathematical knowledge (Clements and Sarama, 2009). By modelling and fostering math-talk throughout the day, teachers can provide the math language that allows students to articulate their ideas.

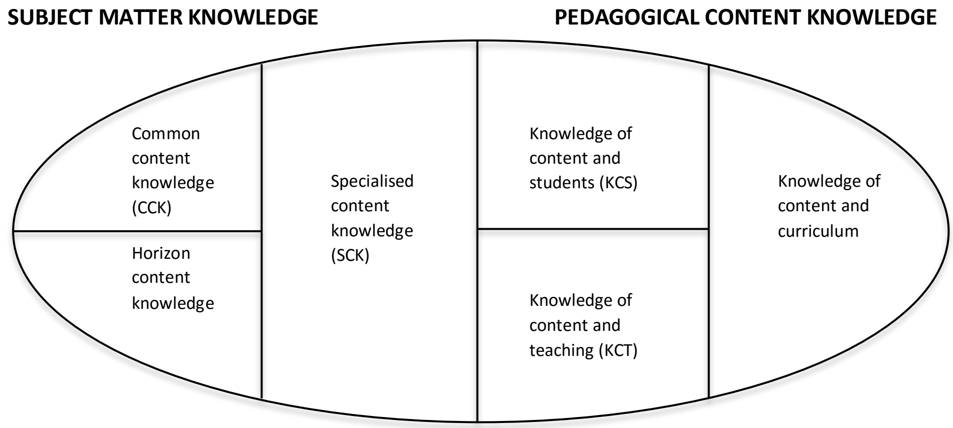
Research Reports 17 and 18 (Dunphy *et al.*, 2014; Dooley *et al.,* 2014) highlight mathematization as pivotal to the development of mathematical proficiency, and proffer that its key processes (connecting, communicating, reasoning, argumentation, representing justifying, problem-solving and generalising) should permeate any new mathematics curriculum. These processes are also core to computer science and many of its applications such as programming and coding. Mathematization strongly supports the computational skills that are also essential to proficiency in computer science and coding. Mathematization is important in building children's capacity to think flexibly and creatively and also contributes to fluency in other disciplines such as science and engineering, among others.

Dooley *et al*. (2014) highlight that mathematization takes dedicated, integrated and sustained time, and so if it is to be central to the new mathematics curriculum, significant changes in curriculum, pedagogy and curricular supports will be demanded, thus posing wide-ranging and systemic challenges. Teachers, too, will be asked to engage in mathematics teaching that is qualitatively different than what they themselves experienced. Ultimately, if teachers are to promote good mathematics learning, they must not only have an openness to and facility with the processes of mathematization, but critically, they must possess good Mathematical Knowledge for Teaching (MKT).

Mathematical Knowledge for Teaching (MKT)

Teachers' knowledge and understanding of mathematics can influence the tasks they select, their level of questioning, and how and to what extent concepts are developed within their classroom (Zopf, 2010). Using Shulman's concept of pedagogical content knowledge (PCK), Ball *et al*. (2008) specifically analysed the work of teaching from a mathematical viewpoint, and developed a theory termed mathematical knowledge for teaching (MKT) defined as …*the mathematical knowledge needed to carry out the work of teaching mathematics* which includes absolutely *everything that teachers must do to support the learning of their students, including planning, assessment, parent-teacher meetings, homework and much more* (p.395). They refined Shulman's (1986) idea of Pedagogical Content Knowledge into at least two subdomains, that of knowledge of content and students (KCS), and knowledge of content and teaching (KCT), and additionally included Shulman's idea of curricular knowledge in this section. Furthermore, they subdivided Shulman's domain of subject matter knowledge into common content knowledge (CCK) and specialised content knowledge (SCK), as well as horizon content knowledge (Figure 3).

Figure 3: Adapted from Ball *et al*., (2008)



CCK is needed by teachers and non-teachers alike, while SCK is unique to the work of teaching. Horizon content knowledge refers to an awareness of how mathematical topics are related over the span of mathematics included in the curriculum (Ball *et al.,* 2008, p.403). From this research, Ball *et al*. developed measures to assess teachers' mathematical knowledge for teaching (MKT). Delaney (2008) has adapted these measures for use in the Irish context and his research findings reveal that Irish primary teachers' levels of MKT vary substantially, with particular strengths and weaknesses (Table 3).

Table 3: Strengths and weaknesses in the MKT of Irish primary teachers (Delaney, 2010)

Strengths	Weaknesses
▪ Identifying and classifying children's mistakes ▪ Matching fraction calculations with representations ▪ Algebra	▪ Attending to explanations and evaluating understanding ▪ Identifying and applying properties of numbers and operations ▪ Matching word problems with fraction calculations

This variation in Irish primary teachers' levels of MKT is important to note since teachers frequently teach in isolation, and so, children are learning in classrooms where their teachers bring very different resources of MKT to their teaching, ultimately impacting children's learning. Teachers also need good MKT to appraise and modify mathematics textbooks (Delaney, 2010). Developing good MKT should enable teachers to provide higher quality mathematics instruction and concomitantly, increase children's achievement. Furthermore, it should enable teachers to find teaching mathematics more professionally fulfilling (Delaney, 2010), including areas they find difficult such as problem-solving.

Problem-solving

It is generally acknowledged that solving problems is vital for mathematical proficiency. Problem-solving generally refers to engagement in mathematical tasks that have the potential to provide intellectual challenges that enhance students' mathematical development (Cai and Lester, 2010). The centrality of problem-solving to mathematical learning is clear from the outset in the PSMC. The following paragraph exemplifies how problem-solving was contextualised within that document:

> *Developing the ability to solve problems is an important factor in the study of mathematics. Problem-solving also provides a context in which concepts and skills can be learned and in which discussion and co-operative working may be practised. Moreover, problem-solving is a major means of developing higher-order thinking skills. These include the ability to analyse mathematical situations; to plan, monitor and evaluate solutions; to apply strategies; and to demonstrate creativity and self-*

reliance in using mathematics. Success helps the child to develop confidence in his/her mathematical ability and encourages curiosity and perseverance. Solving problems based on the environment of the child can highlight the uses of mathematics in a constructive and enjoyable way. (DES, 1999, p.8)

While the import of problem-solving was emphasised in the PSMC, evidence suggests a mismatch between what was intended and the experience of children in many Irish classrooms. An evaluation of curriculum implementation by the DES (2005) revealed *an over-reliance on traditional textbook approaches, which did not promote the development of specific problem-solving skills* (p.29). Additionally, national and international assessments and evaluations (for example, NA, 2009; TIMSS, 2011) highlighted problem-solving as an area in which Irish children continued to underperform. *Literacy and Numeracy for Learning and Life (2011),* while acknowledging that the PSMC provides clear guidance on what children should learn, also highlights weaknesses in the implementation of problem-solving approaches in Irish classrooms. It emphasises the need *to use open-ended challenging tasks that motivate young people to engage with problem-solving in a meaningful way* (2011, p.31), and suggests additional guidance should be provided for teachers on the best approaches to teaching and learning in this area. Similarly, Dooley *et al*. (2014) argue that while problem-solving is afforded a central role in the PSMC, in reality the impression given is that children first have to learn the mathematical procedures before they can apply them to practical situations, rather than problem-solving being the context in which to learn mathematics. Notwithstanding, while the PSMC and *Literacy and Numeracy for Learning and Life* (DES, 2011) highlight the importance of problem-solving for children's mathematical proficiency, neither provide details as to how problem-solving can best be implemented in the classroom context.

Research suggests that problem-solving should not be taught as a separate topic in the mathematics curriculum but rather should be an integral part of mathematics learning (Cai and Lester, 2010). Teachers need to see beyond correct or incorrect answers, and instead look at children's mathematical understanding (Kelly, 2003). Problem-solving requires a long-term approach and commitment at every class level, in every mathematical topic, and in every lesson. Teachers need to allow sufficient time for problem-solving activities, should not over-simplify the problem for their children and need to pose questions that ensure sound classroom discourse (Cai and Lester, 2010). Engaging in problem-solving activities not only helps develop children's higher-order thinking skills but also reinforces positive attitudes to mathematics.

Irish teachers' reliance on textbooks is not conducive to the development of children's problem-solving abilities, since, as Delaney (2012) highlights, many of the problems in Irish mathematics textbooks are of poor quality. He emphasises that there is little evidence to suggest that the use of problem-solving strategies, such as RUDE[8], work and proffers that the best way to learn problem-solving is through practice and the use of problems which children can approach at different levels. A popular method of solving problems is that advocated by Pólya (1945). He enunciated four basic stages in the problem-solving process:

1. Understand and explore the problem
2. Make a plan
3. Carry out the plan
4. Look back and reflect.

Problem-solving is therefore, an iterative, cyclical process. By engaging in problem-solving, children's mathematical understanding deepens. However, learners need opportunities to regularly engage in worthwhile problem-solving activities that are open-ended and connected to real-life contexts. Worthwhile problems provide a level of challenge that is intriguing and invites speculation and hard work. Problems can have multiple solutions; the solutions should not be immediately apparent but it should be possible to solve the problems within a realistic timeframe. Problems should require decision-making beyond mathematical operations and should encourage collaboration in seeking solutions. They should offer learning experiences linked to key concepts as per grade-specific curriculum expectations. Problem-solving skills can be developed in various ways, for example, through constructive play, games, puzzles, role-play, classroom situations, robotics, coding, etc. In response, the new primary mathematics curriculum could provide a repository of mathematics problems to encourage teachers to move away from textbooks and to engage in richer problem-solving activities with

[8] Read, underline, draw and estimate.

the children in their classrooms which involve looking at the real and designed world opening up great opportunities for computational thinking.

Lifelong learning in mathematics

Neale (1969) suggests that a predominant attitude to mathematics is multidimensional and includes *a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics and a belief that mathematics is useful or useless* (p.632). This definition encompasses constructs such as self-confidence, motivation, beliefs and general attitudes towards mathematics. Successive TIMSS studies have shown *a strong positive relationship within countries between children's attitudes towards mathematics and their mathematics achievement. The relationship is bidirectional, with attitudes and achievement mutually influencing each other.* (Mullis *et al.,* 2012, p.19). If children are 'good' at mathematics they are more likely to enjoy doing mathematics. This has implications for children's mathematical learning, and indeed their lifelong learning in general. The *National Literacy and Numeracy Strategy* (DES, 2011) suggests that the curriculum should not only define the knowledge and skills that children are expected to acquire in school, but also the attitudes. It emphasises that *the development of positive attitudes and motivation are vital for progression in literacy and numeracy* (p.43) and recommends the promotion of better attitudes to mathematics among children, young people and the general public. Similarly, both NCCA research reports (Reports 17 and 18, 2014) also emphasise the importance of children's attitudes and disposition to their mathematical learning and development.

In addition to helping children develop positive attitudes towards mathematics, it is also important that they develop the skill of self-regulation. Self-regulated learning (SRL) is a key characteristic of effective learning and an important skill children need to develop in order to meet the demands of 21st century learning, and ultimately lifelong learning. In general, researchers proffer that SRL includes goal-setting, motivation, metacognition (thinking about one's thinking), and the use of cognitive and metacognitive strategies (Andrade, 2013; Vrugt and Oort, 2008; Zimmerman, 2000). A growing body of evidence suggests that SRL is learnable

(Andrade, 2010; Pintrinch, 1995; Zimmerman and Schunk, 2001). Additionally, in the past decade or so, researchers increasingly suggest that SRL can be developed through the use of Assessment for Learning (AfL) practices (Andrade, 2010; Baas, Castelijns, Vermeulen, Martens and Segers, 2014; Black and Wiliam, 2009; Brookhart, 2013; Clark, 2012; Heritage, 2013; Wiliam, 2014). These experts believe that through engagement in effective AfL principles, strategies and techniques, children become more autonomous in their learning and ultimately equipped with a wide range of cognitive and metacognitive strategies, thus enabling them to self-regulate their learning.

Assessment

The centrality of assessment to inform and support good teaching and learning is widely recognised, and a combination of good Assessment for Learning (AfL) and appropriate Assessment of Learning (AoL) practices are recommended (DES, 2011; NCCA, 2007). Research suggests that using AfL on a day-to-day basis is one of the most powerful ways to improve learning in mathematics and increase children achievement (for example, Black and Wiliam, 2003; Wiliam, 2007). In the AfL literature, myriad experts mention the positive effects of using AfL on both students and teachers (Florez and Sammons, 2013; Hodgson and Pyle, 2010), and numerous reviews synthesising thousands of research studies have provided quantitative evidence of the positive impact AfL practices can have on children’s learning and achievement (Black and Wiliam, 1998; Crooks, 1988; Natriello, 1987; Nyquist, 2003).

Table 4: Data from *Visible Learning* (Hattie, 2009) and *Outstanding Formative Assessment: Culture and Practice* (Clarke, 2014, p.4)

Influences on Learning	No. of Studies	Effect Size
Assessment literate students (students who know what they are learning, have success criteria, can self-assess, etc.)	209	1.44
Providing formative evaluation	30	0.90
Lesson Study	402	0.88
Classroom Discussion	42	0.82
Feedback	1310	0.75
Teacher-student relationships	229	0.72
Meta-cognitive strategies	63	0.69

Additionally, major research projects developing AfL practice have found that when teachers truly embrace AfL practices, not only is children's learning enhanced but professional and organisational learning is too (Swaffield, 2011). Furthermore, related data extracted from Hattie's (2009) synthesis of over 900 meta-analyses suggest AfL significantly impacts learning (Table 4). Regarding the role of AfL (or formative assessment) in mathematics, the National Council of Teachers of Mathematics (NCTM, 2013) in the US recently clarified their position stating:

> *Through formative assessment, students develop a clear understanding of learning targets and receive feedback that helps them to improve. In addition, by applying formative strategies such as asking strategic questions, providing students with immediate feedback, and engaging students in self-reflection, teachers receive evidence of students' reasoning and misconceptions to use in adjusting instruction. By receiving formative feedback, students learn how to assess themselves and how to improve their own learning. At the core of formative assessment is an understanding of the influence that assessment has on student motivation and the need for students to actively monitor and engage in their learning. The use of formative assessment has been shown to result in higher achievement. The National Council of Teachers of Mathematics strongly endorses the integration of formative assessment strategies into daily instruction.*

In the Irish context, the importance of regularly using AfL to enhance the teaching and learning in mathematics is also recognised by the NCCA (2007) and the DES (2011), as well as post-graduate researchers of mathematics (for example, McDonnell, 2013). However, detailed advice and support will be needed if teachers are to make effective use of AfL in teaching and learning.

Regarding Assessment of Learning (AoL) (or summative assessment), it was noted earlier that reporting standardised test results in second, fourth and sixth classes to parents and to the DES, is now mandatory. While, teachers recognise the importance and usefulness of standardised tests to aid the diagnosis of mathematical difficulties, they also have reservations regarding an over-emphasis on standardised testing, which to them represents a somewhat narrow view of learning that could negatively impact children learning and achievement (INTO, 2015).

Finally, regarding assessment in general, a criticism levelled at the PSMC was the apparent disconnect between curriculum and assessment. Therefore, it is important that the new mathematics curriculum be aligned with an assessment framework so that they can mutually support and scaffold curriculum understanding and implementation. Indeed, most countries internationally now articulate clear expectations for children's mathematical learning at specific points in their schooling and it is suggested that Ireland should follow suit.

Towards a new Primary Mathematics Curriculum

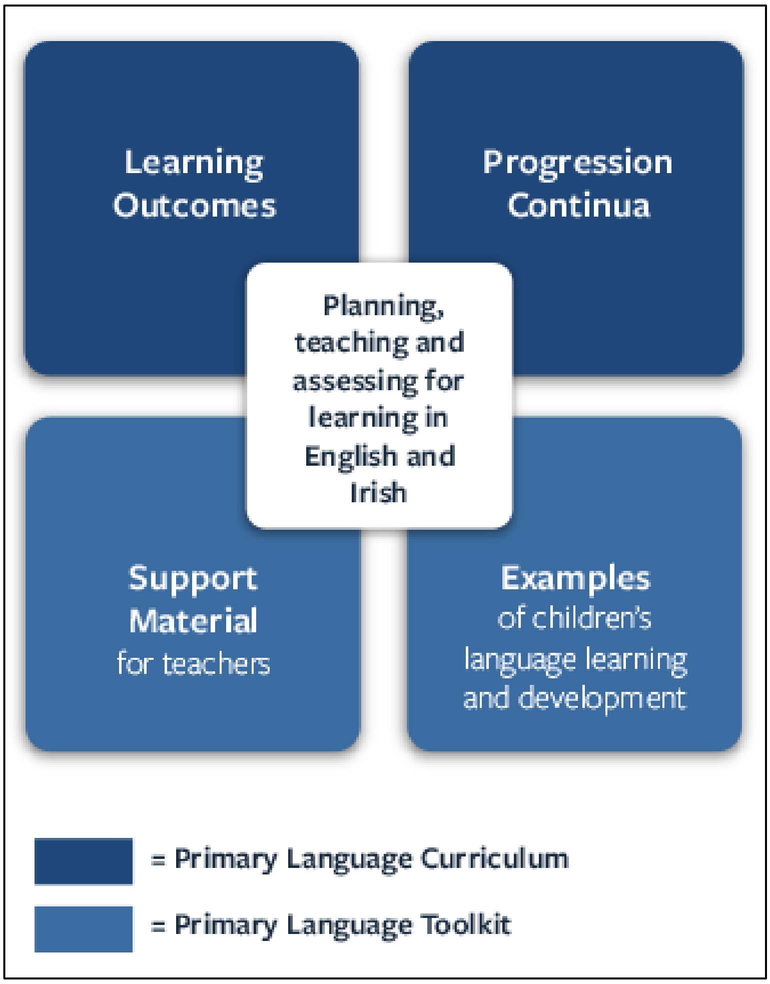
Discrepancies between the intended curriculum and the enacted curriculum are strongly evidenced by reviews of classroom implementation in the Irish context. As the enacted curriculum can be seen as a key mediating variable separating education policies from children's learning achievement (Clune, 1993; Smith and O'Day, 1991), it is critical when presenting the new primary mathematics curriculum that it not only communicates clearly the key aims and objectives of the curriculum but also supports teachers to translate the conceptual perspectives underpinning the curriculum into their own practice.

Irish teachers have expressed strong concerns about curriculum overload, leading to calls for curriculum content to be reduced and the curriculum to be re-presented as a coherent whole (INTO, 2015). Textbooks, large volumes of educational initiatives and the presentation of curriculum have exacerbated teacher's experiences of curriculum overload, among other reasons. Teachers suggested that curriculum overload can be addressed through professional autonomy and integration but acknowledge that *for integration to be successful, teachers need a very good knowledge of curriculum content* and subsequently welcome guidance around this (INTO, 2015, p.41).

Curriculum cohesion

With the dissemination of the new Primary Language Curriculum for junior infants to second class (DES, 2015), mathematics is the second area for curriculum review and redevelopment. In light of this recent development and given the importance of curriculum cohesion, it is useful to see how the Primary Language Curriculum is structured (Figure 4). The Primary Language Curriculum includes four interconnected components—Learning Outcomes, Progression Continua, Support Material and Examples of children's learning and development. Learning Outcomes describe the expected language learning and development for children at the end of a two-year period while the Progression Continua describe, in broad terms, milestones and steps in a child's journey in his/her language learning and development. Support Materials include a range of guides, podcasts and videos to support teachers' use of the Primary Language Curriculum in the school's first and second languages. The Examples of children's learning and development have been developed by teachers and children and show children's language learning and development across the three strands and across a range of school contexts.

Figure 4: The four interconnected components of the Primary Language Curriculum



Building on the work on the new language curriculum, the specification for the PMC will include the following curriculum components:

1. Introduction
2. Rationale
3. Aims
4. Strands
5. Elements
6. Expectations for learners
    a. Learning outcomes
    b. Progression continua
7. Toolkit
    a. Examples of children's learning and development

b. Support Material for teachers.

Organisation of curriculum

In considering the organisation of the new PMC, it might be useful to analyse how curricula in other jurisdictions are organised. The audit commissioned by NCCA (2014) revealed significant commonalities in how 13 countries organised their mathematics curricula. These mathematics curricula invariably included the domains of Number, Measures and Geometry, and Data and Statistics. Additionally, most included Algebra as a stand-alone strand, while some included it with Number. Table 5 compares strands from the PSMC with strands from the NCCA Audit (2014), the Jump Maths Programme[9] (Eivers *at al*. 2014), suggested strands from Report 18 (NCCA, 2014), and those from the post-primary junior cycle syllabus which is the syllabus children will be using once they transfer to second-level.

Table 5: Comparison of strands/content domains

PSMC	JUMP Maths	NCCA Report 18	NCCA Audit (No.=13)[10]	Maths Post-Primary)
1. Number 2. Algebra 3. Shape and Space 4. Measures 5. Data	1. Number Sense 2. Measurement 3. Geometry 4. Patterns and Algebra 5. Probability and Data Management	1. Number 2. Measurement 3. Geometry and Spatial Thinking 4. Algebraic Thinking 5. Data and Chance	1. Number (all) 2. Measures (all) 3. Geometry (all) 4. Data Handling and Statistics (12) 5. Algebra [stand-alone] (9) 6. Processes in Maths (5) 7. Other (2)	1. Number 2. Algebra 3. Functions 4. Geometry and Trigonometry 5. Statistics and Probability

Atweh and Goos (2011, p.223) noted that the categorisation of content into traditional mathematical fields (or strands) *may be convenient in a syllabus but it does not lend itself to dealing with real-world applications that often require cross-disciplinary approaches.* Despite the intentions of new curriculum initiatives in the US (The Common Core State Standards for

[9] . JUMP Math (Junior Undiscovered Math Prodigies) is a project co-funded by the Department of Education and Skills, Accenture, and Science Foundation Ireland. JUMP is a Canadian-designed programme intended to help children succeed at, and enjoy, learning mathematics. Information about its underlying philosophy is available at http://www.jumpmath.org/cms/

[10] Numbers in brackets indicate how many of the 13 countries audited organised and labelled strands as listed.

Mathematics, 2010) and Australia (The Australian Curriculum: Mathematics, 2012), they have been widely viewed as *lost opportunities* (Atweh and Goos, 2011; Atweh, Miller and Thornton, 2012; Hurst, 2014a). This is because curriculum content in these publications are still presented in the same linear fashion as they were in previous curriculum documents. Long lists of mathematical content that are *a mile wide and an inch deep* (Schmidt *et al.*, 2001) do little to give teachers reason to consider that mathematics may be more than unconnected bundles of information and, as a consequence, many teachers continue to teach it in the same unconnected way and inevitably, many children learn it in the same unconnected way. On discussing the inability of adults to transfer what has been learned in one situation to a different situation, Clark (2011) commented that this is because *they have been programmed to think linearly, inductively, and in little boxes* (p.34).

How children learn mathematics

Recent theories of mathematics learning have moved away from seeing learning as acquisition of knowledge towards seeing learning as the understanding of the practice of doing mathematics. This change in perspective implies the need for new learning goals for mathematics education. In supporting children's learning in mathematics, there is a strong case for balancing process and content goals. This contrasts with the design of the PSMC where content and processes are presented separately, and content is emphasised over processes. Clements, Sarama and DiBiase (2004) state that equally as important as mathematical content are general mathematical processes such as problem-solving, reasoning and proof, communication, connections, and representation; specific mathematical processes such as organising information, patterning, and composing; and habits of mind such as curiosity, imagination, inventiveness, persistence, willingness to experiment and sensitivity to patterns (p.3). Research Report 17, commissioned by the NCCA, proposes that processes and content should be clearly articulated as related goals since mathematization can be regarded as both a process and as content. For example, just as children engage in processes such as connecting, they simultaneously construct new and/or deeper understandings of content.

Mathematization goals will need to be broken down for planning, teaching and assessment purposes. This can be done through identifying critical ideas or shifts in mathematical reasoning required for the development of mathematical concepts (for example, Simon, 2006; Sarama and Clements, 2009). Such a framework provides opportunity to present children's learning as a progression towards enhanced mathematical proficiency. The specification of goals is an issue that is closely linked to pedagogy since different practices support different goals (Gresalfi and Lester, 2009) and it is acknowledged that pedagogical support will be needed to help teachers shift their thinking and practice in achieving mathematization goals.

In the classroom, children engage in mathematization by working collaboratively in groups and pairs, working on rich mathematical tasks, investigating and reasoning about problems, exploring ideas and strategies to solve these problems, and sharing and communicating their learning and thinking in a variety of ways. In providing these learning experiences for children, the teacher plays a proactive role in creating zones of proximal development where learning is scaffolded and meaning co-constructed based on awareness and understanding of the child's perspective (Bruner, 1996). Mathematization is thus contingent on a pedagogy of 'math talk', argumentation and discussion designed to support effective conceptual learning (Corcoran, 2012).

Reconceptualising content knowledge for teaching mathematics

New curriculum documents for teaching mathematics that were developed to raise standards in both Australia and the USA—the Australian Curriculum: Mathematics (ACARA, 2012) and The Common Core State Standards for Mathematics (NGA Center, 2010)—have led to much international discussion about teacher content knowledge for teaching mathematics and how mathematics should be taught (Ireland - Delaney, 2010; Australia - Callingham *et al.*, 2011, and Clarke, Clarke and Sullivan, 2012; New Zealand - Anakin and Linsell, 2014; and USA - Thanheiser *et al.*, 2013, and Green, 2014). Rather than being concerned with the amount of mathematical knowledge needed by primary teachers, some researchers (Hill and Ball, 2004, cited in Clarke, Clarke and Sullivan, 2012) suggest it may be more appropriate for policy makers to consider *how* the knowledge is held.

Recently, researchers (Charles, 2005; Clarke, Clarke and Sullivan, 2012; Siemon, Bleckley and Neal, 2012) have suggested that presenting mathematical content from the perspective of the *foundational concepts* of mathematics is key to developing teachers' mathematical content knowledge and their capacity to respond effectively to curriculum documents. Such a focus would enable teachers to make use of the many connections and links within and between such *foundational concepts* and to make them explicit to students.

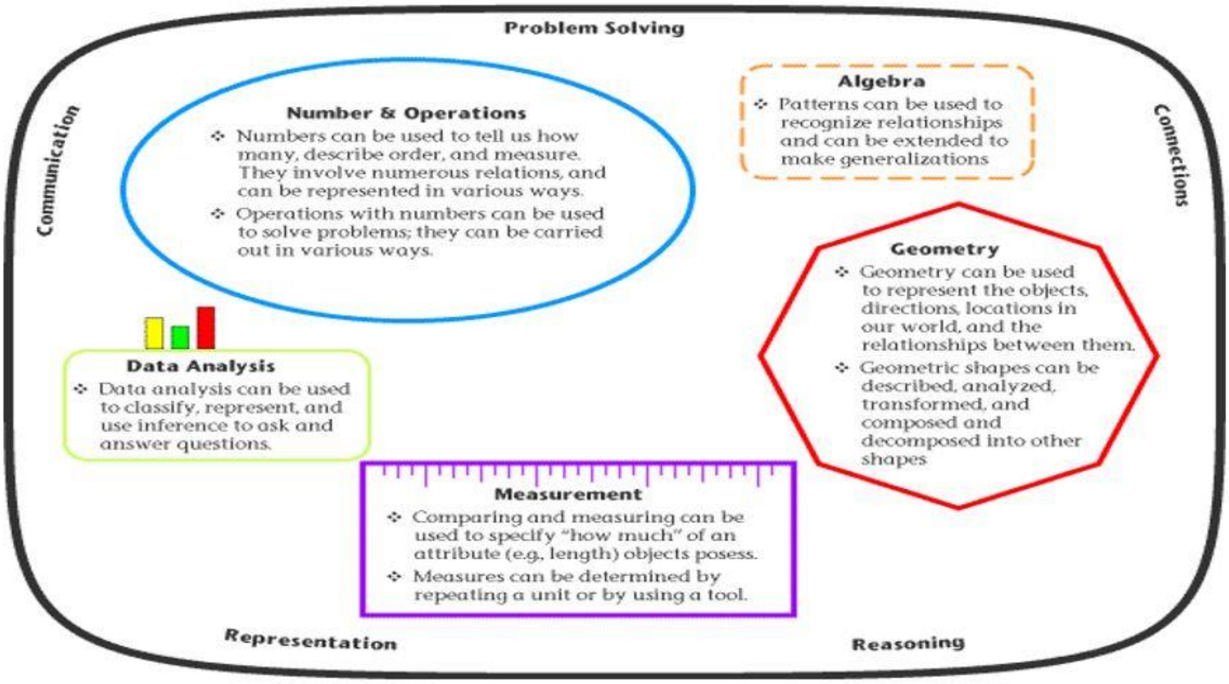
The way one views mathematics is not inconsequential and has been linked to success in mathematics. Boaler (2012) observed that people who make connection within mathematics and see it as a connected subject tend to do well in mathematics, whereas people who see mathematics as a bundle of isolated topics tend not to do so well. Presenting mathematical content and processes in terms of foundational concepts stresses the importance of conceptual understanding as the building blocks to scaffolding 'Big Ideas' in mathematics. Moreover, this presents an opportunity to support teachers to reconceptualise their ideas about mathematics teaching and learning as well as the development of their pedagogical content knowledge.

Making the case for 'Big Ideas'

The National Council of Teachers of Mathematics (NCTM) claim that teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise (NCTM, 2000, p.17). In research studies, where teaching and learning in maths was found to be most successful, teachers' mathematical content knowledge and teaching practices were anchored around a set of 'Big Ideas' in mathematics which enabled students to develop a deeper understanding of mathematics (Ma, 1999; Stigler, 2004; Weiss, Heck and Shimkus, 2004; Charles, 2005).

The notion of 'Big Ideas' of mathematics has been afforded prominence within the literature in recent time (Clements, Sarama, and DiBiase, 2004; Charles, 2005; Clarke, Clarke and Sullivan, 2012; Siemon, Bleckley and Neal, 2012) though it is still considered an elusive term. Clements and Sarama (2009) equate learning goals as the big ideas of mathematics. These big ideas are clusters of concepts and skills that are mathematically central and coherent, consistent with children's thinking and generative of future learning. For example, one 'Big Idea' is that counting can be used to find out how many there are in a collection, another would be, geometric shapes can be described, analysed, transformed and composed and decomposed into other shapes. 'Big Ideas' are the foundations of children's learning compounded by the notion that the degree of understanding is determined by the number and strength of the connections (Hiebert and Carpenter, 1992, p.67) and furthermore that we understand something if we see how it is related or connected to other things we know (Hiebert *et al.*, 1997, p.4). Charles (2005) contends that 'Big Ideas' are important because they enable us to see mathematics as a coherent set of ideas that encourage a deep understanding of mathematics, enhance transfer, promote memory and reduce the amount to be remembered (p.10). When one understands 'Big Ideas', mathematics is no longer a set of unconnected bundles of content and skills (see Table 6). Put simply, 'Big Ideas' help children to make connections with their learning in mathematics and effective teaching helps to make these connections explicit (Charles, 2005).

Table 6. Example of 'Big Ideas' underpinning different mathematical strands (Clements, Samara and Di Biase, 2004)



Reported benefits of adopting a 'Big Ideas' approach include:

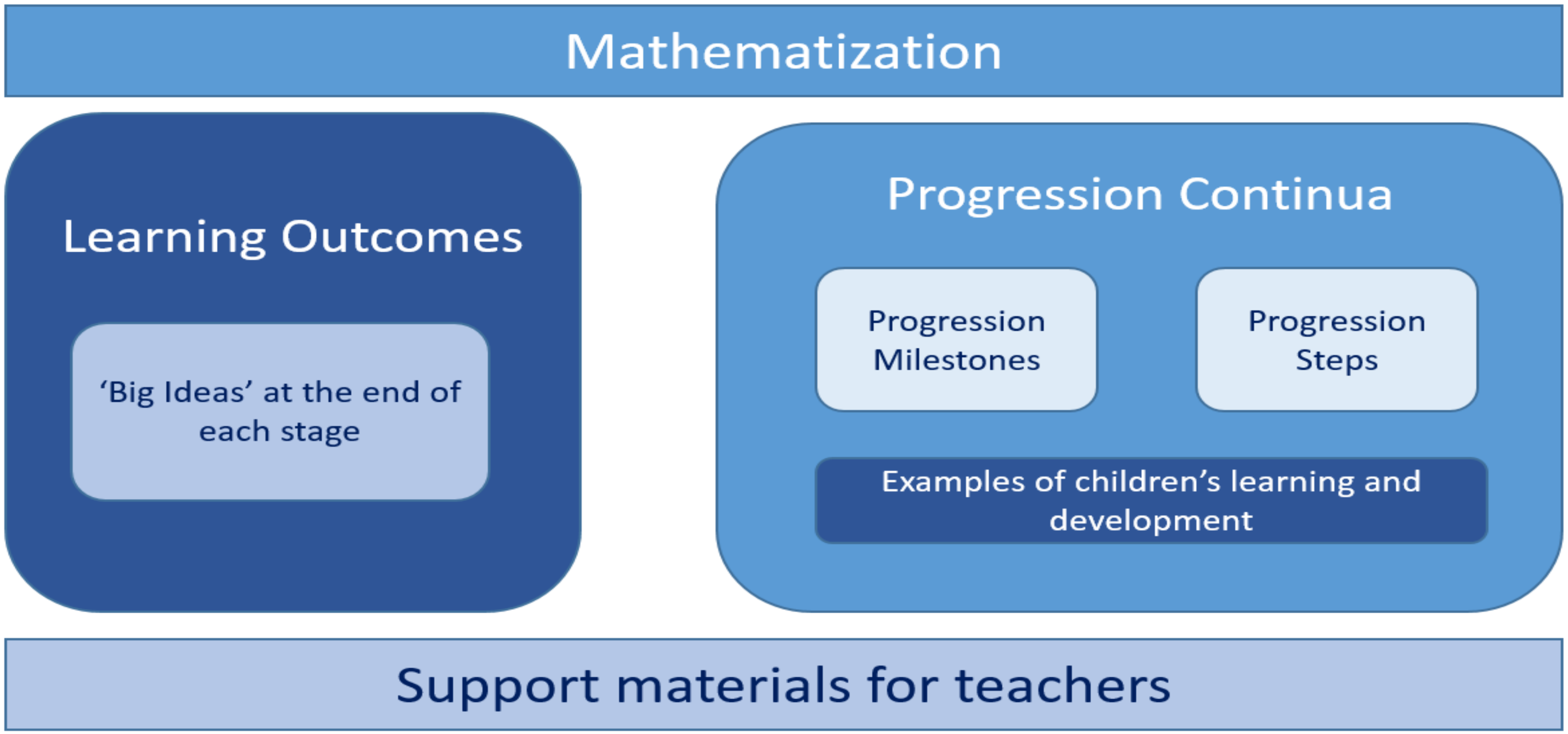
- Promotes understanding (Charles, 2005; Reys, 2008)

- Promotes memory, motivation, transfer and the development of autonomous learners (Lamdin, 2003)
- Thins an overcrowded curriculum (National Curriculum Board, 2009; National Mathematics Advisory Panel, 2008)
- Increases the number and strength of the connections that are made to other ideas and strategies (Charles, 2005)
- Supports further learning and problem-solving (Siemon, 2007; AAMT, 2009)
- Maximises progress for all by targeting teaching to key ideas and strategies (Siemon *et al.*, 2006)
- Provides curriculum coherence and articulates the important mathematical ideas that should be the focus of curriculum (Charles, 2005).

There is not necessarily any one particular way in which content ideas can be linked around 'Big Ideas' or even how these links might be presented. Hence, how 'Big Ideas' thinking can be incorporated into the curriculum will need deliberation. Notwithstanding, the literature suggests that a focus on 'Big Ideas' with their myriad links and connections would greatly enhance pedagogies for delivering mathematics curricula (Hurst, 2014). Such deep and connected knowledge would be likely to lead to more effective concept-based teaching rather than a reliance on teaching procedures. 'Big Ideas' give a new perspective to curriculum development that shows strong potential for supporting teachers in negotiating curriculum intentions, promoting a more connected view of mathematics as well as offering promise in 'thinning out' an overcrowded curriculum (Siemon, Bleckley and Neal, 2012).

The following model outlines a conceptual framework for development of the new primary mathematics curriculum aligned with NCCA curriculum specifications. This model illustrates how the relationships between the different curriculum components may be conceptualised in a new curriculum specification for primary mathematics. The model is an adaptation of the emerging curriculum model offered in Research Report No.18, (Dooley *et al.*, 2014).

Figure 1: A developing curriculum model



Brief for the development of a new Primary Mathematics Curriculum

The background paper, as evidenced by research, teacher voices and new perspectives in mathematics both nationally and internationally, has signposted the need to reconceptualise approaches to teaching, learning and assessment of mathematics for primary school children. Moreover, the background paper offers perspectives on presenting the primary mathematics curriculum in a new way that emphasises depth of learning, understanding and application of mathematical concepts and supports children to develop positive dispositions to mathematics. The following brief reflects key implications for the development of the new primary mathematics curriculum arising from the background paper as well as from the NCCA Research Reports 17 and 18 (2014).

Guiding principles

The following guiding principles offer direction and focus for the development of the new primary mathematics curriculum. Curriculum developments should aim to:

1. Reconceptualise a new curriculum to reflect new aims, learning goals and emphases

A fresh and coherent vision (blueprint) for children's learning in mathematics is necessary to guide the development of the new primary mathematics curriculum. The curriculum should be coherent in terms of aims, goals relating to both processes and content, and pedagogy. Mathematical proficiency as defined in the US context—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition (National Research Council [NRC], 2001)—provides a good starting point for the development of aims for the new PMC in the Irish context. Notwithstanding, the aims of the new primary mathematics curriculum will need to be re-contextualised and redefined for the Irish context and recognise the development of mathematical proficiency as building on pre-school and home experiences of learning mathematics as promoted within *Aistear: the Early Childhood Curriculum Framework* (2009).

The structure and presentation of the new primary mathematics curriculum will require careful deliberation and planning so as to amplify new emphases. Key emphases in the new primary mathematics curriculum will include conceptual development, mathematization, problem-solving, application of knowledge, teaching 'Big Ideas' and fostering positive dispositions to mathematics. Big ideas are a departure from a view of mathematics as a set of disconnected concepts, skills, facts and procedures and rather serve to foster integration and facilitate children to make connections within their learning in mathematics as well as other contexts. The curriculum will promote authentic application of mathematical content, ideas and skills within appropriate and relevant contexts, such as real-life situations and children's play.

Children will engage with foundational concepts in mathematics organised according to the five content domains – Number, Measurement, Geometry and Spatial Thinking, Algebraic Thinking, and Data and Chance. Early Mathematical Activities will be integrated into these five content areas. Mathematical processes such as communicating, reasoning, argumentation, justifying, generalising, representing, problem-solving, and connecting, will be foregrounded in curriculum documentation through the articulation of related mathematization goals (critical ideas). Critical ideas will indicate shifts or milestones in children's mathematical development in each foundational concept across stages, for example, two years. Critical ideas will function to support teachers to help make children's learning visible and present children's learning as a progression towards 'Big Ideas'. Narrative descriptors of mathematical content and processes will indicate progression steps in children's understanding and application of mathematical (foundational) concepts. These learning paths and narrative descriptors will be broadly specified and will outline the journey towards achieving learning outcomes. Moreover, they will serve as reference points for teachers in their planning, teaching and assessment.

2. Support children to meet the demands of 21st century learning and life

The new primary mathematics curriculum will recognise the role of early mathematical learning as a vital life skill and a foundation for citizenship in the 21st century. It will nurture the fundamental skills of conceptual development, critical reasoning, analytical thinking and problem-solving. Moreover, it will lay the foundations for children to acquire the basic language structures and foundational concepts in mathematics to enable them to interact, understanding and conceptualise the world around them. The new primary mathematics curriculum will aim to support young children to acquire a set of skills and competencies in order to meet the demands of 21st century learning and life, to create new knowledge and to navigate their way through change, uncertainty and opportunity.

3. Ensure continuity and progression across sectors

Work on the new mathematics curriculum will take cognisance of developments at both early childhood (*Aistear*) as well as at junior cycle in order to ensure continuity and progression across sectors. The development of the mathematics curriculum for junior infants to second class, in particular, will need to build on and align with the pedagogical emphases in *Aistear*.

A common language for communicating curriculum goals and principles will need to be established to facilitate cross-sectoral communication and transitions so that parents and educators across early childhood settings can communicate about children's mathematical experiences and the features of pedagogy that support children's learning.

4. Support and build understanding and application of 'Big Ideas' in mathematics

Learning outcomes will describe the expected learning and development for children at the end of a stage in terms of critical ideas in mathematics. Critical Ideas will indicate shifts in children's mathematical development towards understanding and applying the 'Big Ideas' in mathematics. The progression continua will describe, in broad terms, children's mathematical learning and thinking towards these 'Big Ideas'.

Key starting points for the development of an outcome-focused curriculum might be:

- Defining ‘Big Ideas’ – Drawing on research, what are the ‘Big Ideas’ we want children to understand and how should we present this within the context of the curriculum?
- Identifying desired results according to stages of learning – For each ‘Big Idea’, what are the critical ideas we want children to understand and use by the end of each stage of learning?
- Planning learning experiences – What foundational concepts and learning activities will facilitate understanding of the ‘Big Ideas’?
- Determining assessment evidence – How will we know children have understood the ‘Big Ideas’?

Similarly, planning and assessment approaches will be aligned with learning outcomes and progression milestones. Learning outcomes or critical ideas will serve as starting points for planning, teaching and assessing children’s mathematical learning. Progression milestones and steps will further scaffold the planning, teaching and assessment processes. For the purpose of supporting progression in children’s mathematical learning and development, support materials will be provided to offer multiple, diverse and appropriate opportunities for children to demonstrate learning and achievement.

5. Promote the principles of inclusion, equity and access

The curriculum will be developed in line with the principles of universal design for learning and as such, promote the principles of equity and access for children with a diverse range of abilities. For children with special educational needs and in particular, those with severe and profound and low moderate needs, the curriculum will outline what is appropriate and relevant for them to know and provide differentiated support so they can access this learning. The curriculum will support children who attend Irish- and English-medium schools, and acknowledge and support children from different language backgrounds where neither English nor Irish is their first language. It will be considerate of the wide range of diverse backgrounds that children come from and their differing starting points as they enter primary school, including children from socio-economically disadvantaged backgrounds.

Furthermore, the curriculum will support teachers to recognise children's development in mathematical conceptual understanding and application, and decide how this can be extended further through mathematical experiences.

6. Outline changes in pedagogy and curriculum supports

While foregrounding mathematical proficiency as the aim of the mathematics curriculum has the potential to change the kind of learning that children experience in primary schools, it also demands significant changes in pedagogy and necessitates curriculum supports to scaffold this change. The curriculum should inform teachers about goals, learning paths and critical ideas in developing understanding around the 'Big Ideas' of mathematics. Accordingly, teachers should be encouraged and enabled to develop content knowledge and pedagogical knowledge for teaching primary mathematics.

Given the complexities involved, teachers will require appropriate support material to develop the knowledge, skills, and dispositions required to teach mathematics well. Support material will draw on research and practice to provide teachers with practical support in using a range of pedagogies evidenced in research as being effective in mathematical teaching and learning.

Support material might include:

- Lesson Study or research lessons focused on connecting practice and 'Big Ideas' to allow teachers to interrogate and negotiate the new primary mathematics curriculum with colleagues as it relates to their setting and content.
- Video tutorials, for example, on initiating the planning process.
- A bank of rich tasks devised by teachers and linked to learning outcomes and the development of foundational concepts.

7. Address the need for appropriate resources to support teaching, learning and assessment including the promotion of digital learning and technology

In promotion of the centrality of mathematical proficiency and 'Big Ideas' within the new primary curriculum, particular consideration will need to be taken in addressing the issue of over-reliance on traditional textbooks. Curriculum developments will need to address the need for appropriate resources to support the teaching, learning and assessment of the curriculum. Collaborative and active learning in a mathematics-rich environment, along with the use of concrete learning resources and digital technology for all classes, will be embedded in curriculum material.

Curriculum supports will exemplify how tools, including digital tools, can scaffold and enhance learning and assessment. Support material developed in line with the curriculum should also aim to deepen children's mathematical understanding, provide a level of challenge, be open-ended and connected to real-life contexts.

8. Consider the role of external factors, wide-ranging and systemic challenges impacting curriculum implementation

External factors outside the curriculum development space but nonetheless significantly impacting on effective curriculum implementation, will need to be considered in the development of the new primary mathematics curriculum. As noted in the background paper, amongst these factors are

- Standardised testing
- Textbooks
- Curriculum dissemination and professional development
- Messaging across the system, including open communication and dialogue with parents and the wider community focusing on the importance of mathematics learning in the early years, the goals of the mathematics curriculum and ways in which children can be supported to achieve these goals.

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