NCCA

Designing activities, tasks and problems for Mathematical Modeling

The design principles listed below can be used by teachers to develop or adapt tasks for mathematical modeling purposes.

Design Principles for Model-Eliciting Activities
Adapted from Lesh and Doerr (2003)

Principle	Description
Reality	The context should be realistic or imaginable for children. They should be able to make sense of the situation based on extensions of their current knowledge.
	A problem can be fictional or drawn from real-life but must be imaginable for children.
Model construction	The activity should involve constructing, describing or predicting a structurally significant system, i.e., the underlying mathematics must be important and relevant for children's learning at the relevant class level
	Think about how the Cookie Monster task highlighted mathematical structure. Where problems are drawn from real-life contexts, teachers should ensure the mathematics addressed is significant.
Self-assessment	Criteria embedded in the activity should allow children to self-evaluate and refine their model where necessary.
	Checking if the solution makes sense in the real world allows for self-assessment.
Model documentation	The task should set children up to produce documentation of their thinking
	Younger children may explain their thinking orally and record through mark-making or drawing.
Simple prototype	The problem should be as simple as possible while still creating a need for a significant model.
	Significant models with complex mathematical ideas can be generated from simple problems- see integrated STEM examples above.
Generalisability	Children should be challenged to produce models that are reusable, shareable and modifiable.
	Children should be encouraged to identify generalisable ideas from modeling activities and, where appropriate, have opportunities to trial these in new problems.

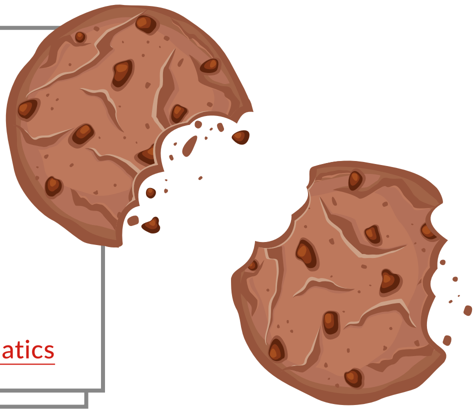
Modeling Mathematical Operations

Children should be given opportunities to develop their understanding of mathematical concepts and operations through mathematical modeling activities. A teacher wishing to develop understanding of addition and subtraction might use the sample task shown below. There are a number of different ways that children may approach this task. Children may draw pictures, use base ten blocks or place value counters, use hundred squares or empty number lines. They may also record their work as number sentences. The openness of this approach allows for multiple representations and solution methods, where some children may 'count up' from 17 to 30 (effectively answering 17 + __ = 30) and others may ‘count back’ or subtract (solving 30 - 17 = __ or 30 - __ = 17). Classroom discussion should aim to help children make conceptual links between addition and subtraction and associated counting strategies, and also evaluate the effectiveness of different representations, discussing for example how drawing individual biscuits and crossing them out may be effective for smaller numbers, but not so helpful when bigger numbers are involved. Children who have used less efficient methods should have opportunities to trial new approaches and refine their ideas. This may occur within individual lessons or across a series of lessons.

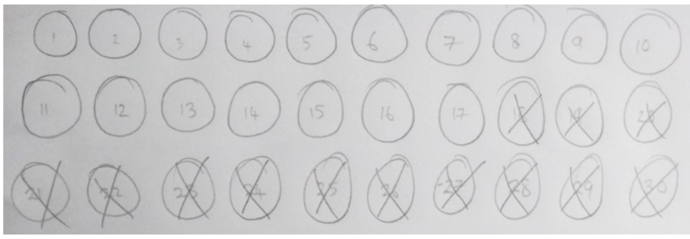
Sample Task

There are 30 biscuits in a packet. The Cookie Monster ate some and now there are only 17 left. How many biscuits did he eat?

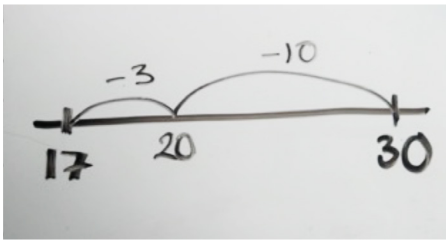
Note: *This task is an adaptation of the Cookie Monster 3-Act task available at* https://gfletchy.com/.
A video of the task being enacted in a US classroom is available on https://www.teachingchannel.com/blog/modeling-with-mathematics



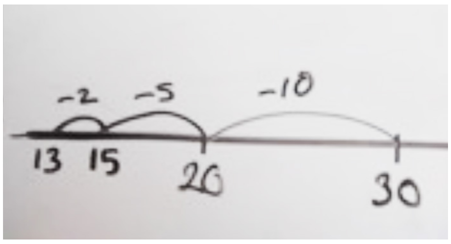
Sample Solutions – Counting back from 30



Drawing cookies and counting back from 30 to 17

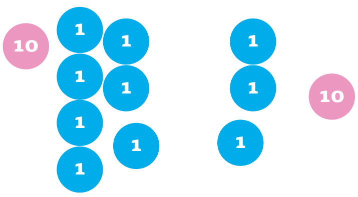


Counting back from 30 to 17 on the empty number line



Counting back 17 from 30

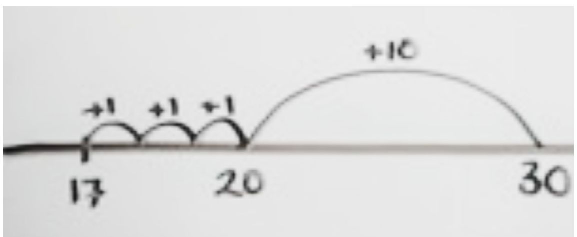
Sample Solution – Counting on



Using Place Value Counters to count up from 17

Síle: There’s 17 left so I made my counters look like 17 cookies. Then I counted 18, 19, 20 and then I knew I just needed 10 more to make 30.

Teacher: OK, I’m going to draw out what you said so that everyone can see. You started at 17, then you counted, 18, 19, 20, and then you counted 10 more to make 30. Is that right?...Does anybody want to ask Síle a question? ... So how many did the cookie monster eat?



Teacher’s use of an empty number line to reflect the child’s strategies

While this approach to teaching shares many similarities with a problem-solving approach, developing generalisable or transferable knowledge is also a key part of mathematical modeling. For this reason, discussions should aim to help children identify general ideas that can be used again in similar problems, e.g., bridging to the nearest tens number, or choosing counting on or back methods to suit the particular numbers given. It is recommended that teachers' use of models will serve to reflect children's thinking rather than direct it - teachers may use models to clarify and make children's ideas explicit but should not insist on children using a particular model or representation.

Teachers may use models to clarify and make children's ideas explicit but should not insist on children using a particular model or representation. Instead they should support children in making connections between models.

Children should have opportunities to develop their understanding of multiplication and division in similar ways, through engaging in problems that they can make sense of and relate to their current understanding. The appendices of the Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) Report contains examples of early division modeling tasks. See also the Interpreting Multiplication and Division lesson on the Mathematics Assessment Project website for ideas on how to support children's modeling of multiplication and division of fractions.

Modeling Complex Problem Situations

In the Cookie Monster task, the teacher's main goal is to develop children's understanding of addition and subtraction and some of the messiness of applying mathematical ideas to real life has been stripped away from the problem. Mathematical modeling also lends itself to linkage and integration and holds great potential for underpinning STEM and other integrated activities. In such cases, it is often preferable to consider how to support children in grappling with the messiness of real-life issues rather than removing the complexity completely. Warm-up activities, such as reading books or relevant newspaper articles, should be used to familiarise children with novel problem-contexts.

Authentic real-world problems provide rich opprtunities for mathematical modeling. In such cases, it is necessary to spend time formulating the problem that will be answered and identifying relevant assumptions and variables. See the modeling process, page X.

Sample Task 1

This fruit salad looks like a healthy snack to eat. Is it also healthy for our planet? Carry out some research and design a snack that is healthy for you and for the planet. Justify your decisions.

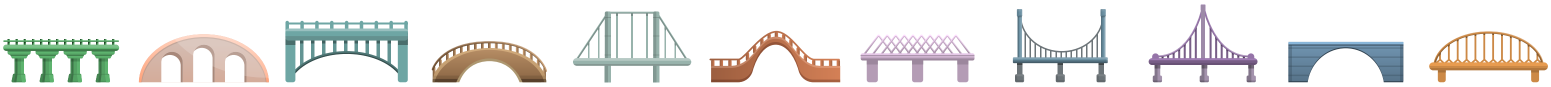
Note: *This task has been adapted from the Healthy People, Healthy Planet activity from the Rowett Institute. The level that children engage with the task will depend on their mathematical and other knowledge. For example, in deciding what a healthy snack is, some children will use general rules such as minimizing sugar or fat. Others will be able to analyse nutritional details in a more mathematical way and make recommendations based on, for example, the proportion of energy, protein or vitamins per serving. In deciding whether foods are 'healthy' for the planet, children may focus on some of the following: whether the packaging used is recyclable, how far and by what means the food has been transported, or whether it is organic or not. To ensure that children bring mathematical reasoning to this task, they should be encouraged, at an age-appropriate level, to sort, quantify and rank relevant data wherever possible, for example, sorting into sets according to whether the packaging is recyclable or not, or ranking fruits by the distance travelled or by how environmentally-friendly the mode of transportation used was. This data should then be used to justify their decisions and recommendations. Follow-up activities could explore mathematically global-issues related to food production and transportation (see https://www.trocaire.org/documents/creating-futures-resources/ for ideas).*

Many STEM activities involve children measuring or collecting data of some kind. To encourage deep engagement with any data collected, modeling problems focusing on subjective words, such as best, efficient, or robust, should be used (Bliss et al., 2014). During the modeling activity, children then have to decide how to interpret these words but must also attend to mathematical elements and consider how they can create a systematic ranking that includes quantifiable or measurable elements. There are no perfect answers to the sample activities shown here. Instead, the intention is to provide opportunities for the children to use mathematics to explore and describe each situation. Having opportunities to consider other people's models and to test and refine the models they develop is crucial. Children should also have opportunities to use and refine the mathematical models developed in the context of other problems- for example, while the context of deciding the best kitchen towel and best bridge are quite different, similar mathematical ideas could be used to approach both tasks.

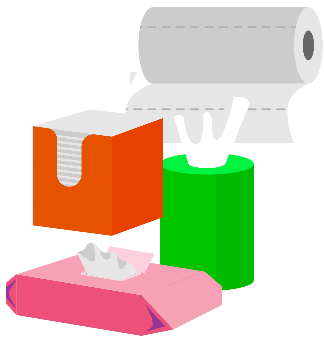
Sample Activity 2a

Our class has made some fantastic bridges this week. I would like to give a prize to the best bridge. What would be a fair way to decide on the prize? Is there some way that I could assign points to the bridges? What would I assign the points for?

This is an extension of the Discover Primary Science 'Design a Bridge' challenge. This challenge encourages children to use a 'fair test' to investigate the strength of bridges they construct by putting coins or other weights on the bridge. While deciding on the 'best' bridge children will likely consider how much weight the bridge can hold but should also be encouraged to consider other features such as the length of the bridges, and the amount of materials used in construction.



Sample Activity 2b



The principal wants to order new kitchen towels for teachers and children to use when cleaning up spills in the classroom. She has a choice between three different options.

Carry out some research and then write a letter detailing which one you think is best and why.

This is adopted from the Best Kitchen Paper Towel Investigation (Murphy, Broderick & Kenny, 2015) which includes directions for how a 'fair test' investigation of absorbency might be carried out. Investigating the 'best' kitchen towel means that children have to combine their findings about absorbency with consideration of other features or variables that they consider important, for example, number of sheets per towel, price, whether the towels are made of recycled/renewable materials etc.

For each of these activities, children will have to identify the features or variables that they think are most important and find ways to quantify and combine this data. This may involve creating and combining rankings, e.g., most to least absorbent and least to most expensive, or weighting data, e.g., deciding that the amount a bridge holds is the most important feature so marking this out of 100, while the length of the bridge is less important so marking this out of 50.

While the tasks and examples given in this section may appear more suitable for senior primary classes, complex real-life tasks can be tackled across the primary age-range at different levels of mathematical complexity. The GAIMME report suggests that the same problem context, such as what to bring for lunch or how to plan for the class party, can and should be revisited at different class levels with increasing expectaions of mathematical thinking. For example, the context of how toys or sports equipment might be used at breaktime might involve direct counting or 'matching' between children and resources initially, before taking into account paired and group games. In senior classes, the same context can be explored using more sophisticated mathematical approaches and children may survey and measure the yard and make recommendations based on the space available for different class levels/and or play activities.

Three-Act Tasks

Three-act tasks which were originally developed to support problem solving, have been also used as a means to teach mathematical modeling. The three-act structure lends itself to developing mathematical questions, constructing and refining solutions. The first act involves using a video or photo to elicit children's own mathematical questions or problem-posing about real world scenarios. The second act involves identifying information needed to answer the relevant mathematical question. The third act involves reflecting on and revising the mathematical models that have been developed. It is recommended that three-act tasks should be used to support planned learning of target mathematical content rather than as standalone problem-solving activities.

I wonder...

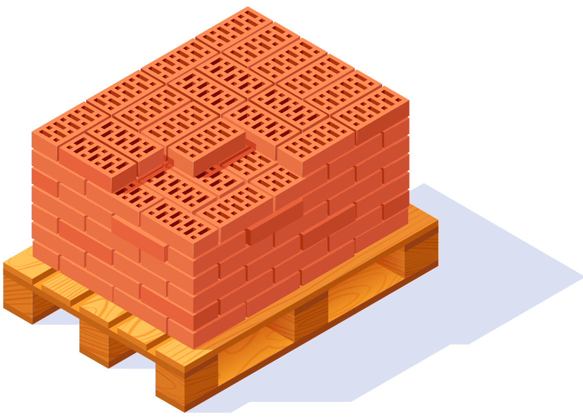
How many bricks would be there if the pallet was full?
How many are missing?

How big is that pallet of bricks?
Would it fit under the table/through the door?

How heavy is that pallet?

How high/long could I build a wall with those bricks?

How many pallets would I need to rebuild my room/house/school?

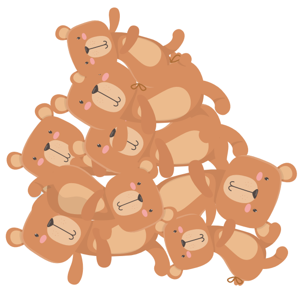


Note: *These questions may be generated by the children in the first act. The teacher may decide to ask the class to focus on one particular question or alternatively allow students to pursue their own questions.*

This task might be selected as part of a unit of work on proportional reasoning as it allows children to relate one or more attributes of a brick to the attributes of the pallet, wall or other structure. It may also be suitable in a unit of work on volume.

Fermi Problems

Fermi problems have also been proposed as a possible vehicle for mathematical modeling. These are non-standard problems focused on estimation. For example, children might be asked to estimate how many 10-year-olds live in Co. Cork. Formulating a solution will likely involve simplification, estimations and proportional reasoning based on known or researched facts. Problems involving, for example, estimation of water use or mass of waste produced by school communities have been used to raise awareness of environmental and sustainability issues. Fermi problems are also suitable for younger children. For example, how many teddies could we fit on our table, or how many blocks would you need to stack to reach your height? Children can be encouraged to provide estimates and should have opportunities to refine these estimates as more information is gathered. For example, the child may refine their estimate of how many teddies fit on the table when they see it half full.



Resources

Ideas from Suh and Seshaiyer (2017), including the '*Modeling Mathematics Ideas Toolkit*', available from http://modelmath.onmason.com/. The related website, http://sparkstem.onmason.com, contains ideas and samples of children's work on integrated STEM activities.

Information on and examples of three-act tasks see, https://blog.mrmeyer.com/category/3acts/ or https://gfletchy.com/

Integrated STEM activities focused on global citizenship issues with potential for mathematical modeling are available at https://practicalaction.org/stem/

www.Maths4All.ie

Model-eliciting activities for upper primary and secondary students available at https://unlvcoe.org/meas

References

Bliss, K., Fowler, K., & Galluzo, B. (2014). *Math Modeling: Getting Started & Getting Solutions*. Philadelphia, PA: SIAM. Retrieved from https://m3challenge.siam.org/sites/default/files/uploads/siam-guidebook-final-press.pdf

Lesh, R. & Doerr, H. (2003). Foundations of models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H.M. Doerr (Eds.), *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching* (pp. 3–33). Mahwah, NJ: Lawrence Erlbaum.

Suh, J. & Seshaiyer, P. (2017). *Modeling Mathematical Ideas: Developing Strategic Competence in Elementary and Middle Schoo*l. London: Rowman & Littlefield Publishers.