Draft Background Paper and Brief for the Review of Applied Mathematics

For consultation

NCCA

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1. Introduction

Applied Mathematics is one of 33 subjects examined annually in the Leaving Certificate examination. Written over 40 years ago, the LC Applied Mathematics syllabus (Appendix 1) mirrors a section of the LC Physics syllabus known as Mechanics. With its emphasis on content and in the absence of any aim or rationale, it is difficult to ascertain what group of students' needs the syllabus aims to meet. Over the years, possibly because of the content overlap between it and LC Physics, the subject has become a popular option for male students taking Higher level Mathematics and Physics. Although Leaving Certificate subjects are prepared for a 180 hour course of study, a practice has emerged with Applied Mathematics where students rarely study the course for 180 hours. Most students' study the subject either in school 'off timetable' or with a private teacher outside school; their experience of learning in Applied Mathematics is often reduced to exam preparation.

> *The only way to get good at Applied Maths is to practice as often as possible and as many different questions as you can get your hands on. To help you with this, the school has a stock of Applied Maths papers going all the way back to the 1970s*
>
> School website

With the examination regarded as predictable,

> *Almost every question which appeared in recent years was similar to at least one other question on an older paper – the natural conclusion being that if you cover all the old papers along with the recent ones you really should see very little new material in the leaving cert exam.*
>
> The Physics Teacher website

> *Based on my experience (LC 2008), the papers followed very predictable patterns from year to year over a very long period of time (most past questions are variations on a small number of problems), so diligent practice of exam questions should prepare students for most of what the paper will ask them to do. No substitute for practice in Applied Maths or Maths in my own experience. I don't think Applied Maths helped with my Maths work in a direct way but definitely bolstered my Physics - made the maths questions in Physics a walk in the park by comparison!*
>
> Engineering student, NCCA third level survey

Applied Mathematics is seen by many taking Higher level Mathematics and Higher level Physics as an efficient way to secure additional CAO points.

Chosen by less than 3% of the Leaving Certificate cohort annually, less than 500 of whom are girls, approximately 90% of Applied Mathematics candidates sit the examination at Higher level, and in excess of 25% of these students achieve an A grade.

Year	Total Number of candidates taking Applied Mathematics	Number of Higher level candidates	Percentage of total Applied Mathematics cohort	Percentage of total Leaving Certificate cohort
2008	1395	1288	92.3	2.44
2009	1446	1333	92.2	2.46
2010	1329	1213	91.3	2.22
2011	1427	1274	89.3	2.34
2012	1490	1344	90.2	2.56

Table 1: Extract from the SEC Chief Examiners' Report 2012[1] showing the number of Leaving Certificate candidates taking Applied Mathematics (Higher level) 2008 -2012

Year	Grade	A1	A2	B1	B2	B3	C1	C2	C3	D1	D2	D3	E	F	NG	Total
2009	**Number**	**182**	**134**	**136**	**150**	**120**	**125**	**105**	**89**	**95**	**62**	**55**	**55**	**18**	**7**	**1333**
	%	**13.7**	**10.1**	**10.2**	**11.3**	**9.0**	**9.4**	**7.9**	**6.7**	**7.1**	**4.7**	**4.1**	**4.1**	**1.4**	**0.5**	
2010	**Number**	**256**	**103**	**111**	**108**	**87**	**96**	**63**	**94**	**48**	**57**	**95**	**60**	**29**	**8**	**1213**
	%	**21.4**	**8.3**	**9.2**	**8.8**	**7.2**	**7.8**	**5.2**	**7.7**	**4.0**	**4.7**	**7.8**	**4.9**	**2.4**	**0.7**	
2011	**Number**	**190**	**154**	**118**	**144**	**118**	**116**	**79**	**79**	**87**	**46**	**65**	**57**	**18**	**3**	**1274**
	%	**15.2**	**11.8**	**9.3**	**11.4**	**9.3**	**9.0**	**6.2**	**6.2**	**6.8**	**3.6**	**5.1**	**4.5**	**1.4**	**0.2**	
2012	**Number**	**248**	**126**	**127**	**143**	**125**	**80**	**92**	**104**	**61**	**60**	**68**	**69**	**30**	**11**	**1344**
	%	**18.5**	**9.4**	**9.4**	**10.6**	**9.3**	**6.0**	**6.8**	**7.7**	**4.5**	**4.5**	**5.1**	**5.1**	**2.2**	**0.8**	

Table 2: Extract from the SEC Chief Examiners' Report 2012: Summary of outcomes for Leaving Certificate Applied Mathematics (Higher level) 2009 -2012

[1] Chief Examiners' Reports provide a review of the performance of candidates in the examinations and detailed analysis of the standards of answering. The Applied Mathematics report is published every 5 years.

As noted on careersportal.ie aside from niche languages such as Latin, Russian, and Japanese, this means that Applied Maths has the **highest A percentage** in the Leaving Cert. This record leads to the perception that only the top students in the country choose Applied Mathematics and The Irish Applied Mathematics Teachers Association (IAMTA) note that This perception will stand favourably to students when attending interviews for high-profile jobs in the near future. http://www.iamta.ie/docs/brochure.pdf

Leaving Certificate Applied Mathematics is also promoted as a subject that enhances students' problem solving skills. However, with its emphasis on content as opposed to the development of skills and mathematical reasoning students' are not problem solving per se but rather, learning to solve particular problem types in mathematical physics.

> *I knew the procedure for the questions so knew that I could make a decent attempt even if I didn't get the right answer.*
>
> Engineering student, NCCA third level survey

> *Notes were handed to us in Applied Maths of the key points and pieces of information needed in each chapter. Each chapter was approached in the exact same manner - notes, examples, practice, homework, correct. With maths there was a little more freedom for discussion.*
>
> Science student, NCCA 3rd level student survey

Although LC Applied Mathematics has not been reviewed since it was developed over 40 years ago, mathematics education in Ireland has been the subject of a major review over the last 6 years. The *Project Maths* initiative, which began in 24 schools in September 2008 and was rolled out in all schools nationally two years later, has changed the mathematical landscape in Ireland. With its emphasis on skills development, in particular reasoning and problem solving, this initiative has started to change how students engage with mathematics in the classroom. In addition, broader senior cycle review is also ongoing so it is timely, now that the developmental stage of the *Project Maths* initiative has entered its final phase in all schools, that LC Applied Mathematics is reviewed and aligned with these developments.

- Under the remit of the previous Council, a course committee for LC Applied Mathematics had begun to look at the future of the subject at senior cycle. Their view was that

- A new specification should aim to widen the appeal of Applied Mathematics by including topics other than mechanics.
- In this context, they suggested a core and options structure. Options such as programming, game theory and financial mathematics were investigated as possibilities. However, finding a way to widen the appeal without diminishing he relevance to mathematical physicists was identified as a challenge for the core.
- The syllabus should be designed for 180 hours of study in line with all Leaving Certificate subjects.
- Applied Mathematics should be afforded full subject status on the school timetable.

Following considerable debate about the contents of the core, a view emerged that Applied Mathematics should complement and be taken concurrently with LC Mathematics, thus creating, in effect, an extended mathematics course.

The Leaving Certificate Applied Mathematics committee ceased to exist when the last council of the NCCA ended. Following the formation of the new council new structures were established for curriculum and assessment development involving the formation of specialist development groups. A new development group for Mathematics will be convened shortly to continue the development work on LC Applied Mathematics; the new group will comprise nominees from teacher and management bodies, the Department of Education and Skills (DES), the State Examinations Commission (SEC), the subject association, higher education and the various interests represented on the previous Leaving Certificate Applied Mathematics committee . The group's work will be undertaken in line with the process set out in *Structures and processes* (NCCA 2012). This background paper is part of that process. It is designed to act as a stimulus to the work of the development group. In addition, the final section of the paper proposes a brief for the review of LC Applied Mathematics that will guide the work of the development group.

2. Context

It is universally accepted that a country's success in the 21st century will depend on the ideas and skills of its population. As the world becomes increasingly technological, the value of these national assets will be determined in no small measure by the effectiveness of science, technology, engineering, and mathematics (STEM) education. STEM education, with its emphasis on mathematical knowledge and skills, will help Ireland to generate the capable and flexible workforce needed to compete in a global marketplace. It will ensure
our society will continue to make fundamental discoveries and to advance understanding
of ourselves, our planet, and the universe. It will generate the scientists, technologists, engineers, and mathematicians who solve old and new problems to create the new ideas, new products, and entirely new industries of the 21st century. It will provide the technical skills and quantitative literacy needed for individuals to participate in the workforce and make better decisions for themselves, their families, and their communities. And it will strengthen our democracy by preparing all Irish citizens to make informed choices in an increasingly technological world.

However, as summarised in Review of Mathematics in Post-Primary Education (NCCA, 2005) research has highlighted a number of problem areas in mathematics education in Ireland:

- Postu primary mathematics education in Ireland features a highly didactic pedagogy, with mathematics being taught in a procedural fashion that places relatively little emphasis on problem solving (Oldham. E, 2001, Lyons. M, Lynch. K, Close. S, Sheerin. E, & Boland. P, 2003).
- Across several cycles of PISA over the last decade, Ireland has consistently been ranked among the middle OECD countries in terms of mathematical literacy.
- Irish 15 year olds find items involving shape and space and problem solving particularly challenging and the proportion of higher achieving 15 year olds is below the OECD average. (OECD reports for PISA 2000–2012).
- Chief examiner reports on state examinations in mathematics over a number of years have consistently pointed to the overu reliance by candidates on roteu learned procedures and the lack of deeper understanding of basic mathematics concepts. There is evidence that students

are not able to apply and transfer mathematical knowledge and skills, except in the most practised way and in familiar contexts.

The report of the *Expert Group on Future Skills Needs* (2008) argues that Ireland must raise its level of mathematical achievement to ensure that it continues to successfully compete with rival economies. It was against this backdrop, and informed by the *International Trends in Mathematics Education* research (Conway and Sloane, 2006), that the NCCA proposed the innovative reform initiative called *Project Maths*. The aim of this initiative is to empower Irish mathematics students to develop essential reasoning and problem-solving skills and to raise their mathematical attainment. It is envisaged that this will be reflected in improved performance in the state examinations, improved achievement in internationally comparable tests such as PISA and TIMSS; and a more positive attitude towards mathematics among students. *Project Maths* began on a phased basis[2] in 24 schools in September 2008 and rolled out to all schools nationally two years later. In September 2012 a notable landmark was reached when phase 3 began in all schools. There are other imminent landmarks:

Spring 2014	**June 2014**	**June 2015**
The final, full-day workshop for teachers in a series of ten will be delivered by the Project Maths Development Team.	All five strands will be examined in the Leaving Certificate examinations.	The phasing of the examination changes to both Junior Certificate and Leaving Certificate mathematics examinations will be complete.

Initial evaluation work has been undertaken by researchers at the National Foundation for Educational Research (NFER), based in the UK. They explored the impact of Project Maths on student achievement, learning and motivation and their findings show that there is emerging evidence of positive impacts on students' experiences of, and attitudes towards, learning mathematics. At this early stage in Project Maths, Irish students are performing well in aspects of the revised syllabuses

[2] The Project Maths curricular development initiative involved the phased development and introduction of new syllabuses in Mathematics for both Junior Certificate and Leaving Certificate students. In each case, the five strands of the revised syllabuses were introduced in three phases. Phase 1 involved the introduction of Strand 1 (Statistics and probability) and Strand 2 (Geometry and Trigonometry); Phase 2 involved the additional introduction of Strand 3 (Number) and Strand 4 (Algebra); and Phase 3 involved the additional introduction of Strand 5 (Functions). The content of the previous syllabus was phased out as the new material was phased in. Corresponding changes were made to the examinations at each level.

and, overall, the introduction of the revised syllabuses does not appear to be associated with any specific deterioration or improvement in students' achievement. The recently published Chief Inspector's report noted that teachers' strengths lie in their ability to explain and communicate concepts and skills but found there is scope for developing how resources are used in the teaching and learning of mathematics. Where resources were used, inspectors reported that their use was generally to enhance the presentation of the lesson content rather than to actively engage the students in their own learning. (*Chief Inspector's Report*, DES 2013)

International Context

In senior cycle education in Ireland, uniquely, mathematics—although not compulsory— is studied by the vast majority of students (Leaving Certificate Mathematics: a comparative analysis Pope, 2013).

Table 3: Extract from table 6: What are the participation rates in upper secondary mathematics education? (Hodgen et al 2010a:38)

Country	Not studying mathematics	Studying any mathematics	Studying advanced mathematics	Compulsory /proportion of curriculum
Ireland (NCCA data)	Negligible 0-5%	All 95-100%	Low 0-15%	N, one of seven
Finland	Negligible 0-5%	All 95-100%	Medium 16-30%	Y, one of seven or eight
Massachusetts	Few 6-20%	Most 81-94%	Medium 16-30%	Y, one of seven
New Zealand	Some 21-50%	Many 51-80%	High 31-100%	Y, one of eight
Scotland	Many 51-80%	Some 21-50%	Medium 16-30%	N, one of five
Singapore	Some 21-50%	Many 51-80%	High 31-100%	N, one of six

As a consequence, LC Mathematics focuses on preparing ***all*** students to participate fully in modern society while at the same time ensuring that those who wish to progress to further study in mathematics have the conceptual understanding they need to serve as a foundation for further and deeper engagement. For those who believe that a mathematics syllabus should prepare students for further study in mathematics and mathematics-related disciplines, this is an unsatisfactory compromise.

The removal of some content[3] from the revised LC Mathematics syllabus under the Project Maths initiative to allow students time to develop skills essential to the development of mathematical proficiency has been criticised in some quarters. Critics characterise the changes as 'dumbing down' the syllabus and they express concern that students will be under-prepared for third-level courses with a significant mathematical component, such as mathematics, science, engineering, actuarial studies, computer science and economics.

> *...the Project Maths syllabus contains an ill–advised balance of topics, with a strong emphasis on commercial mathematics, applied probability and Euclidean geometry, and a minimum of calculus and linear algebra. This is particularly unsatisfactory for the Higher-Level students who wish to study technical subjects (engineering, any of the sciences, or even economics) at third level. As a result of this lack of preparation, technical colleges and science faculties at universities will have to lower their standards.*
>
> Kirkland, Stack et al (2012)

An international solution to these tensions between competing areas of mathematics for inclusion in curriculum specifications has been to provide different 'curriculum pathways' which allow students to choose mathematics courses that more closely match their future mathematical needs. Kirkland, Stack et al have explored and discussed this issue in their paper *Major flaws in Project Maths* (2012). It should be noted that, while Ireland currently offers advanced mathematics in the form of *Leaving Certificate Higher level Mathematics*, it does not provide any extra time for students to engage with the more advanced material. The curriculum time allocation for Ordinary level mathematics is 180 hours. Ordinary level mathematics is a sub-set of Higher level mathematics yet the curriculum time allocation for Higher level mathematics is still only 180 hours. As with all other subjects, this arrangement arises from a belief that students working at Higher level should be able to progress more quickly, even with additional content, than their counterparts working at Ordinary level. This, however, is inconsistent with the belief in other countries where separate pathways for advanced mathematics with extra time allocation are offered. The tables overleaf set out curriculum pathways available to students internationally, the aspirations of the learners whose needs they aim to address and the classroom time allocated to each pathway.

[3] Matrices , vectors, and elements of calculus have been removed to allow time for the development of skills and conceptual understanding

Singapore

Name of Pathway	Geared to the needs of learners who…	Time allocation
Higher 1 ***H1***	wish to follow a broad curriculum and acquire foundational knowledge and skills in mathematics which will support their future studies at university level. [Comparable in intellectual difficulty and rigour with H2 Mathematics]	125 hours
Higher 2 ***H2***	wish to continue to third level courses with a significant mathematical component such as mathematics, physics and engineering.	250 hours
Higher 3 ***H3***	have an aptitude and passion for mathematics; the H3 course allows these exceptional students to pursue mathematics at a higher level and in greater depth.	Up to 460 hours, since it must be taken concurrently with H2. Total time for H3 is between 112 and 210 hours.

Finland

Mathematics is compulsory in upper secondary education in Finland and is offered at two levels: Basic and Advanced.

Name of Pathway	Geared to the needs of learners who…	Time allocation
Basic Course	do not wish to study science or medicine at university	228 hours
Advanced Course	wish to study science or medicine at university	371 hours

Scotland

Name of Pathway	Geared to the needs of learners who…	Time allocation
Higher Mathematics	wish to continue to third level courses with a significant mathematical component, such as mathematics, commerce, engineering and science.	160 hours
Advanced Higher Mathematics	have completed Higher Mathematics. Taken in the final year of school or the first year of college, Advanced Higher Mathematics is designed to provide a broadened understanding of algebra, geometry and calculus for those candidates wishing to develop the experience they gained through the Higher Course.	120 hours Must have completed Higher Mathematics

Massachusetts

Mathematics is compulsory in upper secondary education in Massachusetts.

Name of Pathway	Geared to the needs of learners who…	Time allocation
Traditional Algebra 1, Algebra II and Geometry	wish to progress to third level. Successful completion of either pathway is a prerequisite for university entrance.	500 hours
Integrated Maths I, Maths II, Maths III		500 hours
Pre-Calculus	wish to progress to a third level course with a significant mathematical component. An advanced model pathway is taken concurrently with either the traditional or integrated pathway	Minimum time allocation 171 hours
Quantitative Reasoning		Minimum time allocation 171 hours

International Baccalaureate

Name of Pathway	Geared to the needs of learners …	Time allocation
Mathematical Studies Standard Level ***SL***	with varied backgrounds and abilities. It is designed to build confidence and encourage an appreciation of mathematics in students who do not anticipate a need for mathematics in their future studies. Students taking this course need to be already equipped with fundamental skills and a rudimentary knowledge of basic processes.	150 hours
Mathematics Standard Level ***SL***	who expect to need a sound mathematical background as they prepare for future studies in subjects such as chemistry, economics, psychology, and business administration. Students taking this course will already possess knowledge of basic mathematical concepts, and be equipped with the skills needed to apply simple mathematical techniques correctly.	150 hours
Mathematics Higher Level ***HL***	with a good background in mathematics and strong analytical and technical skills. Students taking this course will be expecting to include mathematics in their university studies, either as a subject in its own right or within courses such as physics, engineering and technology. The course is also for students who have a strong interest in mathematics and enjoy meeting its challenges.	240 hours
Further Mathematics ***HL***	with a very strong background in mathematics who have attained a high degree of competence in a range of analytical and technical skills, and who display considerable interest in the subject. Students who take this course will expect to study mathematics at university, either as a subject in its own right or as a major component of a related subject. The course is designed specifically to allow students to learn about a variety of branches of mathematics in depth and also to appreciate practical applications.	390 hours, since it must be taken concurrently with Mathematics Higher Level

Adopting a similar pathways approach to mathematics education in Ireland would mean seeing the review of LC Applied Mathematics as more than simply a review of a syllabus and preparation of a new specification. It would involve looking again at mathematics education through the lens of the students whose needs it aims to address, and providing post-primary curriculum pathways appropriate to a variety of post-school aspirations. The table below outlines a set of potential pathways, while Fig. 1 offers an overview of how the mathematical landscape could look if a pathways approach were adopted.

Name of Pathway	Geared to the needs of learners who may proceed…	Time allocation
Foundation level Mathematics	to use and apply mathematics in their future careers This pathway would lead learners towards further studies in areas in which specialist mathematics is not required.	180 hours
Ordinary level Mathematics	to use and apply mathematics in their future careers. This pathway would lead learners towards the use of academic mathematics in the context of further study.	180 hours
Higher level Mathematics	to third level courses where mathematics would be an advantage. Ordinary level Mathematics is a subset of Higher level Mathematics.	180 hours
The new specification	to courses at third level with a significant mathematical component. Would be taken concurrently with Higher level Mathematics	360 hours, since it must be taken concurrently with HL Mathematics

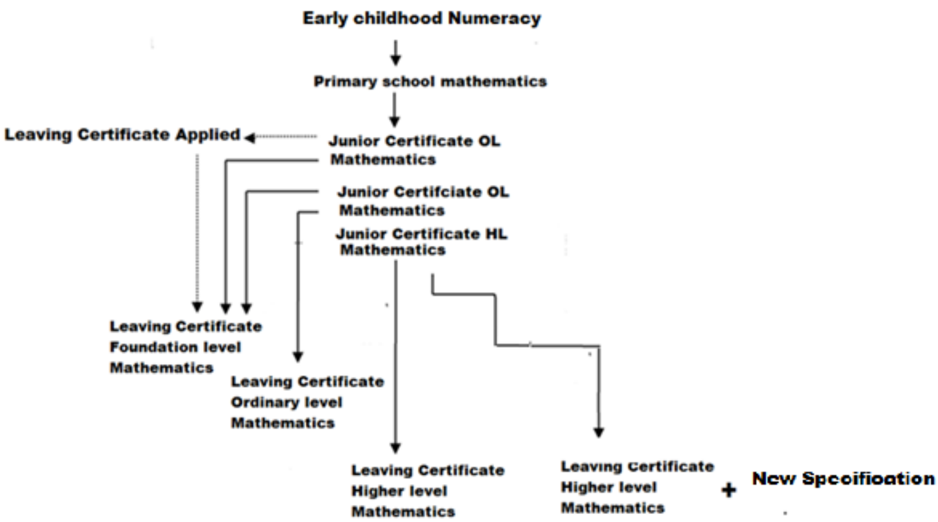


Fig. 1 Overview of potential pathways

Readiness for post-school courses

Much research has been conducted into student preparedness for post-school courses (Marchall, Summers, Woolnough,1999; Conley, 2003). One of the most dominant themes raised by participants in these studies is the importance of the habits of mind students develop in post-primary school and bring with them to post-school courses. These habits are considered by many university academics to be more important than specific content knowledge.

> *The habits of mind include critical thinking, analytic thinking and problem solving; an inquisitive nature and interest in taking advantage of what a research university has to offer; the willingness to accept critical feedback and to adjust based on such feedback; openness to possible failures from time to time; and the ability and desire to cope with frustrating and ambiguous learning tasks. Other critical skills include the ability to express one's self in writing and orally in a clear and convincing fashion; to discern the relative importance and credibility of various sources of information; to draw inferences and reach conclusions independently; and to use technology as a tool to assist the learning process rather than as a crutch.*
>
> (Conley, 2003, p. 8)

The development of habits of mind informs the senior cycle vision of creative, confident and actively involved young people who are prepared for a future of learning. *Towards Learning* (NCCA 2009) sets out that vision, the values on which it is based and the principles that are shaping the review and development of senior cycle curriculum and assessment. Developments at senior cycle, whether new subjects and courses or a review of existing subjects, place the learner as the focus of the educational experience, enabling them to become resourceful and confident, to participate actively in society, to build an interest in learning and the ability to learn throughout their lives. By embedding the five keys skills—critical and creative thinking, communicating, information processing, being personally effective and working with others—in the learning outcomes of each subject, learners will be assisted in reaching their full potential, both during their time in school and in the future.

To improve its suitability in offering students different paths through mathematics at senior cycle and preparing them well for third level courses containing a significant mathematical component such as engineering, science, actuarial studies, economics and computer science the new specification will need to be written in terms of learning outcomes with an emphasis on deep mathematical learning and skills development. Perhaps it should not only introduce students to mathematical content they are likely to encounter should they proceed to mathematics-related disciplines at third level, but support them in the development of the habits of mind described by

Conley (2003). It might also provide them with the opportunity to consolidate their understanding of LC Mathematics in a deep and meaningful way and further develop their synthesis and problem-solving skills. A critical factor will be the way in which the assessment of the course supports both the content and skills and these mathematical habits of mind

3. Syllabus Development Issues

Learning from Project Maths

Any review of a mathematics syllabus in Ireland must take account of the experience of the *Project Maths* initiative. The student-centred, connectionist model of learning proposed by *Project Maths* challenges the traditional transmission approach to mathematics education prevalent in Irish classrooms, with its strong allegiance to 'teacher exposition' and student 'drill and practice'.

This new model, however, requires a change in mind-set among all the stakeholders in mathematics education: teachers, students, parents, inspectors, examiners, text-book authors and those with an interest in the STEM agenda from both third level education and industry. A student-centred, connectionist model requires a re-conceptualisation of three important aspects – *mathematics*, *teaching,* and *learning:*

- Mathematics must be re-conceived as an interconnected body of ideas and reasoning processes rather than a given body of knowledge and standard procedures to be 'covered'.
- Teaching must be re-conceived as an activity that involves
    - exploring meaning and connections through non-linear dialogue between teacher and learners
    - presenting problems before offering explanations
    - making misunderstandings explicit and learning from them

rather than one that involves

- structuring a linear curriculum for learners
- giving explanations before problems have been posed and explored, and checking that these have been understood through practice exercises
- correcting misunderstandings.

- Learning must be re-conceived as a collaborative activity in which learners are challenged and arrive at understanding through discussion, as opposed to an individual activity based solely on watching, listening and imitating until fluency is attained.

Although the process of introducing Project Maths remains at an early stage, the research evidence emerging from numerous reports: Lubienski, S., (2011), O'Donoghue, J., O'Keeffe, L.,(2011), Grannell, J. et al (2011) ,NCCA (2012), Cosgove. J.,et al (2012), IMTA (2013) , Jeffes, J.,et al (2013) and DES (2013) suggests that making this transition will take longer and require more teacher support than was first envisaged. There is no doubt that the pervasive emphasis placed on external, high-stakes examinations is constraining reform efforts in the Irish mathematics classroom. Teachers see their role, at least in part, as that of exam coach and feel under huge pressure now that they can no longer predict the questions that will appear in the examinations. They lack confidence in their students' ability to solve problems that they have not '*covered*' in class.

> *I live in fear that a question will come up in the exam that I haven't covered*
>
> Mathematics teacher, NCCA Maths In Practice group 2013

Seeing their role in this way places mathematical authority with the exam looming in the future, instead of with the teacher in the classroom now, and so provides less motivation for teachers to engage their students in daily classroom activities not directly linked to examinations. They feel under pressure to 'cover' the syllabus and disappointed if the classroom time spent on a particular topic is not reflected in the question allocation on the examination paper, feeling limited time has not been well spent. Their related reliance on the *definitive textbook* and the generation of *more examination style questions* suggests that teachers, who are so intrinsic to the reform underway, need ongoing support in reconceptualising their role.

Traditional conceptions of *mathematics, teaching* and *learning* are not only institutionalised in the minds of teachers, they are also institutionalised in texts. The 2011 NCE-MSTL report *A Review of School Textbooks for Project Maths* (2011) page 21: draws attention to the fact that textbooks support the transmission approach to mathematics education.

> *The most significant overall finding is the mismatch between textbook expectations and Project Maths expectations.*

With their traditional conceptions of *mathematics, teaching* and *learning* teachers have been challenged by the *Project Maths* syllabus expressed in learning outcomes. This is to be expected as there is ample research evidence to suggest that not only teachers, but also text book authors and students will modify the impact of a new syllabus by adapting it to fit their existing practice (Thompson, 1992; Haug, 1999; Hill, 2000). This must be taken into consideration in the review of *Applied Mathematics*. Any new specification will be viewed through the lens of the previous syllabus, a list of content, and interpreted in terms of the related examination, which was generally predictable and very closely aligned to the content (see Appendix 2). If the comfort of predictability is removed, time to 'cover the content' will become an issue. The drafting of the new specification for Applied Mathematics must, therefore, be cognisant of the considerable difficulty experienced by teachers interpreting the *Project Maths* syllabuses. Overall, the new specification and related material must support teachers struggling with their re-conceptualisation of *mathematics, teaching* and *learning*.

Perspectives on content

The increasing rate of change of knowledge is a challenge for new curriculum specifications and it is no surprise, therefore, that there should be tensions and debate among stakeholders about what should be included in a revised Applied Mathematics specification. In her paper *Leaving Certificate Mathematics: A comparative analysis* (Pope, 2013), Dr Sue Pope notes that *the mathematics expectations*[4] *in Ireland are comparable to those in the other countries that offer a broad upper secondary curriculum*. In her work she compared the aims and objectives, the content strands and the approach to problem solving and assessment. Tables showing the outcomes of these comparisons are available at http://www.ncca.ie/tableofcomparison. Pope points out that *differential equations* as well as areas of content removed from the LC Mathematics syllabus as a result of the *Project Maths* initiative, *in particular, vectors and matrices*, are usually included in the more advanced syllabuses in the other countries and jurisdictions she examined and suggested that this content *may be worth considering in the review of… Applied Mathematics.*

[4] Pope compared the *Leaving Certificate Mathematics* syllabus revised under the *Project Maths* initiative with the mathematics syllabuses in five other jurisdictions.

Providing different 'curriculum pathways' which allow students to choose mathematics courses that more closely match their future mathematical needs is a challenge for *Applied Mathematics* and for the system as a whole. Employing a core and options structure, as suggested by the previous course committee represents a suitable compromise. Perhaps material not currently offered in mathematics could be included in the core in a way that would accommodate mathematical physicists as well as those with other interests. Students could then have a choice of *options* from a comprehensive list. This echoes the perspective of O'Reilly (2002) who suggests that *new or extended areas of content might include: differential equations, difference equations, vector algebra, trigonometry, graph theory, data handling, simulation and numerical analysis.* Three possibilities along these lines are illustrated below.

1. All students study the core and select ***one*** option from a comprehensive list

	Units	**Weighting**
Core	– Matrices and their applications – Vectors and their applications – Linear programming – Further calculus – Differential equations	70% 125 hours
Options	Mechanics	30% 45 hours
	Computer programming	30% 45 hours
	Networks and graph theory	30% 45 hours
	Business mathematics	30% 45 hours
	Game theory	30% 45 hours

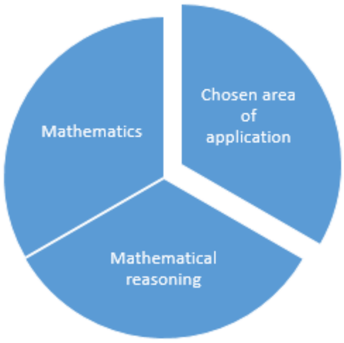
2. The weighting of the core is reduced; all students study the core and select two options from a comprehensive list.

	Units	**Weighting**
Core	– Matrices and their applications – Vectors and their applications – Linear programming – Further calculus – Differential equations	40% 80 hours
Options	Mechanics 1	30% 45 hours
	Mechanics 2	30% 45 hours
	Computer programming	30% 45 hours
	Networks and graph theory	30% 45 hours
	Business mathematics	30% 45 hours
	Game theory	30% 45 hours

A significant challenge for the development group working on the specification will be to present the material from the core in a way that will motivate and engage learners with a range of interests.

Another possibility is to abandon the core and options structure altogether and simply provide curriculum pathways representing a number of specialised areas where mathematics is applied. Thus providing a structure for students to combine their interest in mathematics and mathematical reasoning, with an interest in a specific field of intellectual activity.

3. Students engage with the material in the context of a chosen area of application.



Possible areas of application include Physics, Chemistry, Computer Science, Business, Social Sciences, Medical Sciences and Engineering.

Supply and demand may be a challenge for schools which may not have the capacity to provide all the options or areas of application for their students. Collaboration and judicious use of technology may have a contribution to make to overcome this challenge and that contribution will be considered later in this paper. Although it is probably advisable that the specification complement and be taken concurrently with LC Mathematics, there are issues for the development group to consider. One issue is its level on the National Framework of Qualifications (NFQ) and the challenge of specifying the options, these must be *ab initio* so that students progressing from junior cycle can engage with the material yet progress to a suitable level on the NFQ. If the specification is geared to the needs of learners who wish to proceed to courses at third level with a considerable mathematical component, the question also arises as to whether it should be developed at one level (Higher level) only.

The specification and CAO points

Any revision of a Leaving Certificate subject will give rise to discussion about CAO points and, with the perception that, currently, LC Applied Mathematics represents an efficient way for a specific cohort 'to get points', there is likely to be much interest and debate about the value of the specification in the points system. Although decisions around the allocation of CAO points to Leaving Certificate subject grades are not the remit of the NCCA, possibilities could be discussed and suggestions made. Since the revised specification, albeit an extension subject which must be taken concurrently with Higher level LC mathematics, comprises completely separate content it should attract full regular points in the first instance. Students taking two mathematics subjects have the same type and scale of advantage as those doing two language subjects. The added complication in the case of mathematics is that currently bonus points are awarded for A-D grades in Higher level LC mathematics. The award of bonus points is currently the subject of review and the outcomes of that review can inform any advice NCCA might give on how and if LC Applied Mathematics may or may not attract bonus points. The intention of the bonus was to encourage students to take mathematics *at* Higher Level; if Applied Mathematics is offered at only Higher level, the same incentive is not necessary.

Naming the subject

Given the considerable emphasis on application in LC Mathematics, the question remains as to whether LC Applied Mathematics would be a suitable name for the new specification. LC Mathematics is taught in contexts that allow students to see connections within mathematics, between mathematics and other subjects, and between mathematics and its application to real life, and in part its assessment is devoted to contexts and applications of mathematics. Some consideration might be given to adopting a different name for this specification, perhaps *Extended Higher Mathematics* or *Applications of Higher Mathematics* are possibilities.

Perspectives on assessment

Since expectations of what learners should achieve have changed since the *Applied Mathematics* syllabus was developed over 40 years ago there is a need for reflection on how best to design assessment approaches for the new specification in ways that will give students fairer and more varied opportunities to provide evidence of their achievements, and that will go some way towards measuring the broader competencies set out in the *Key Skills Framework* (NCCA 2009). Assessment in education has been the subject of research for decades, and the impact of high-stakes testing such as the Leaving Certificate examination on teaching and learning is well documented. Since the early 1980's there have been several reviews of such research (Linn et al., 1982; Crooks, 1988; Koretz, 1988; Koretz et al., 1991; Shepard, 1991; Kellaghan et al., 1996; Black & Wiliam, 1998; Stiggins, 1999; Linn, 2000). A strong, common theme emerges from the findings: high-stakes testing has a backwash effect into daily learning and teaching. This effect resonates strongly with the practices prevalent in Irish mathematics classrooms: teachers focusing on the content of the exams, administering repeated practice tests, training students in the answers to specific questions or types of question, and adopting transmission styles of teaching. In such circumstances teachers sometimes make little use of formative assessment to assist students in the learning process (Broadfoot et al., 1998; Reay & Wiliam, 1999; Osborn et al., 2000; Pollard et al., 2000). The vision of assessment articulated in *Towards Learning* is that, in whichever context it is used, assessment can and should be supportive of learning. A concern for equity and fairness is central to this. For assessment to be meaningful, equitable and fair it must be well-aligned to the type of learning that is valued. Factual knowledge tests are well suited to assessing the outcomes of traditional teaching approaches based on rote learning and knowledge transfer. But such tests are less adequate when it comes to assessing complex competencies (Biggs, 1996, 1999) or the outcomes of student-centred connectionist approaches to teaching and learning.

The development of assessment approaches and components for the new specification will need, therefore, to consider a range of approaches – along similar lines to those in Appendix 3 – that offer students multiple and appropriate opportunities to achieve and in so doing use the backwash effect of high stakes testing to support learning. A core and options structure can offer flexibility in approaches to assessment with the possibility of the core being assessed by a written paper and the chosen option in an alternative way such as one described in Appendix 3.

Perspectives on the role of technology

Technology is contributing to a re-conceptualising of learning, to expand beyond linear, text-based learning and to engage students who learn best in other ways. Its role in schools has evolved from a contained 'computer class' into a versatile learning tool with the potential to change how concepts are demonstrated, projects assigned and progress assessed. Digital simulations and models are freely available that help teachers to explain concepts that are too big or too small, or processes that happen too quickly or too slowly to demonstrate in a physical classroom. Technology that allows teachers to access data from students' performance on tasks is widely available. They can access what each student did to reach his or her end result and thereby understand whether trial-and-error or actual knowledge of the concept led to the answer. Epistemic games can empower students to think like innovative professionals by placing them in professional roles such as engineer, scientist or mathematician and asking them to solve real world problems. Despite these opportunities, reports indicate that only a minority of EU teachers make considerable or extensive use of ICT in their daily teaching and learning (Conway & Brennan, 2009; Cosgrove & Marshall, 2008; DES, 2008; European Schoolnet and University of Liege, 2013).

In her consultative paper *Building towards a Learning Society: A National Digital Strategy for Schools* Dr Deirdre Butler notes that the concept of teaching and learning through the use of ICT is highly complex and that there is a *relationship between teachers' general philosophical beliefs about teaching and learning, their pedagogical practices, and their use of ICT.* The rich and growing selection of open educational resources available free online, for all levels of education, has the potential to dramatically change the role of the educator in the classroom and support a reconceptualization of learning. Flip teaching (or flipped classrooms) is just one form of blended learning which challenges the traditional conception of teaching. Flipped teaching encompasses any use of technology to leverage the learning in a classroom, so a teacher can spend more time interacting with students instead of lecturing. This most commonly happens using teacher-created videos that students view outside of class time. The student first studies the topic by himself, typically using video lessons created by the teacher or shared by another educator. In the classroom, the student then tries to apply the knowledge by solving problems and doing practical work with peers and teachers. The role of the classroom teacher is then to tutor the student when they get stuck, rather than to impart the initial lesson. This allows time inside the class to be used for additional learning-based activities, including use of differentiated instruction and project-based learning. *(Bergman, J. and Sams. A. 2012)*

Potential exists to provide at least some of the options in the new specification for Applied Mathematics as a blended learning experience for students combining online and face-to-face elements. This would present an opportunity not only to use technology in teaching and learning, but also to increase the collaboration between second-level schools and the second-level and third-level sectors. Since one of the aims of the new specification may be to provide a curriculum pathway for second level students who want to progress to third level courses with a significant mathematical component – such as mathematics, science, computer science, engineering, actuarial studies, and economics – a blended learning environment would allow second-level schools an opportunity to share their expertise and give third-level experts an opportunity to provide the stimulus material: videos, teaching and learning tasks, and assessment projects for the specification. This level of collaboration would be beneficial to the system as a whole and the new specification could play a similar transitional role, at a different level, as the Common Introductory Course in mathematics plays in spanning primary and post-primary education[5]. This blended learning approach might also have potential in situations where a specialist mathematics teacher may not be available, or where there is only a handful of students in a school who wish to avail of this subject.

[5] *Moodle in the Community* is an example of a successful collaboration between DCU and four local second-level schools. This community outreach initiative not only has the potential to be scaled up to facilitate collaboration between a third-level institute and a consortium of second-level schools, but also the potential to facilitate collaboration between second-level schools that wish to share their resources. The technology offers flexibility and supports teachers in their mentoring role. Though they may not be directly involved in delivering a particular option, teachers have access to their students' record, they can monitor their progress, their on-line engagement and can export all their activities, assignments and assessments to an e-portfolio which would assist in guidance and reporting to parents.

4. Brief for the review of LC Applied Mathematics

The development of the specification will contribute to clarifying the paths that students can take through mathematics at senior cycle, and into related fields at third level education. It will address continuity and progression by categorising possible post-school pathways and considering mathematics curriculum pathways at senior cycle appropriate to each category. In other words, it will examine the current mathematics provision through a pathways lens and possibly reassign learning outcomes to more suitable pathways. It will examine the provision for exceptional students in light of the PISA 2012 findings and address questions such as:

- Are students following mathematics curriculum pathways that provide them with a suitable mathematical challenge?
- Should some of the proposed *options* be made available as short courses to exceptionally able students at junior cycle?
- What would be a suitable senior cycle pathway for students who take an *options* short course at junior cycle?

Some consideration will be given to naming the subject/specification and to whether it should be developed at a single level (Higher) only.

In line with current developments, the specification will be student-centred and outcomes-based, and follow the format of other senior cycle specifications:

- Introduction and rationale
- Aim
- Objectives
- Structure
- Key skills
- Learning outcomes
- Assessment
    - Assessment components
    - Assessment criteria.

In general terms, the specification should be aligned with levels 4/5 of the National Framework of Qualifications. If developed as a Higher level only specification, that alignment should predominantly be towards level 5.

More specifically, the development of the new/revised specification will address

- how its presentation can support teachers in re-conceptualising mathematics, teaching and learning
- whether it should have a core and options structure and related issues
- how to widen the appeal of the core
- whether the core should comprise mathematics not currently offered, such as matrices, vectors and differential equations
- the inclusion of motivating and engaging options
- how student work will be assessed; the provision of multiple, diverse and appropriate opportunities available for students to achieve
- how to embrace technology and collaboration in the learning, teaching and assessment associated with the specification
- the broader context of ‘pathways in mathematics at senior cycle and how this specification relates to those pathways and to existing mathematics specifications.

The work of the Mathematics Development Group will be based, in the first instance, on this brief. In the course of its work and discussions, elaborations of some of these points and additional points may be included in the brief.

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Appendix 1: Syllabus for Leaving Certificate Applied Mathematics

APPLIED MATHEMATICS

Ordinary and Higher Level Courses

NOTE: SI units to be used throughout.

Candidates will be expected to know the dimensions of any physical quantity dealt with. Knowledge of the relevant parts of the Mathematics course is assumed. Candidates will be required to deal only with such cases as can be treated in two dimensions.

N.B. Those parts of the syllabus which are printed in *italics* belong to the Higher Level course only. The Higher Level course includes the Ordinary Level course treated in greater depth.

1. Motion of a particle. Displacement, velocity as vectors. Applications of the vector addition law. Description of vectors in terms of unit perpendicular vectors. Elementary treatment of relative motion.

2. Newton's laws. Mass, momentum. Acceleration and force as vectors. Units and dimensions.

3. Motion in a straight line under uniform acceleration e.g. motion under gravity, motion on smooth and rough inclined planes. Work, potential energy, kinetic energy, power. Application of energy conservation. Motion of connected particles.

4. Equilibrium of a particle under concurrent forces, including friction.

5. Centre of gravity of simple bodies and systems of particles Moments and couples. Equilibrium of a rigid body *or bodies.*

6. Liquid pressure. Thrust on a horizontal surface. Archimedes' Principle.

7. Projectiles. *Projectiles on inclined plane.*

8. Angular velocity. Uniform motion in a circle without gravitational forces. Conical pendulum. Circular orbits.

9. Conservation of momentum. Collisions. Direct collisions, elastic ($0 < e \leq 1$) and inelastic ($e = o$). *Oblique collisions of smooth elastic spheres in two dimensions.*

10. *Simple harmonic motion of a particle in a straight line. (Application of simple harmonic motion to include the simple pendulum*.)

11. *Motion of a rigid body about a fixed axis:*

a) Calculation of moments of inertia for a rod, rectangular lamina, circular lamina and compound bodies formed of those. (Sphere is excluded). Application of parallel and perpendicular axes theorems, with proofs done as classwork. Idea of radius of gyration. Application of the conservation of energy principle to a rotating body.

b) Application of the principle of angular momentum: rate of change of angular momentum about a fixed axis equals the total external moment about that axis. Compound pendulum. Simple applications.

12. Ordinary *differential equations and applications:*

a) first order, variables separable;

b) Second order reducing to type (a)

Format of examination papers:

Ordinary Level: six questions to be answered out of nine

Higher Level: six questions to be answered out of ten.

Appendix 2: LC Applied Mathematics examinations

Extracts from the Chief Examiners reports showing the popularity of questions in the 2007 and 2012 LC Applied Mathematics examinations.

Note

- the similarity in question popularity between the 2 years
- the close alignment between the list of content

Ordinary Level							
Question	Topic of Question	**Average Mark % 2007**	**Response Rate(%) 2007**	**Rank Order 2007**	Average Mark % 2012	Response (%) Rate 2012	Rank Order
1	Linear Motion	91.4	100	1	89.2	97.9	1
2	Relative velocity	84.4	90	5	65.6	87	5
3	Projectiles	73.4	95	3	68.2	87.7	4
4	Connected Particles	80.8	92.5	4	81.0	92.5	3
5	Collisions	73.6	97.5	2	86.2	94.5	2
6	Centre of Gravity	78.2	37.5	6	73.8	63.7	6
7	Statics	71.2	35.0	7	59.8	21.9	8
8	Circular Motion	78	22.5	9	60.6	25.3	7
9	Hydrostatics	58	25.0	8	47.0	19.2	9

Higher Level							
Question	Topic of Question	Average Mark % 2007	Response Rate (%) 2007	Rank Order	Average Mark % 2012	Response Rate (%) 2012	Rank Order
1	Linear Motion	71.2	90.6	4		95.1	1
2	Relative velocity	69.2	65.0	6		68.6	6
3	Projectiles	71.4	92.5	3		92.0	4
4	Connected Particles	72.0	94.4	1		94.2	2
5	Collisions	69.0	93.1	2		92.4	3
6	Circular Motion and SHM	60.2	12.5	9		22.2	8
7	Statics	45.8	19.4	8		12.2	9
8	Rigid Body Motion	64.4	33.8	7		27.3	7
9	Hydrostatics	49.2	8.8	10		9.9	10
10	Differential Equations	70.6	90.0	5		79.0	5

Appendix 3: Sample assessment approaches

1. Extended Modelling and Problem Solving Task

An extended modelling and problem-solving task is an assessment instrument developed in response to a mathematical task. It may require a response that involves mathematical language, appropriate calculations, tables of data, graphs and diagrams, and English prose. Students may provide a response to a specific task or issue that could be set in a context that highlights a real-life application of Mathematics.

The students would be required to

- analyse information and data from a variety of sources
- process information to identify assumptions and parameters
- interpret and synthesise data
- apply known mathematical procedures to solve a problem
- explain relationships to develop and support mathematical arguments
- reflect on and evaluate data collected, propositions, results and conclusions
- communicate ideas.

Teachers and/or school management would be required to implement strategies to ensure authentication of students' work such as

- students keeping a log/blog of their progress on the task but the task report is written in a controlled setting
- students complete the task and are required to answer questions about the task in a controlled setting
- teachers provide judicious feedback and assistance and report on the assistance given.

2. A practical or investigative task

The student would be required to write an extended response to a practical or investigative task. The report and the activities leading to a report could be done individually and/or in groups; in class time and/or in the students' own time. A report could typically be in written form, or a combination of written and oral multimedia forms. The report will generally include an introduction, analysis of results and data, conclusions drawn, justification, and, when necessary, appendixes and a bibliography and/or reference list.