Case study

The students in 5th class were asked to solve the following problem by drawing a clearly labelled diagram.

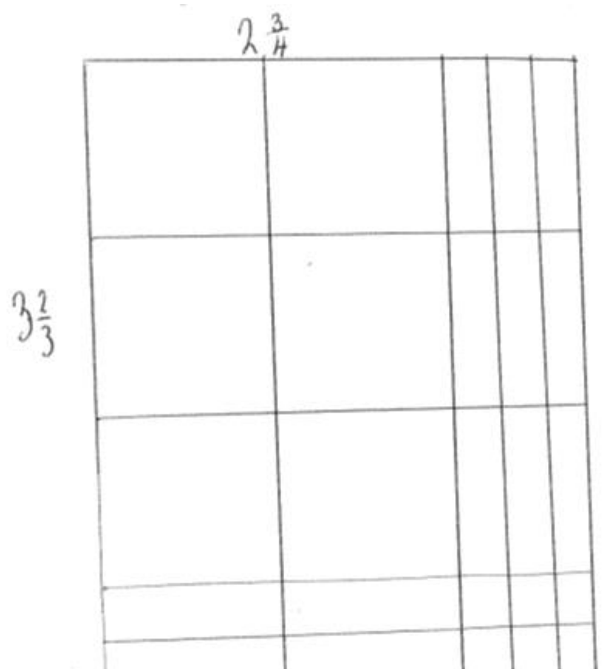
What is the area of a rectangle that has a width of $2\frac{3}{4}$ and a length of $3\frac{2}{3}$?

As I was circulating around the class, listening to and observing the students working on the problem, I overheard Tomás

Tomás: *Usually all you have to do to find the area is to multiply the length by the width, but we can't do that 'cos we have fractions.*

> We had spent a lot of time working on area and perimeter problems, so the students were familiar with finding the area by counting the amount of units or in the case of rectangles by multiplying the length by the width. Why did Tomás think this method would not apply with fractions?

When we started the whole class discussion Darragh volunteered to come to the board to discuss his strategy for solving the problem. He carefully drew this diagram on the board



Darragh: *You can get some of the area but not all of it.*

Teacher: *What part of the area can you get?*

Darragh: *I know the length times the width is the area so* $2 \times 3 = 6$

Teacher: *Where is the* 2×3 *or the 6 in the diagram?*

Darragh: *The big squares are the whole and you can just count 6.The smaller ones you can count too, but…eh they aren't wholes*

Teacher: *Why not?*

Darragh: *Those pieces aren't whole squares the way the other ones are, because of the fractions. So [starts counting rectangles on the top right] there are 9 of those* $\frac{1}{4}$ *s that is 2 wholes and* $\frac{1}{4}$ *left. There are 4 of those [points to rectangles at the bottom left] and that is* $1\frac{1}{3}$*. But I don't know how to count the others.*

Teacher: *Why not?*

Darragh: *I don't know; it's like they are pieces of pieces of something.*

John: *Like fractions of pieces when the pieces are fractions.*

There was a pause in the class and students started to think about that one. I let them discuss it for a minute or two then

Teacher: Can anyone explain what John is saying?

Joseph: *I think what he means is that those pieces [points to the smaller rectangle in the bottom right of Darragh's diagram] are fractions of fractions, but…what is that?*

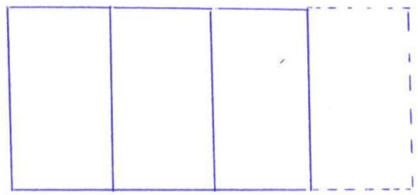
Tomás: *There is* $\frac{2}{3}$ *on one side and* $\frac{3}{4}$*on the other*

Padraig: *It's like* $\frac{2}{3}$ *of* $\frac{3}{4}$ *but you can't have that*

Tomás: *Yeh, there's no way you could have* $\frac{2}{3}$ *of* $\frac{3}{4}$

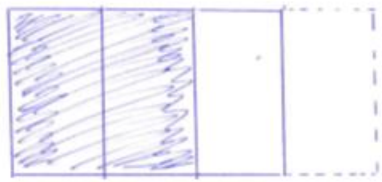
At this stage I decided to introduce an idea that might ease the confusion that the students were having. I used the example that Padraig had posed, but I put the idea into a simple and meaningful context.

Teacher: *Think about this; someone gave me* $\frac{3}{4}$ *of a leftover chocolate bar [I drew a diagram on the board]*



I ate $\frac{2}{3}$ *of that for little break. What part of the whole chocolate bar did I eat?*

Seya: *That much [comes up and shades in the diagram] It's* $\frac{1}{2}$



I said nothing for a few seconds and let the class think about what Seya did. Then Padraig said 'or it could be this…

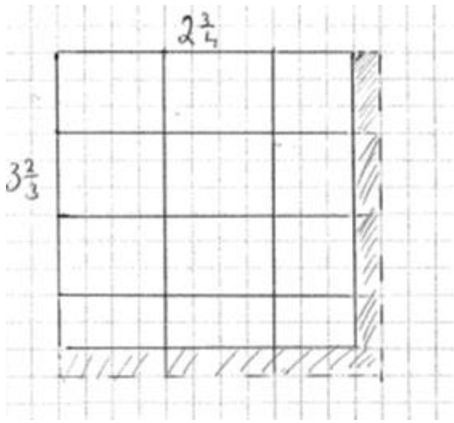
Tomás: *Well I think it's* $\frac{6}{12}$

[comes to the board and draws this]

We spent some time deciding which diagram was “right”. There were interesting discussions which I allowed to continue until the idea that both diagrams were equivalent was more comfortable for them. When we returned to the original problem it was decided unanimously that you could indeed find the area of the entire region by naming all the little bits and counting them.

This is what Darragh had to say.

Darragh: I can see now how you know what the names are ‘cos if you extend the diagram to show all the missing part it’s easy look.



Look it’s so easy 6 [points to the 6 full squares] there’s three $\frac{3}{4}$*’s [points to the top 3 rectangles in the right-hand column]which is* $2\frac{1}{4}$ *and here [points to the 2 bottom right rectangles] two* $\frac{2}{3}$*’s which is* $1\frac{1}{3}$ *and these are the ones I couldn’t do before but it’s easy now:* $\frac{6}{12}$ *‘cos I can see what the whole is.*

So the area is $6 + 2\frac{1}{4} + 1\frac{1}{3} + \frac{6}{12}$

Teacher: *Could we write this in another way?*

There was a lot of discussion.

Tomás: $\frac{6}{12}$ is $\frac{1}{2}$ and that is $\frac{2}{4}$ so the area is $9\frac{3}{4}$ and $\frac{1}{3}$

Teacher: *Can everyone see Tomás's answer in the diagram?*

At this stage there was a lot of discussion as the students tried to show the $9\frac{3}{4}$ and $\frac{1}{3}$ in the diagram. Then Seya said:

Seya: *When you look at the diagram it's easier just to say* $\frac{121}{12}$*, just one number. Llook it's easier [points to the diagram]*

John: *Yeh, the diagram gives you Seya's and the sums give you Tomás's.*

Thoughts for teachers:

- What prior knowledge should your students bring to the task?
- Are your students ready for this task?
- How would you use this task with your students?
- What mathematics do you want your students to learn from engaging in this task?
- What do you think your students might find difficult about this task?
- What questions might you ask as your students as they are working on the task