Question ***Exponential equations***

..can you clarify whether exponential equations (equations with variables in the index) are on the syllabus for Junior Cert and Leaving Cert, and at what levels please?..

.. Again I wonder if you could clarify if exponential graphs are on the syllabus for the present 2 nd years ie doing their junior cert in 2014 ?...

Answer
Yes, solving exponential equations arises for both JC and LC students. In order to give a more complete answer, I have drawn on the following sections of the syllabus:

- JC: *3.2, Indices; 4.7, Equations and inequalities* ; *5.2, Graphing Functions*
- LC: *3.2, Indices; 4.2, Solving equations; and 5.1, Functions*

At **JCOL** students will meet *exponential relationships* when they

> – use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling)

At this level, students should be engaged in activities that require them to informally solve exponential equations arising from a doubling or tripling context in order to answer questions such as…After how many days will there be 64 bacteria in the tray?...How many sections will there be if I fold the paper in half 4 times? ….When will I have €213 in my account?...

At **JCHL**, students build on this experience when they

> – draw graphs of the following functions and interpret equations of the form $f(x) = g(x)$ as a comparison of functions
> - $f(x) = ax + b$, where $a, b \in \mathbf{Z}$
> - $f(x) = ax^2 + bx + c$, where $a \in \mathbf{N}$; $b, c \in \mathbf{Z}$; $x \in \mathbf{R}$
> - $\boldsymbol{f(x) = ax^2 + bx + c}$, **where** $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$
> - $\boldsymbol{f(x) = a2^x}$ **and** $\boldsymbol{f(x) = a3^x}$, **where** $a \in \mathbf{N}$, $x \in \mathbf{R}$
>
> – use graphical methods to find approximate solutions where $f(x) = g(x)$ and interpret the results

At this level, students should use graphical methods to get approximate solutions to exponential equations, whether expressed informally or written formally using the standard notation. The exponential graphs could be given to the students or they could be asked to draw them themselves.

At **LCOL**, students are expected to work with more advanced exponential equations:

– graph functions of the form
- $ax+b$ where $a, b \in \mathbf{Q}$, $x \in \mathbf{R}$
- ax^2+bx+c where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$
- ax^3+bx^2+cx+d where $a, b, c, d \in \mathbf{Z}$, $x \in \mathbf{R}$
- ab^x where $a \in \mathbf{N}$, $b, x \in \mathbf{R}$

– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions

– use graphical methods to find approximate solutions to
- $f(x) = 0$
- $f(x) = k$
- $f(x) = g(x)$

where $f(x)$ and $g(x)$ are of the above form, or where graphs of $f(x)$ and $g(x)$ are provided

And at **LCHL**, more sophisticated again:

– graph functions of the form
- ax^2+bx+c where $a, b, c \in \mathbf{Q}$, $x \in \mathbf{R}$
- ab^x where $a, b \in \mathbf{R}$
- logarithmic
- exponential
- trigonometric

– interpret equations of the form $f(x) = g(x)$ as a comparison of the above functions

Both **JCHL** and **LCOL** students should be able to

– use and apply rules for indices (where $a, b \in R$, $a, b \neq 0$; $p, q \in Q$; $a^p, a^q \in R$; complex numbers not included):
- $a^p a^q = a^{p+q}$
- $\frac{a^p}{a^q} = a^{p-q}$
- $a^0 = 1$
- $(a^p)^q = a^{pq}$
- $a^{\frac{1}{q}} = \sqrt[q]{a}$, $q \in Z$, $q \neq 0$, $a > 0$
- $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$, $p, q \in Z$, $q \neq 0$, $a > 0$
- $a^{-p} = \frac{1}{a^p}$
- $(ab)^p = a^p b^p$
- $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

– solve problems using the rules for indices (where $a, b \in \mathbf{R}$; $p, q \in \mathbf{Q}$; $a^p, a^q \in \mathbf{Q}$; $a, b \neq 0$):
- $a^p a^q = a^{p+q}$
- $\frac{a^p}{a^q} = a^{p-q}$
- $a^0 = 1$
- $(a^p)^q = a^{pq}$
- $a^{\frac{1}{q}} = \sqrt[q]{a}$ $q \in \mathbf{Z}$, $q \neq 0$, $a > 0$
- $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$ $p, q \in \mathbf{Z}$, $q \neq 0$, $a > 0$
- $a^{-p} = \frac{1}{a^p}$
- $(ab)^p = a^p b^p$
- $(\frac{a}{b})^p = \frac{a^p}{b^p}$

– solve problems using the rules of logarithms
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a x^q = q\log_a x$
- $\log_a a = 1$ and $\log_a 1 = 0$
- $\log_a x = \frac{\log_b x}{\log_b a}$

Proficient mathematicians will make use of structure to help solve problems and, at all levels, students should be encouraged to look for and make use of structure. Consequently, students should develop the practice of writing expressions for functions and equations in ways that reveal their key features. Students, therefore, should explore how the rules of indices can be used to rewrite simple exponential equations such as $8^x = 64$ in ways that allow them to see an algebraic solution to the equation. At **LCHL** students take this further; they are required to not only understand logarithms as functions but also as inverses of exponential functions. At this level, students can think of the logarithms as unknown exponents in expressions with base 10 and use the properties of exponents when explaining logarithmic identities and the laws of logarithms. They should be encouraged to explore algebraic solutions to appropriate exponential equations.

Question ***Transformations of Functions***

I attended a workshop where we looked at scaling and shifting graphs. I have been unable to find this on the syllabus for Leaving Certificate Higher Level 2014. How many types of graphs do the students need to be able to scale? It's not clear in the syllabus.

Could you clarify are Transformatons of Linear, Quadratic, cubie and exponential functions on the syllabus for examination in 2014.If so can you tell me where this is on the syllabus or where it ins inferred on the syllabus.- page number etc

Answer

For students to develop a good understanding of functions, it is important that they are able to move fluidly between different representations of functions (equations, tables and graphs) and to use graphical representations as a way of solving equations. The activities in Workshop 7 dealing with scaling and shifting of graphs are designed to help develop students' conceptual understanding of functions and calculus, and it is really important that students engage in these types of learning experiences.

In section 5.1 of the syllabus you will see that students at all levels are required to be able to interpret equations of the form f(x) = g(x) as a comparison of functions (the functions at each level are outlined; remember also that learning outcomes at LCOL are a subset of LCHL).

At higher level you will see the following two learning outcomes:

– graph functions of the form

- $ax^2+bx + c$ where a,b,c ϵ Q, x ϵ R
- ab^x where a,b ϵ R
- logarithmic
- exponential
- trigonometric

– interpret equations of the form f(x) = g(x) as a comparison of the above functions.

This means that students are expected to be able to interpret equations and make comparisons between functions. Therefore, looking at the effect of, say, adding a constant or changing the coefficients on any of those functions is essential so that students will be able to make these comparisons.

For example, a student should be able to compare the functions

$f(x) = 3x^2 + 2x$ and $g(x) = 3x^2 + 2x + 5$

and describe the impact which adding the constant to produce g(x) has on the graph of f(x).

Question *Implicit differentiation*

Differentiation of the circle is stated on the syllabus but there is no mention of implicit differentiation which is needed to do it. Also it is not mentioned in the text book. Also it doesn't mention whether it is all circles or just circles with centre (0,0). Would be grateful for any clarification.
Implicit differentiation seems to be unavoidable given the sentence in Strand 5 about tangents to circles??

Answer

This specific learning outcome: students should be able to

- use differentiation to find the slope of a tangent to a circle

was included at **HL** to build on the **LCOL** learning outcome: students should be able to

- associate derivatives with slopes and tangent lines.

Students at **HL** should, therefore, first encounter this specific learning outcome with a circle of centre (0,0) and explore how differentiation can be used to find the slope of the tangent line. They can re-arrange the expression to present y as a function of x and then differentiate this, or the chain rule may be used to differentiate the y^2 component and thus obtain a dy/dx element in the result. Hence, they should be able to calculate the slope (of the tangent) at any desired point on the circle.
'***What if***…?' questions could then lead students to wonder how this might change if the circle did not have its centre at (0,0). …How would the differentiation problem change? What challenge might this change present? How could it be overcome?
There is no requirement to develop the concept further or to formally deal with implicit differentiation other than in this specific context.