Key Concepts in Mathematics – Multiplicative Thinking

If these concepts are not fully developed students' will find it difficult to engage meaningfully with core aspects of the Number, Algebra and Functions strands in later years.

Multiplicative Thinking

A capacity to work flexibly with the concepts, strategies and representations of multiplication and division as they occur in a wide range of contexts.

Students who are thinking multiplicatively will be able to

- work flexibly and efficiently with large whole numbers, decimals, common fractions, ratio, and percentages
- recognise and solve problems involving multiplication or division including direct and indirect proportion,
- communicate their solutions effectively in words, diagrams, symbolic expressions, and written algorithms

How does the concept develop? There are several ***ideas*** that support the development of multiplicative thinking. The exploration of these ideas is very important; their development may take many years and according to some researchers, may not be fully understood by students until they are well into their teen years.

1. The ***groups of*** idea. This idea represents an additive model of multiplication and develops when children begin to count large numbers of objects. The one-to-one count becomes tedious and children begin to think about more efficient strategies, they skip count by twos, fives or tens. Some children can find this move from a one-to-one count to a one-to-many count very difficult because they lose sight of what they are actually doing; counting a count. The difficulty is eased if children are given the opportunity to…..
2. Move beyond the ***groups o***f idea to a ***partitioning*** or ***sharing*** idea and focus their attention on the number in each of a known number of shares. Asking children to systematically share collections helps develop this idea. There are documents available outlining tasks that empower children to think about counting by exploring how many ways a number of objects can be shared equally. One of the advantages of the ***sharing*** idea is that it leads to the realisation that a collection may be partitioned in more than one way, e.g. 24 is 2 twelves, 3 eights, 4 sixes, 6 fours, and 12 twos, each of which can be represented more efficiently by an *array* or a *region*.
3. The real strength of the array or region representation is that it provides a basis for understanding fraction diagrams, and leads to the ***area*** idea which is needed to accommodate larger whole numbers and rational numbers. The ***area*** idea is very important and more neutrally represents all aspects of the multiplicative situation, that is, the number of groups, the equal number in each group, and the product. It also demonstrates commutatively of multiplication as well as how multiplication distributes over addition. Read the ***tasks*** and ***case studies*** for ideas on how you can support your students with the ***area idea*** of multiplication.
4. The ***area*** idea generalises to the ***factor-factor-product*** idea which is needed to support fraction representation as well as multiple factor situations such as 24 = 2 x 2 x 2 x 3, exponentiation as in 4 x 4 x 4, and algebraic factorisation as in $(2x + 3)(?) = 2x^2 + 5x + 3$
5. The ***for each*** idea also known as the ***Cartesian Product*** arises in the context of Data in primary school and Strand 1 in post-primary school. It also applies in rate or proportion problems and is evident in the structure of the place-value system, where for example, children need to think about the fact that ***for each*** ten, there are 10 ones, ***for each*** hundred there are10 tens, and ***for each*** one there are 10 tenths and so on. There are documents available outlining tasks that promote the development of the ***for each*** idea.