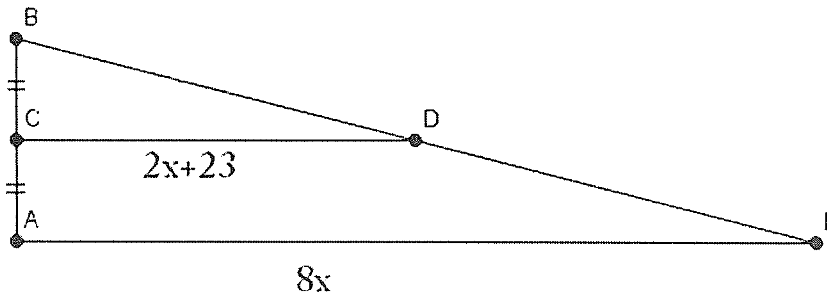
Q1 (LCOL) In the diagram, CD is parallel to AF and equal lengths are marked.

Find the value of x.



$CD \parallel AF \Rightarrow \angle C = \angle A$

$\angle B$ is common to ΔABF and ΔCBD

Remaining angles are equal

Thus ΔABF & ΔCBD are similar

$\Rightarrow$ corresponding sides are proportional

$|BC| = \frac{1}{2}|BA| \Rightarrow |CD| = \frac{1}{2}|AF|$

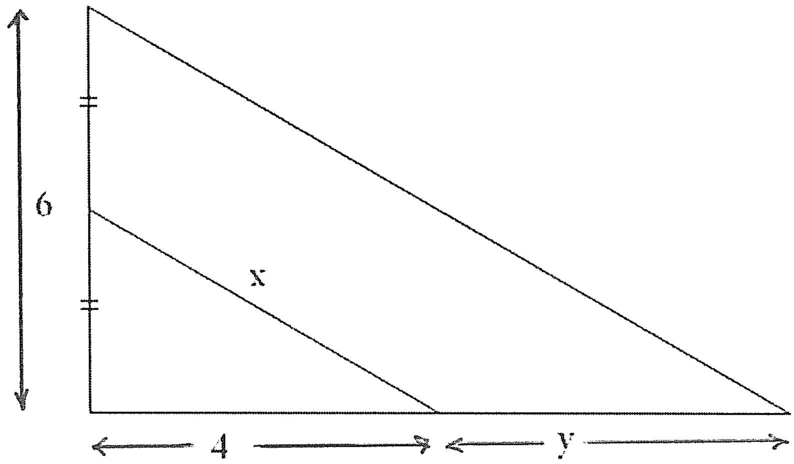
$\therefore \quad 2x + 23 = 4x$

$23 = 2x$

$11.5 = x$

Ans $x = 11.5$

Q2 (JCHL) If the sloped lines are parallel, find the value of x and the value of y.



$$x^2 = 4^2 + 3^2$$
$$= 16 + 9$$
$$= 25$$
$$x = 5$$

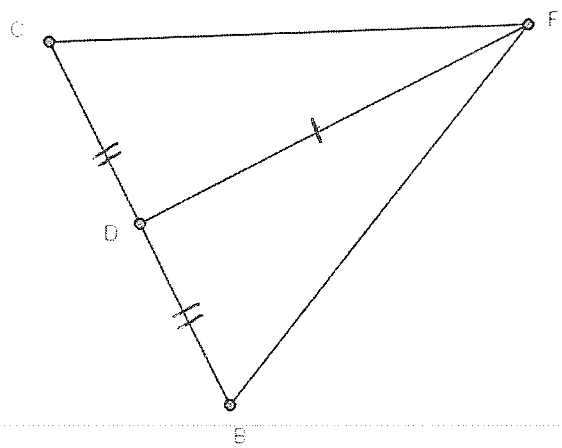
Sloped lines parallel $\Rightarrow$ Δs are similar

$$\therefore \frac{6}{3} = \frac{4+y}{4}$$
$$24 = 12 + 3y$$
$$12 = 3y$$
$$4 = y$$

OR Sloped lines parallel, so sides are divided in the same proportion. Because 6 is divided equally, y must also be 4 in length.

Q3 (JCHL) In triangle FCB $|CD| = |DB|$ and $|\angle FDC| = |\angle FDB| = 90°$

Explain why the triangles FDC and FDB are congruent.



In $\triangle$ FDC and $\triangle$ FDB

$$|CD| = |DB|$$
$$|\angle FDC| = |\angle FDB|$$
$$|FD| = |FD|$$

So (SAS) they are congruent

OR

In $\triangle$ FDC

$$|FC|^2 = |CD|^2 + |FD|^2 \quad \text{(Pythagoras)}$$

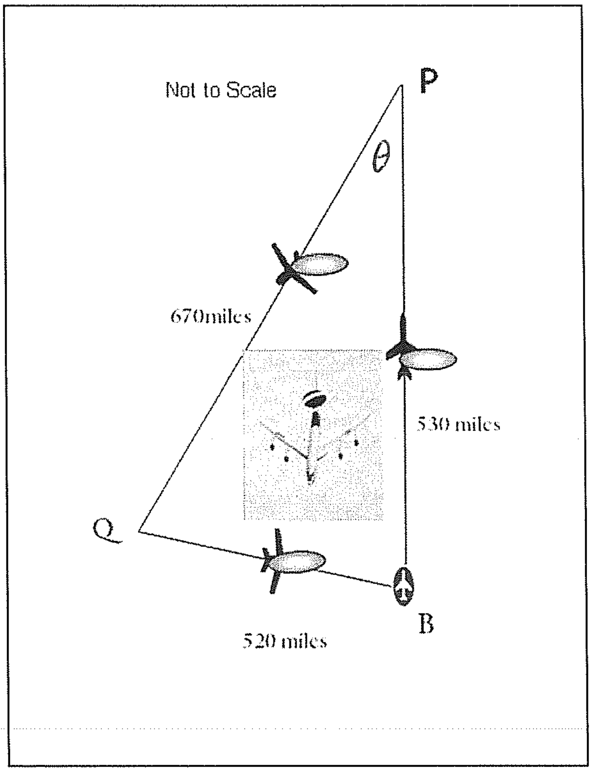
In $\triangle$ FDB

$$|FB|^2 = |DB|^2 + |FD|^2 \quad \text{(Pythagoras)}$$
$$= |CD|^2 + |FD|^2 \quad (|CD| = |DB|)$$

$$\therefore |FB|^2 = |FC|^2$$

Hence the 3 corresponding sides in each $\triangle$ are equal. $\Rightarrow$ congruent

Q4 (LCOL)



An aircraft takes off from Baldonnel **(B)** on a navigation exercise. It flies 530 miles directly North to a point **(P)** as shown. It then turns and flies directly to a point **(Q),** 670 miles away**.** Finally it flies directly back to base, a distance of 520 miles.

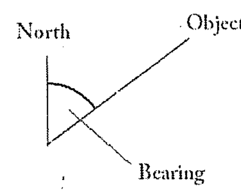
a) Calculate the angle QPB.

$520^2 = 530^2 + 670^2 - 2(530)(670)\cos\theta$

$\cos\theta = \dfrac{530^2 + 670^2 - 520^2}{2(530)(670)} = 0.64686$

$\Rightarrow \theta = 49.7^\circ$

b) If the bearing is defined as the clockwise angle measured from the North direction, calculate the bearing of Q from P.



$49.7^\circ + 180^\circ = 229.7^\circ$

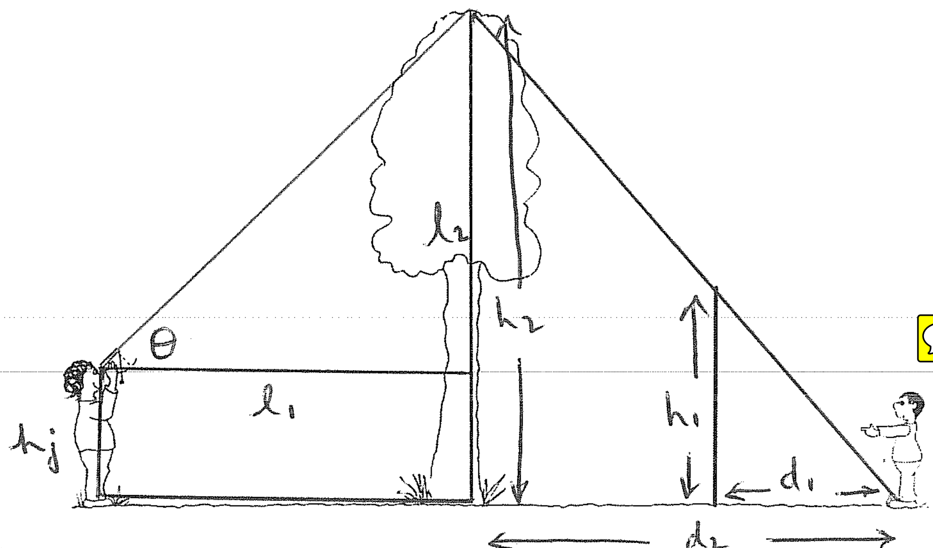
$\Rightarrow$ Bearing is 229.7° from P

Q5 (JCOL) Jane and Stephen want to estimate the height of a tall tree which is vertical and stands on horizontal ground.

Jane has a **clinometer** and Stephen has a 100m measuring tape and a large **stake**.

Explain, using diagrams and your own reasonable measurements, how each of them can make an estimate of the tree's height.

Account for any inaccuracies that might occur and suggest how you could minimise these inaccuracies.



Jane measures the angle of elevation θ and the length l_1 to the tree.

She calculates l_2 from

$$\tan\theta = l_2/l_1$$

and then calculates the tree height by adding her own height (h_j)

$$h_2 = l_2 + h_j$$

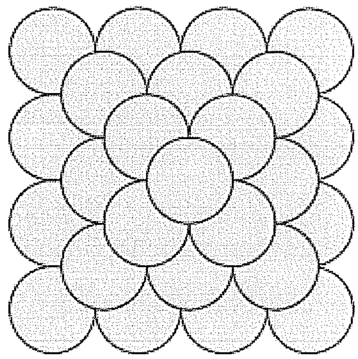
Stephen measures distances d_1, d_2 and the height of the stake h_1 (when the top of the stake coincides with the top of the tree when viewed from a point on the ground where his feet are).

Then $\frac{d_1}{h_1} = \frac{d_2}{h_2}$

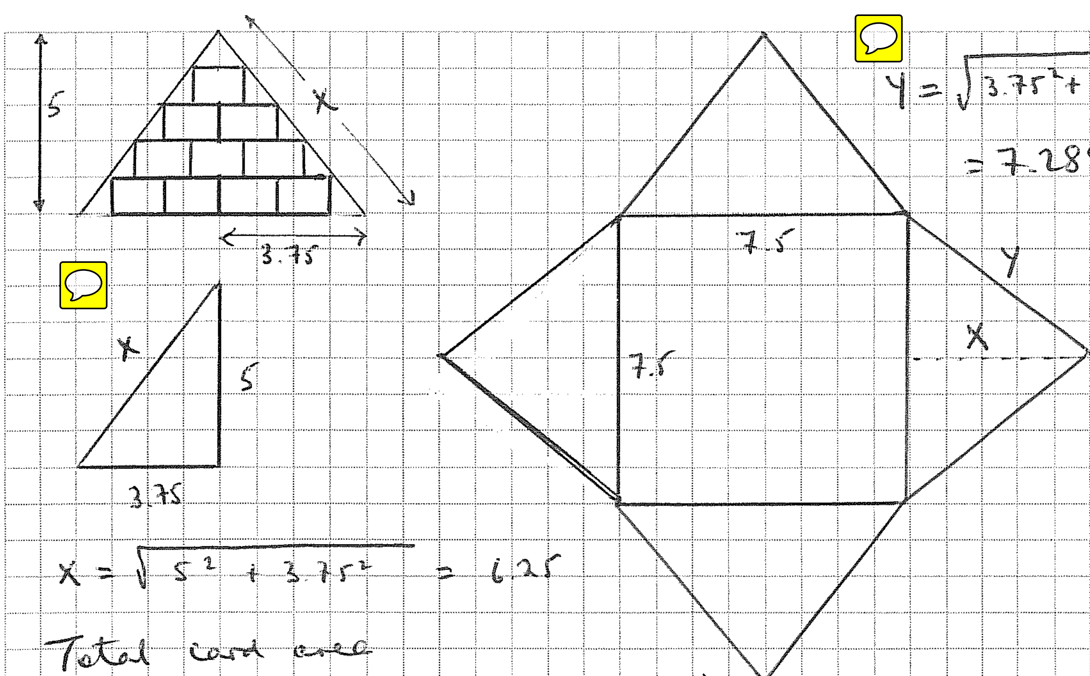
$$\Rightarrow h_2 = \frac{d_2 \times h_1}{d_1}$$

Q6 (LCHL) Joan was asked to design a box for 30 chocolates. Each chocolate is cylindrical with diameter **1.5 cm** and height **1 cm**. She decided the box should be made from card and in the shape of a **square-based pyramid**.

Inside the box the chocolates would be stacked in 4 layers and would look like this when viewed from above.



By sketching a net of the box, without including any joining flaps, calculate how much card the design will need. Show all measurements on your sketch.



$y = \sqrt{3.75^2 + 6.25^2}$

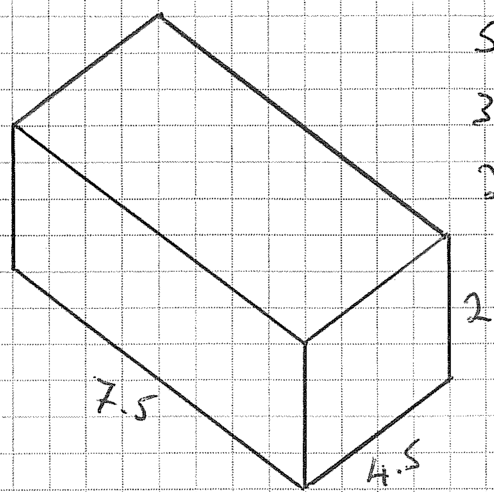
$= 7.289$

$x = \sqrt{5^2 + 3.75^2} = 6.25$

Total card area

$= (7.5 \times 7.5) + 4\left(\frac{1}{2} \times 7.5 \times 6.25\right) = 150 \text{ cm}^2$

Sarah claims that it would need less card if the 30 chocolates were stacked in a closed rectangular box that would hold two layers, each 5 chocolates long by 3 chocolates wide. By calculating the surface area of such a box, decide whether or not the claim is accurate.



5 chocs long ⇒ 7.5 cm

3 chocs wide ⇒ 4.5 cm

2 chocs high ⇒ 2 cm

$(7.5 \times 4.5) \times 2 = 67.5$

$(7.5 \times 2) \times 2 = 30$

$(4.5 \times 2) \times 2 = 18$

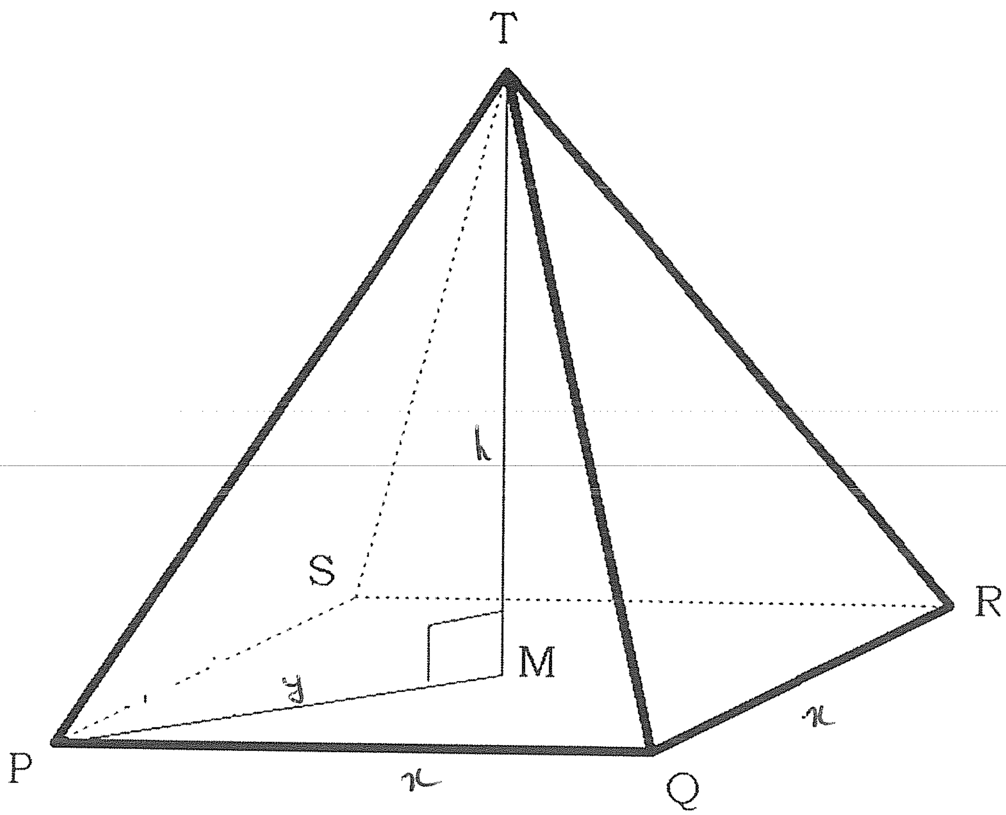
115.5

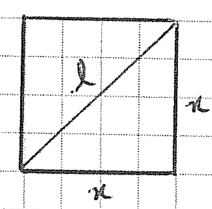
Since $115.5\ cm^2 < 150\ cm^2$

Sarah's claim is accurate

Q7 (LCHL) Show that the length of steel tubing required to make a sculpture in the shape of a square-based pyramid, as illustrated below, is given by the equation:

Length of tubing = $4\sqrt{h^2 + \frac{x^2}{2}} + 4x$, where $x =$ base length and $h =$ vertical height





$PT^2 = y^2 + h^2$

$= \frac{1}{4}x^2(2) + h^2$

$PT^2 = \frac{1}{2}x^2 + h^2$

$\Rightarrow PT = \sqrt{\frac{1}{2}x^2 + h^2}$

$l^2 = x^2 + x^2$

$l = x\sqrt{2}$

$\therefore y = \frac{1}{2}x\sqrt{2}$

Total length $= (4 \times PT) + 4x$

$= 4\sqrt{\frac{1}{2}x^2 + h^2} + 4x$

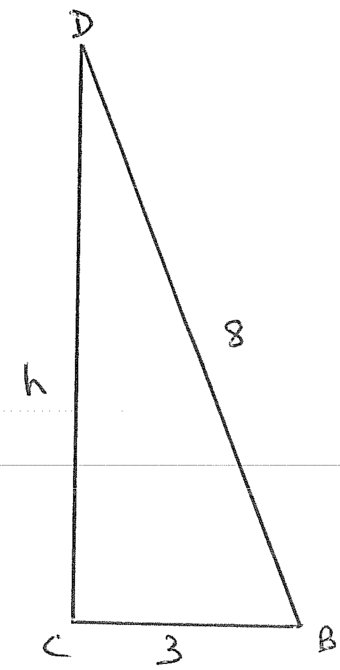
Q8 (LCHL) The diagram shows the structure of a climbing frame. The structure is in the shape of a pyramid on a hexagonal (6 equal sides) base.

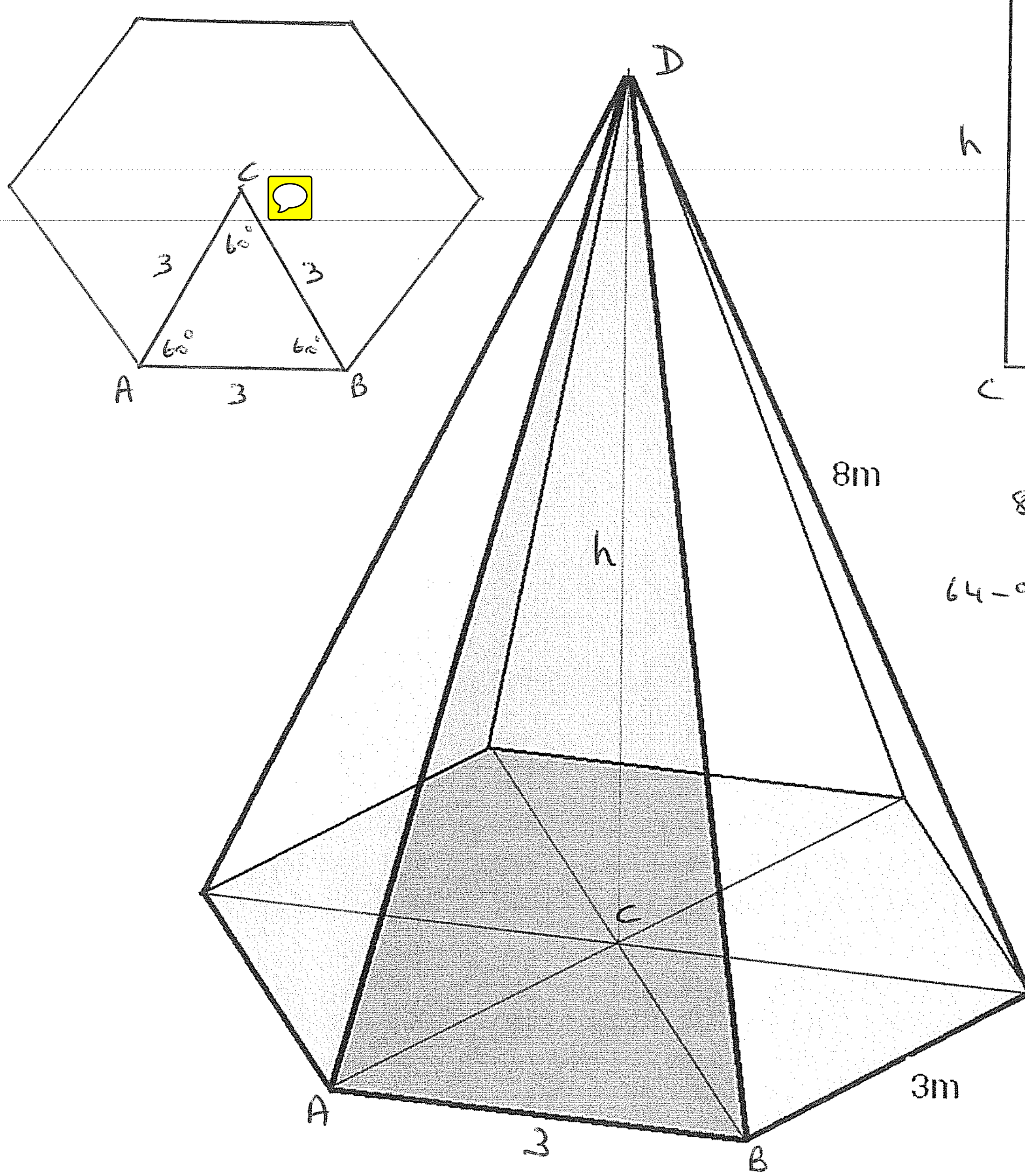
The length of each sloping edge is 8m and the pyramid's base is a regular hexagon with sides of length 3m as shown in the diagram.

The regulations state that the frame cannot exceed 7.5m in height.

Will these dimensions comply with regulations?

Support your answer with calculations.





$8^2 = h^2 + 3^2$

$64 - 9 = h^2$

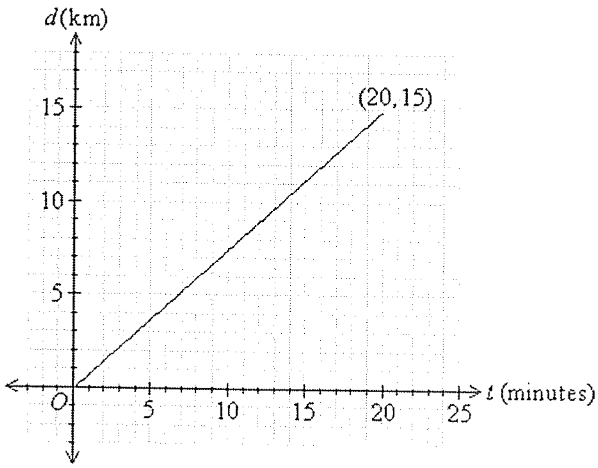
$h = \sqrt{55}$

$= 7.416$

Since $7.416 < 7.5$

it complies with Regulations

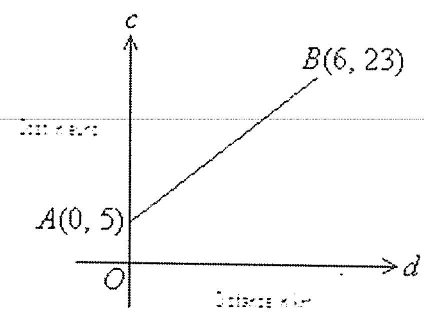
Q9 (LCFL) (a) A cyclist travels for 20 minutes at a constant speed and covers a distance of 15 km, as shown in the diagram. Find the slope of the line and describe its meaning.



$\underline{\text{Slope}} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{15-0}{20-0} = 0.75$

It means that the cyclist covers 0.75 km per minute.

(b) The cost of transporting documents by courier can be represented by the following straight line



(i) What does each point represent?

A is the flat or standing charge for any distance = €5

B shows that it costs €23 for a 6 km journey

(ii) Calculate the slope. What does this represent?

$\text{Slope} = \dfrac{23-5}{6-0} = 3$

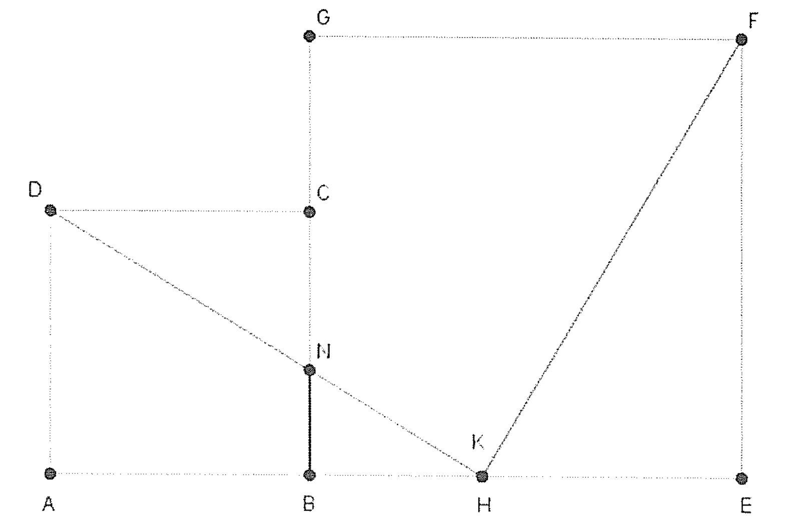
This means it costs €3 for every kilometre on top of the standing charge.

Total cost is €5 plus €3 per km.

Q10 (JCHL) – Geometry

The diagram (Fig. 1) shows two square tiles, ABCD and BEFG placed alongside each other. The point H is chosen along the side BE so that $|HE| = |AB|$.

Fig. 1



(i) Prove that the triangles DAH and HEF are congruent.

$|DA| = |HE|$ (both $= |AB|$)

$|AH| = |AB| + |BH| = |HE| + |BH| = |EF|$ (sides of the square)

$\angle A = \angle E = 90°$

SAS $\Rightarrow$ $\triangle DAH$, $\triangle HEF$ are congruent

(ii) Prove that $\angle$ DHF is a right angle

$|\angle AHD| = |\angle HFE|$ from the $\triangle$s DAH and HEF

$|\angle FHE| + |\angle HFE| = 90°$ Since $\angle E = 90°$

$\therefore$ $|\angle FHE| + |\angle AHD| = 90°$

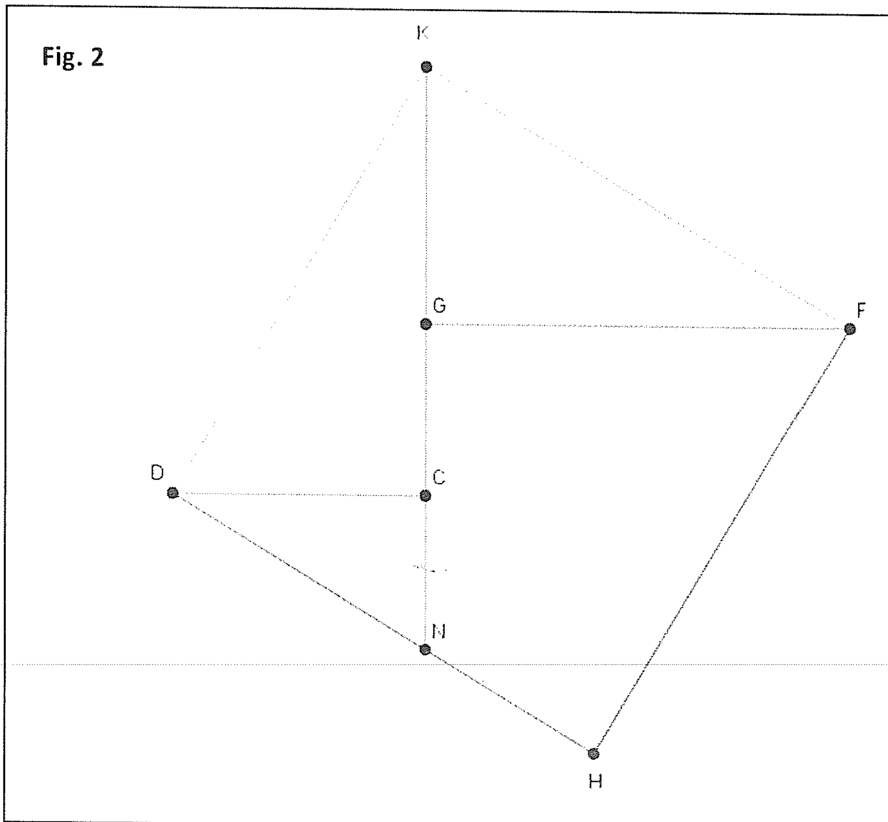
Since AHE is a straight line $\rightarrow$ Remaining angle $= 90°$

$\therefore$ $|\angle DHF| = 90°$

The square tiles are cut along the lines DH and HF as shown and the pieces are moved so that ΔHEF lies in the position DCK and ΔDAH lies in the position KGF (see Fig. 2).

(iii) Prove that the new figure formed, DHFK, is a square.

Fig. 2



$|DH| = |HF| = |FK| = |KD|$ (sides of the congruent triangles)

the new side KD came from FH
and side KF " " DH

The angle DHF is 90° from proof (ii) above

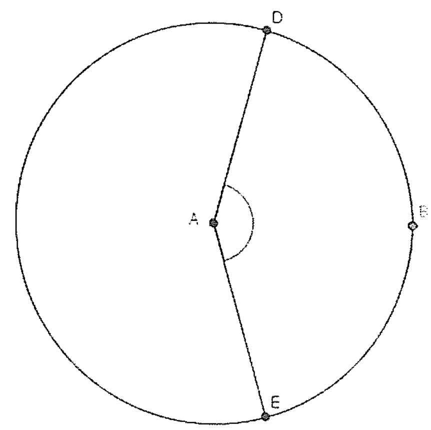
Hence DHFK has 4 equal sides and is right angled.

So it is a square.

Q11 (LCOL)

Fig. 1 shows a circle with centre A.
(a) If the $|\angle DAE| = 150°$ and $|AD| = 12$ cm, find the length of each arc.

Fig 1



Small arc

$$2\pi(12) \times \frac{150}{360}$$

$$= 10\pi \text{ cm}$$

Large arc

$$2\pi(12) \times \frac{210}{360}$$

$$= 14\pi$$

(b) Fig. 2 shows a belt-driven pulley system with pulleys of radii 12 cm and 5 cm respectively. The centres of the pulleys are 24 cm apart.

(i) Find the measure of the angle DAF to the nearest degree.

(ii) Find the total length of the belt needed for this pulley system.

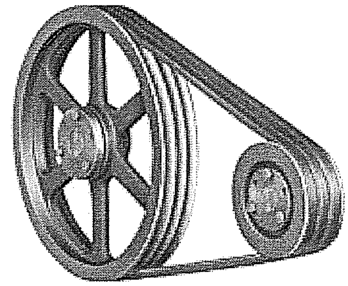
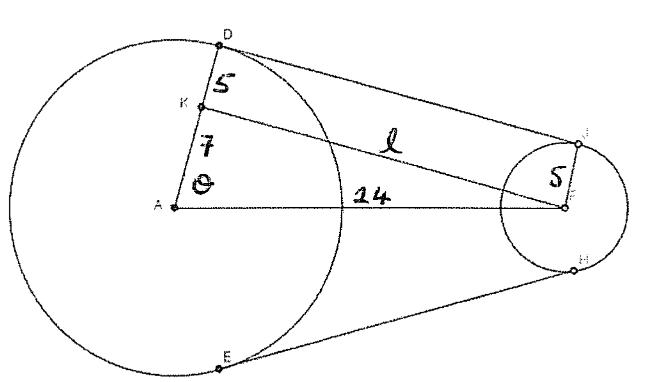


Fig 2



$$l^2 + 7^2 = 24^2$$

$$l^2 = 24^2 - 7^2 = 527$$

$$l = 22.96$$

$$\Rightarrow |DI| = 22.96 \text{ cm}$$

$$\begin{array}{r} 360 \\ 146 \\ \hline 214 \end{array}$$

$$\cos\theta = \frac{7}{24} \Rightarrow \theta = 73^\circ \longrightarrow 2\theta = 146^\circ$$

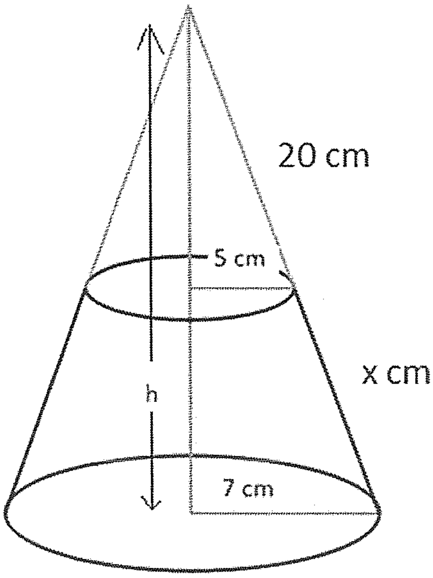
large pulley; large arc $\rightarrow 2\pi(12) \times \frac{214}{360} = 44.82$ cm

small pulley, small arc $\rightarrow 2\pi(5) \times \frac{146}{360} = 12.74$

$\therefore$ belt length $= (2 \times 22.96) + 44.82 + 12.74$

$= 103.48$ cm

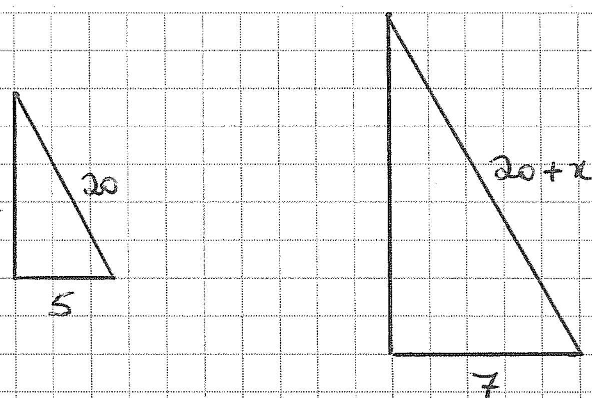
Q12 (LCHL) Geometry/Trigonometry



The figure to the left shows a cone from which a lampshade is to be made.

The smaller cone with base radius 5 cm is cut off the top to form the lampshade.

(i) Calculate the slant height of the lampshade (marked x cm in the diagram).



$\Rightarrow$ Similar triangles

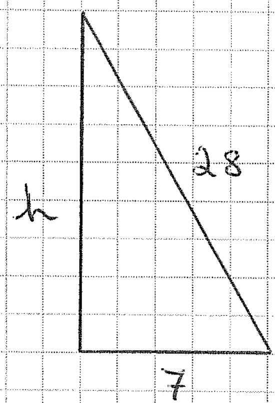
$$\frac{5}{7} = \frac{20}{20+x}$$

$$100 + 5x = 140$$

$$5x = 40$$

$$x = 8 \text{ cm}$$

(ii) Hence calculate the value of h, correct to one decimal place.



$$28^2 = h^2 + 7^2$$

$$28^2 - 7^2 = h^2$$

$$27.11 = h$$

$$h = 27.1 \text{ cm}$$

(iii) Calculate the surface area of the lampshade correct to two decimal places.

Large cone: area $= \pi r l$

$= \pi(7)28$

$= 196\pi$

Small cone:

area $= \pi(5)(20)$

$= 100\pi$

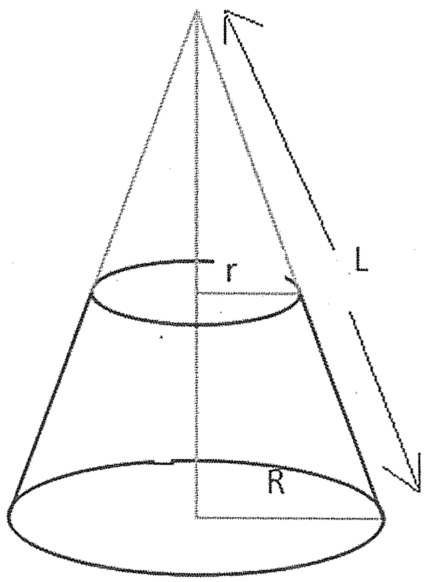
lampshade area $= 196\pi - 100\pi$

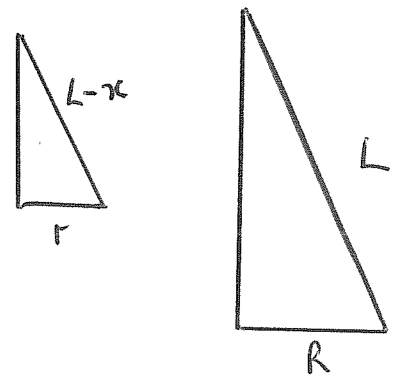
$= 96\pi$

$= 301.5928$

<u>Ans</u> 301.59 cm^2

(iv) By letting r = the base radius of the small cone, R= base radius of the large cone and L = the slant length of the large cone, show that the curved surface area of the lampshade is given by $A = \frac{\pi L(R^2 - r^2)}{R}$





$$\frac{r}{R} = \frac{L-x}{L}$$

$$rL = RL - Rx$$

$$Rx = RL - rL$$

$$x = \frac{L(R-r)}{R}$$

$\therefore$ slant height of small cone

$$= L - \frac{L(R-r)}{R}$$

$$= \frac{RL - RL + rL}{R} = \frac{rL}{R}$$

Large cone area $= \pi R L$

Small cone area $= \pi r\left(\frac{rL}{R}\right) = \frac{\pi r^2 L}{R}$

$\therefore$ area of lampshade $= \pi R L - \frac{\pi r^2 L}{R}$

$$= \frac{\pi R^2 L - \pi r^2 L}{R}$$

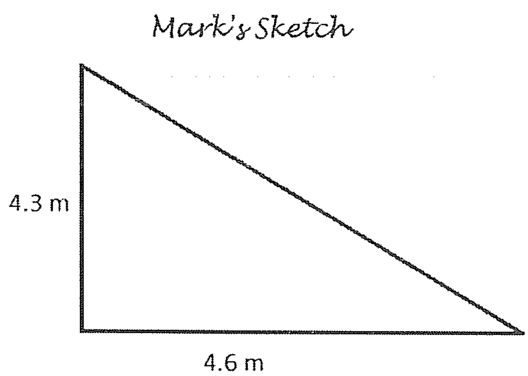
$$= \frac{\pi L(R^2 - r^2)}{R}$$

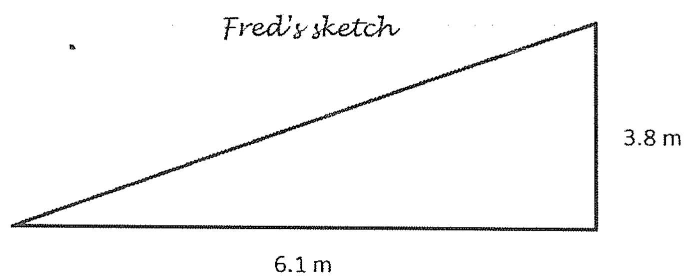
Q13 (JCHL) Trigonometry

Mark and Fred are designing a skateboard ramp. In *Skate Monthly*, they read the following advice

"to make a good skateboarding ramp, you need to find the balance between being too steep and too low. If it's too low, all you end up doing is getting a few inches off the ground, wiping out and looking silly. If it's too steep, you get halfway up, come back down, fall and look even sillier. It's best to keep the ramp angle with the ground between 30 and 45 degrees".

Here are Mark's and Fred's sketches:





(i) Use mathematics to decide which ramp is steeper (that is, has the greater slope).

<u>Slopes</u>

<u>Mark</u>: $= \frac{4.3}{4.6} = 0.935$

<u>Fred</u>: $= \frac{3.8}{6.1} = 0.623$

$\therefore$ Mark's Ramp is steeper than Fred's.

(ii) Which ramp would ensure that the skater travels a greater distance on the ramp?

Mark $l^2 = 4.3^2 + 4.6^2$

$= 39.65$

$\Rightarrow l = 6.3\text{ m}$

Fred $l^2 = 3.8^2 + 6.1^2$

$= 51.65$

$\Rightarrow l = 7.2\text{ m}$

$\therefore$ Fred's ramp will have a greater distance on the ramp.

(iii) Does the angle which each ramp makes with the ground comply with the advice about angles given in *Skate Monthly*? Use mathematics to justify your conclusion.

Mark $\tan^{-1}(0.935) = 43^\circ$

Fred $\tan^{-1}(0.623) = 32^\circ$

Both angles are between 30° and 45° so they comply with the advice given.