Community of Practice:

Session Title: Examining student work

Charmaine, Leona, Anna, Hugh and Ciaran were investigating the commutative property for subtraction. The extract below is from their discussion of the following task.

Task: If you switch around the numbers in a subtraction problem will you get the same answer?

With your colleagues think about these students' ideas
- What prior understanding have the learners brought to this task?
- What problem solving strategies were they using?
- What conjecture did they form?
- How did they represent it that will show it is always true?
- What mathematics are the students learning from engaging in this task?

Now think about the learners in *your* school

- Are your learners ready for such a task?
- Are they as familiar with using representation to show their ides as these students seem to be?

Think about the teacher
- How did the teacher progress the learning?

Together think about how you would manage this task in **your** classrooms
- What students would you do it with?
- How would you introduce it?
- What difficulties do you think your students might have with the task?
- Are your students familiar with making and testing conjectures?
- What do your students understand about mathematical proof?

Charmaine: 5-2 = 3 ..You start here and go back 2 you land on 3

But if you do 2-5 you start here and go back 2 to 0 and then another 3 to -3.
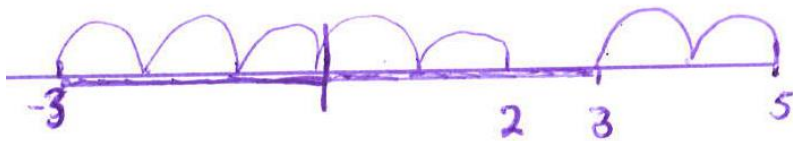
Anna: ye we did it with other numbers too and it's always the same thing

Leona: We used money we thought that if you have some money €10 say and you pay for something that is €7 then you have €3 left so 10-7 =3 ..but if you have €7 and you pay for something that is €10 then you owe €3 cos you'll have to borrow it to pay the whole 10 off….so 7 -10 = -3

Teacher: Can anyone see any pattern when you switch the numbers round in a subtraction problem?

Leona: One is positive and one is negative but the number is the same

Anna: Ye.. that's cos they are both the same distance from 0

Hugh: If the larger number comes first the answer is positive. if the smaller number comes first the answer is negative.

Teacher: Why does this happen? Will it happen for all numbers? Can you explain why?

Ciaran: I think if you use the number line it will prove it ….just get a strip of paper lets say it's to represent 10 then cut a smaller one to represent 7 then place them on the number line like this .
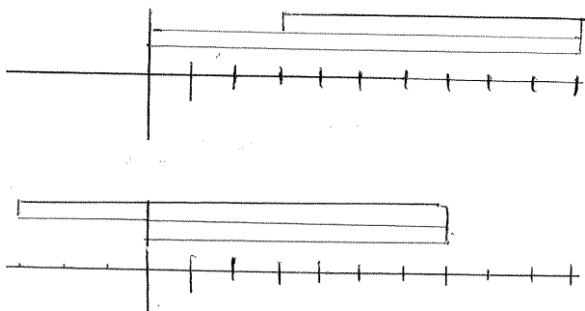
Now to show the switch around just take the same two pieces of paper turn them upside down and push them down the line to show that the answer is the same distance on the other side of 0.These pieces can be any length so it will work for any number

The dialogues that follow are further extracts from the lesson. The teacher is focussing on generalisation in the next extract.

Laura: x- y = the difference …so lets say x –y = z

Hugh: ye so y – x = -z

Ciaran: What tells us which is the larger number?

Laura: eh..nothing

Teacher; So what happens if x isn't the larger number?

Leona; Then z is the negative number

Hugh: Well then what is -z?...that's confusing

- What is the issue the learners are struggling with here?
- How would you help your learners resolve this issue?

Teacher: What can you say about z + -z ?

Ciaran: It equals zero because if you give something… +z and then take it away… –z you really have nothing

Teacher: So z + -z = 0 ?......when this happens we say that z =-z are *additive inverses* that is because when you add them together the overall effect is that nothing has changed

Hugh: Oh ye ..its like when you are doing a big sum on the calculator and you press + something instead of – something you don't have to start again cos you just minus the thing and you are back to where you started.

Teacher: Lets look at this way of writing our idea (x –y) + (y-x) = 0 is this the same idea?

Anna: Mmmm I think we need to try this

Discuss the teacher's move in the discussion above

Here is a piece of work by the same students, this time they were investigating the commutative property of division.

each person gets 4 bars

For 4 bars

each person gets $\frac{1}{4}$ a bar

$16 \div 4 = 4$ $4 \div 16 = \frac{1}{4}$

With your colleagues discuss

- How could you progress the learning?
- How could the previous work help these learners make sense of the commutative property of division
- Is their potential for other learning here?
- How would you engage your students in investigations about the commutative property?
- Would you have everyone investigate each operation? Or would you divide the class into groups and give each group a different operation?
- How does the investigation of commutativity support understanding of the additive inverse? The multiplicative inverse? The reciprocal?

Try some ideas in your class and bring some student work to discuss at the next meeting

Problem solving reminder: If you are going to use these tasks remember, answers are important but what is more important is the mathematics students can learn from engaging with the tasks.