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Mathematics Resources for Students

Junior Certificate – Strand 2

Geometry and Trigonometry

INTRODUCTION

This booklet is designed to supplement the work you have done in Junior Cert geometry with your teacher. There are activities included for use as homework or in school. The activities will help you to understand more about the concepts you are learning in geometry. Some of the activities have spaces for you to fill in, while others will require you to use drawing instruments and paper of your own. You may not need or be able to complete all activities; your teacher will direct you to activities and/or questions that are suitable.

You should note that Ordinary level material is a subset of Higher level and that HL students can expect to be tested on material from the Ordinary level course, but at a greater degree of difficulty. The sequence in which the sections/topics are presented here is not significant. You may be studying these in a different order, or dipping in and out of various sections over the course of your study and/or revision.

In the first topic (synthetic geometry) it is important that you understand the approach taken. Although only HL students are required to present the proof of some theorems, all students are expected to follow the logic and deduction used in these theorems. This type of understanding is required when solving problems such as those given in the section headed 'other questions to consider'.

Each activity or question you complete should be kept in a folder for reference and revision at a later date. A modified version of this booklet and syllabus documents are available at www.ncca.ie/projectmaths and other related materials at www.projectmaths.ie

GEOMETRY 1

SYLLABUS TOPIC: SYNTHETIC GEOMETRY

LEARNING OUTCOME

As a result of completing the activities in this section you will be able to

- complete a number of constructions
- use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies
- investigate theorems and solve problems.

HL learners will

- extend their understanding of geometry through the use of formal proof for certain theorems.

INTRODUCTION

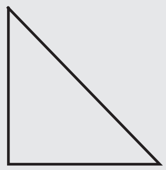
The activities described below and the questions that follow allow you to deepen your understanding of concepts in geometry as well as solve problems using these concepts and their applications.

Activity 1.1 JCFL

The following activity is to help you understand the properties of different triangles. Revise the ideas of acute angle, obtuse angle and right angle. You may want to discuss with someone else the difference between 'could be true' and 'could never be true' before you start.

Read each statement about triangles.

Decide if the statement could ever be true, and tick the correct column in the table. Then draw a diagram of the triangle in the box provided. The first one has been done for you.

Statement	Could be true	Could never be true	Diagram
Triangles can have one right angle and two acute angles	✓		
Triangles can have two right angles.			
Triangles can have one obtuse angle and two acute angles			
Triangles can have two obtuse angles and one acute angle			

Activity 1.2 OL and HL

Geometry has a language all of its own. You are not required to learn all of the vocabulary associated with it but you do need an understanding of the different terms. Fill in the spaces in the activity items below to assess your understanding of what the terms mean. Your teacher will guide you through the differences and the uses of terms in geometry.

What do you understand by the word **line**? Write your answer in one sentence.

What do you understand by the word **triangle**? Write your answer in one sentence.

What do you understand by the word **angle**? Write your answer in one sentence.

What do you understand by the word **definition**? Write your answer in one sentence.

What do you understand by the word **theorem**? Write your answer in one sentence.

What do you understand by the word **axiom**? Write your answer in one sentence.

What do you understand by the word **corollary**? Write your answer in one sentence.

What do you understand by the phrase **geometrical proof**? Write your answer in one sentence.

What do you understand by the word **converse**? Write your answer in one sentence.

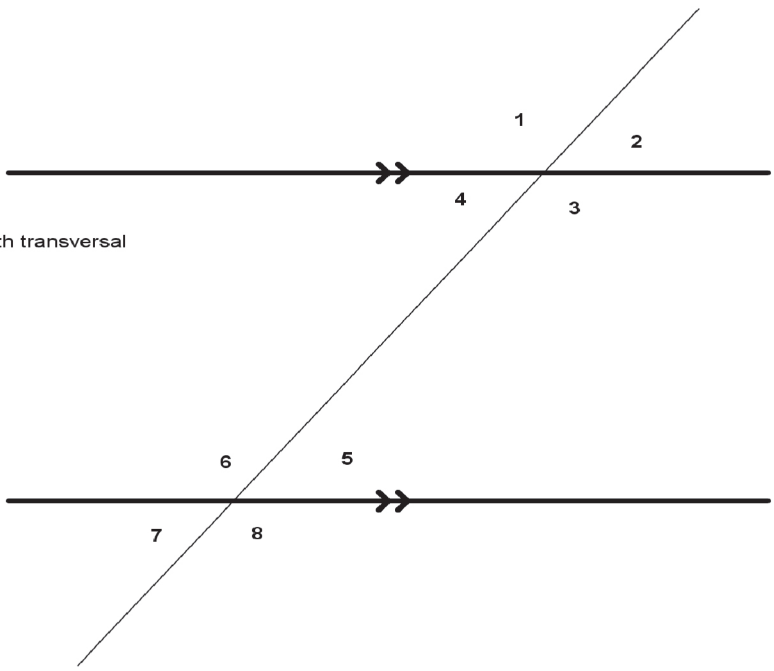
Make out a list of any terms in geometry where you are not sure of their meaning.

1.
2.
3.
4.
5.
6.

Activity 1.3 OL and HL

In geometry we often have to find angles or the lengths of shapes and we can use known 'facts' to establish these or to prove that particular geometrical statements are true. The use of deductive reasoning is important and our ability to piece the clues together makes it easier to do this. This reasoning comes from our ability to build on what we know to be true in order to discover something new.

When examining a pair of parallel lines with a transversal cutting across them, we can see lots of different angles which are equal and we can classify these in different ways.



Parallel lines with transversal

Using the numbers shown on the diagram, list pairs of angles which are equal in measure, using one of the following terms to justify each pair: vertically opposite angles, alternate angles, corresponding angles, supplementary angles.

Use the notation $|\angle 7|$ to mean the number of degrees in angle 7.

a. ___ and ___ are equal because they are ___

b. ___ and ___ are equal because they are ___

c. ___ and ___ are equal because they are ___

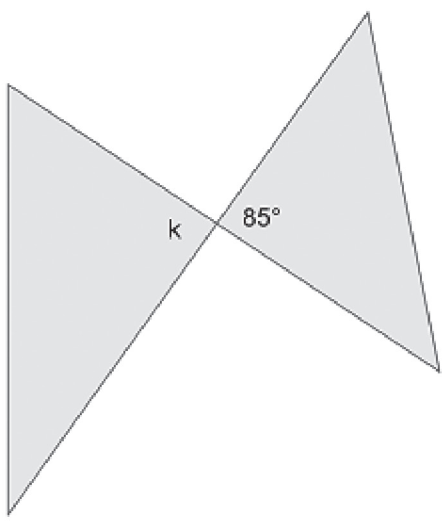
d. ___ and ___ are equal because they are ___

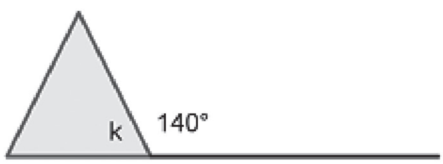
e. ___ and ___ are equal because they are ___

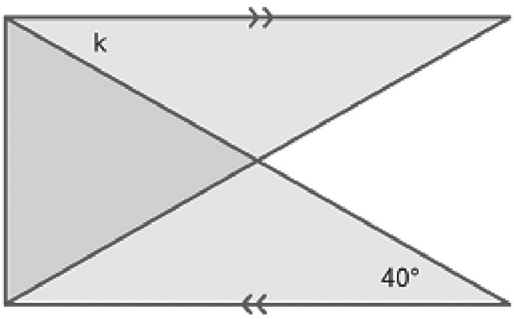
f. ___ and ___ are equal because they are ___

Are there more than six pairs?

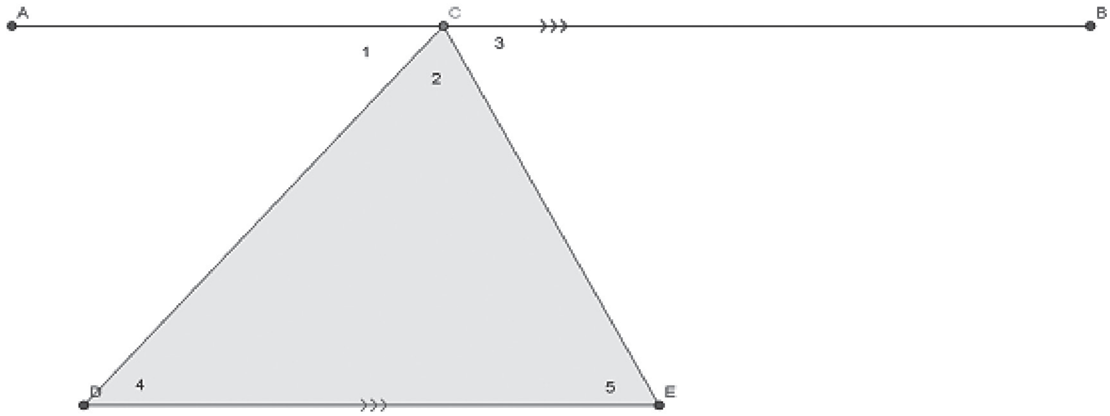
Q. 1 Based on what you have learned above, can you now find the value of the angle k in each of the following diagrams?







Q. 2 Now consider the following diagram and answer the questions below it.



a. What is the sum of the angles in a straight line?

b. Name three angles in the diagram above that make a straight line.

c. Name two pairs of alternate angles.

For HL

d. Can you draw any conclusions from the sum of the angles in the triangle in the diagram?

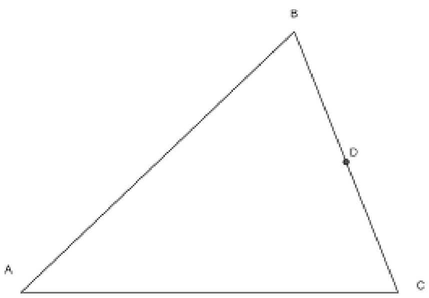
e. If the point C was in a different place on the line segment [AB], would it make any difference?

From this example, what, if anything, can you say generally about all triangles?
Could you show that this is true?

Other questions to consider

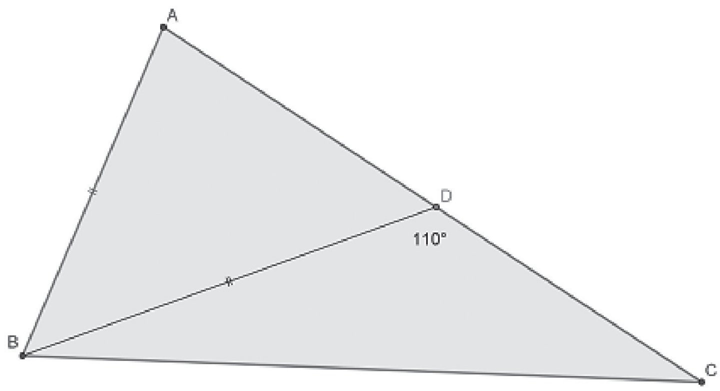
Q. 3 OL and HL

In the diagram |AB|= |AC|, |BD|= |DC|. Show that D is equidistant from AB and AC.



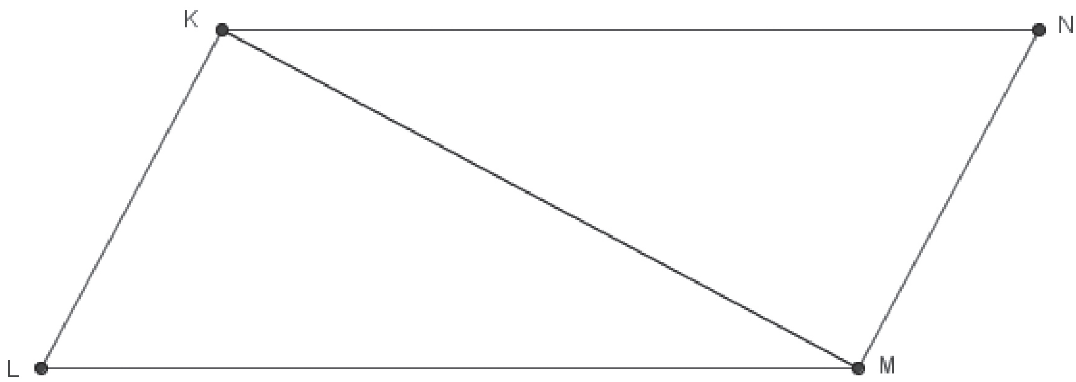
Q. 4 OL and HL

If |BA |= |BD| and |DB |=|DC |, Find the value of | ∠ ABC|.|



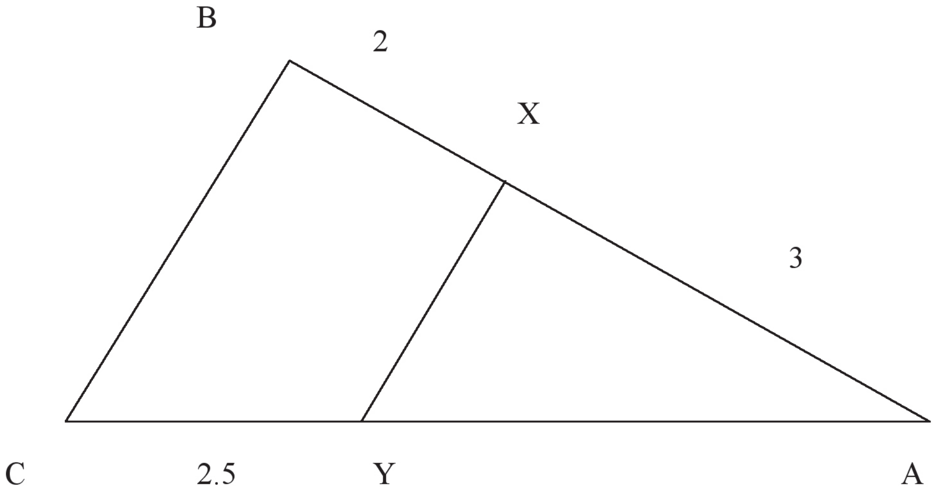
Q. 5 OL and HL

In the diagram KLMN is a parallelogram and KM is perpendicular to MN. If |KM|= 7.5 cm and |LM|= 8.5 cm, find the area of the parallelogram.



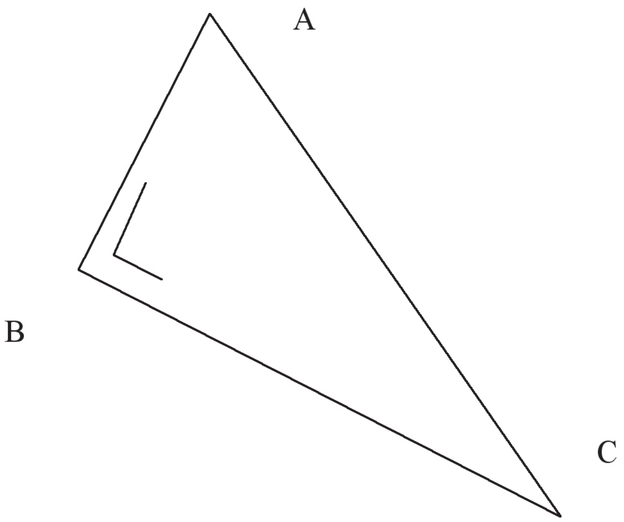
Q. 6 HL

In Δ ABC, XY || BC. Find |AY|



Q. 7 OL and HL

In ΔABC, $|\angle ABC| = 90°$, $|AB| = 7 -$ a number, and $|AC| = 8 +$ the same number. Find $|BC|^2$.

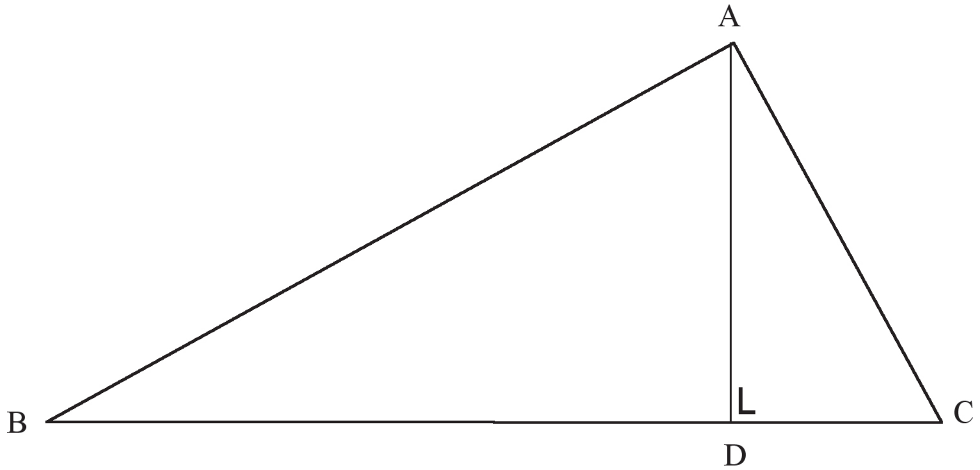


Q. 8 OL and HL

In the diagram $|\angle BAC| = 90° = |\angle ADC|$

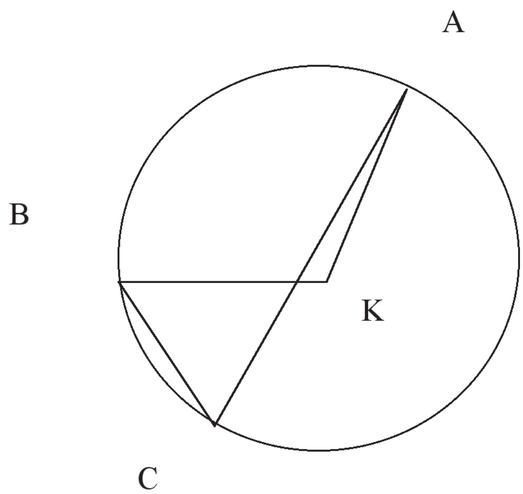
Show that ΔABD and ΔABC are equiangular and that ΔADC and ΔABC are equiangular.

From this, can you show that $|AB|^2 + |AC|^2 = |BC|^2$



Q. 9 HL

The centre of the circle is K. $|\angle AKB| = 100°$. Find $|\angle ACB|$.



Activity 1.4

You are required to carry out a number of constructions and the best way to learn is by doing. If you are studying Technical Graphics, ask your teacher whether there are different ways of doing a given construction that might be easier for you to remember. Trying to learn them off for a test is more difficult than learning them by completing exercises like the ones that follow.

Q. 1 Divide the shape below into four equal parts using only a compass, ruler and pencil.

Q. 2 The following six constructions involve drawing triangles. Try to construct them, but note that not all of them are possible. If it is not possible to construct the triangle, briefly explain why. Also, note if more than one solution is possible.

i. A triangle with sides of length 3 cm, 6 cm and 12 cm.

ii. A triangle with sides of 10 cm each. What kind of a triangle is this? Using this triangle, can you find a way of making two triangles which have a right angle, a side of 10 cm and a side of 5 cm? What do you notice about these two triangles?

iii. A triangle with one side of 4 cm and two angles of 50° each.

iv. A triangle with angles of 55°, 65°and 65°.

v. A right-angled triangle which has two sides the same length. Label the triangle and measure the angles with a protractor. Record their values on the diagram.

vi. A right angled triangle with one side twice as long as the other. Label the triangle and measure the angles with a protractor. Measure the third side as accurately as you can. Record all the measurements on the diagram.

GEOMETRY 2

SYLLABUS TOPIC: TRANSFORMATION GEOMETRY

LEARNING OUTCOME

As a result of completing the activities in this section you will be able to

- locate axes and centres of symmetry
- recognise images, points and objects under transformations

INTRODUCTION

The activities described below and the questions that follow allow you to deepen your understanding of concepts in transformation geometry as well as solve problems using these concepts and their applications.

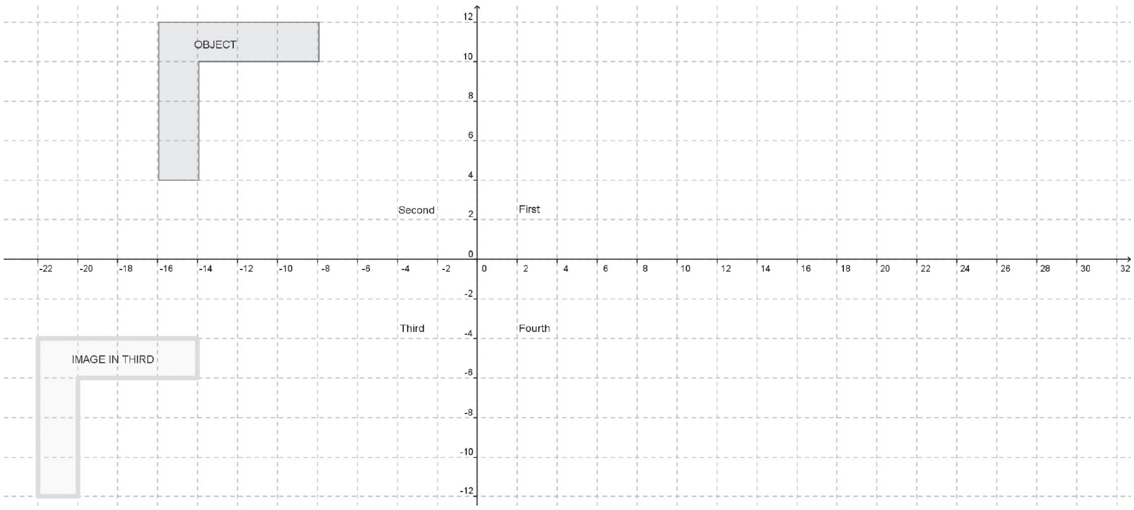
Translation in geometry is about movement of a point or object. The point or object can change its position or an object can change the direction in which it is facing. When we have located the object in a new position we call it the image, because it is like the original.

When we describe movement in the plane we can say

to the left or right, up or down;
to the north, south, east or west;
or, later, we can use co-ordinates to describe the location of the image of an object or point.

Activity 2.1

The object shown in the second quadrant of the plane has been translated into the third quadrant. Your task is to translate it, into the first quadrant and then into the fourth quadrant. After doing so, consider the questions that follow.



i. Does it matter where the image is in each quadrant?

ii. Have the lengths of the sides of the object changed?

iii. Has the area of the shape changed?

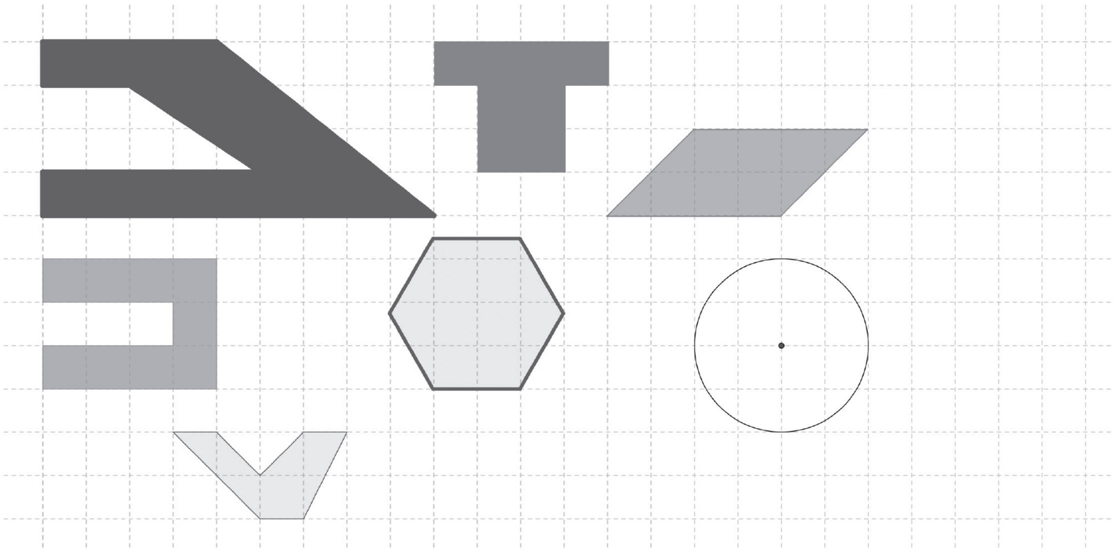
iv. Is there any difference between the image and the object?

v. Comment on the positions of the images?

vi. What can you conclude about the operation of translation on an object?

Activity 2.2

If we can fold one side of a shape exactly onto another it has a line of symmetry. Look at each of the shapes below and draw in the line of symmetry, if it has one. Some shapes may have more than one line of symmetry.



Which of the above are regular polygons? Can you draw any conclusions about the axes of symmetry of these?

Symmetry can occur in the natural world around us. Reflections of ourselves in the mirror or reflections of the sky and landscape in the water on a calm day are usually one of the first ways we experience symmetry.

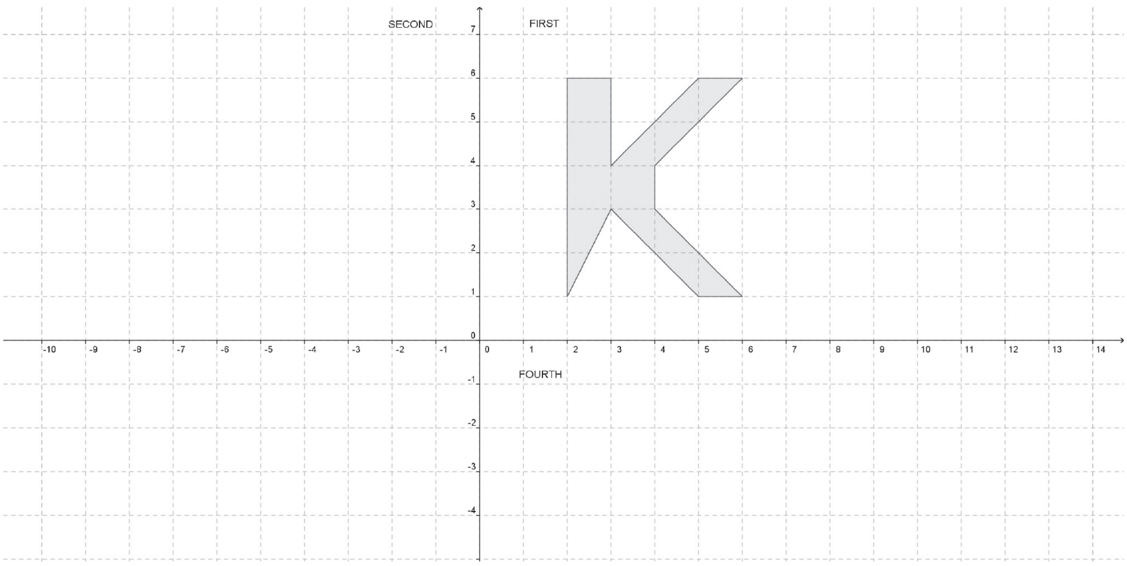
There are some things about symmetry that we should be aware of from looking at our own reflections in the mirror. The closer we stand to the mirror the closer the image appears to be. As you raise your right hand and wave, the image raises its left hand and waves. If you step to the left away from the mirror, the image steps away to its right. Try this yourself in front of a mirror and, if you study science, you can find out more about light, mirrors and images.

There are creatures in the insect world, such as butterflies and beetles, which have symmetrical bodies. You can see some of these at http://www.misterteacher.com/symmetry.html. Snakes may have symmetrical patterns on their skins and flowers can also show symmetry. Some sea creatures such as starfish and ammonites are also well known for their symmetrical and spiral shells.

Axial symmetry

Axial symmetry, or reflection in a line, is another type of transformation. Imagine the axis of symmetry as a line along which you can fold the plane.

If you print off a copy of this diagram you can carry out the activity by folding the sheet of paper along the axes to see where the images will appear. The object is in the first quadrant; find the images of the object in the second and fourth quadrants by reflection in the Y and X axes respectively.



What do you notice about the pointed part of the K as it is reflected into different quadrants?

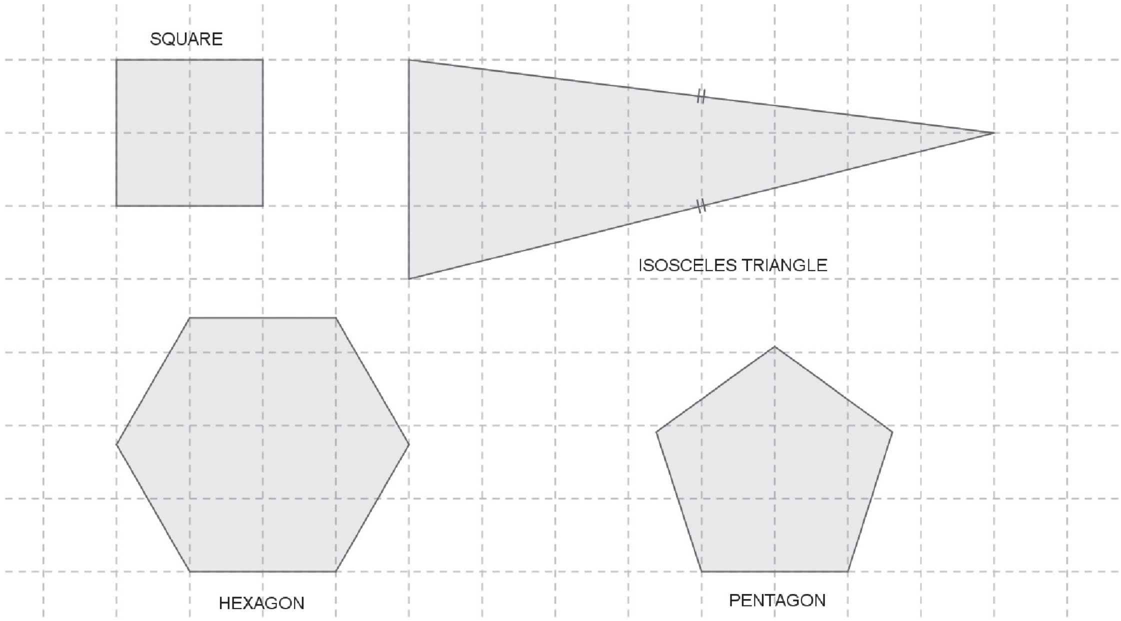
Is the image of K always facing in the same direction?

Describe the image in each quadrant using one short sentence. Focus on what is different from the original object.

Is it possible to find the image of the K in the third quadrant using axial symmetry?

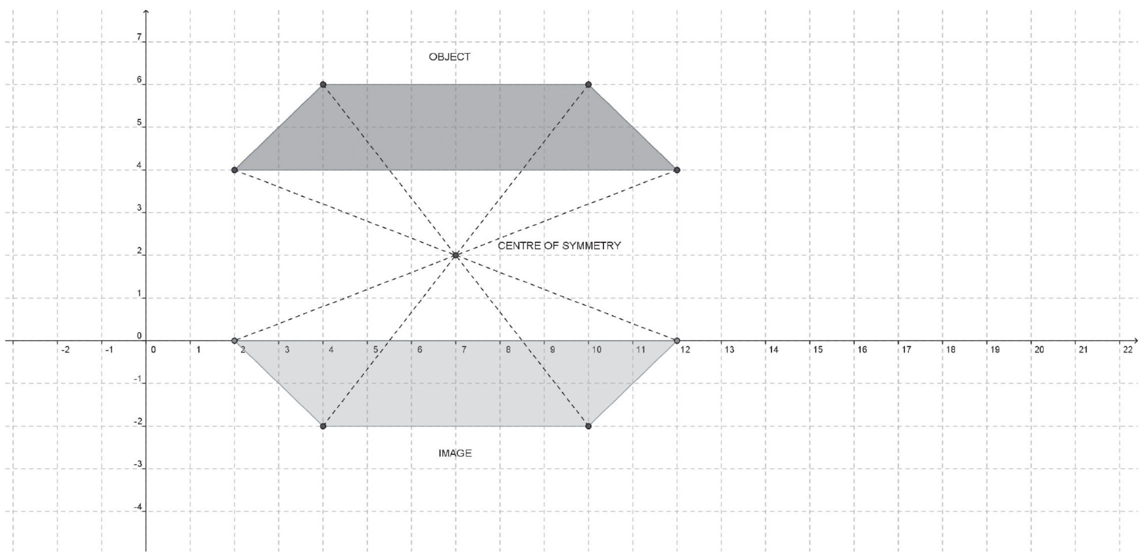
Activity 2.3

Central symmetry or reflection in a point is another type of transformation. Find the centre of symmetry of the following objects, if they have one.

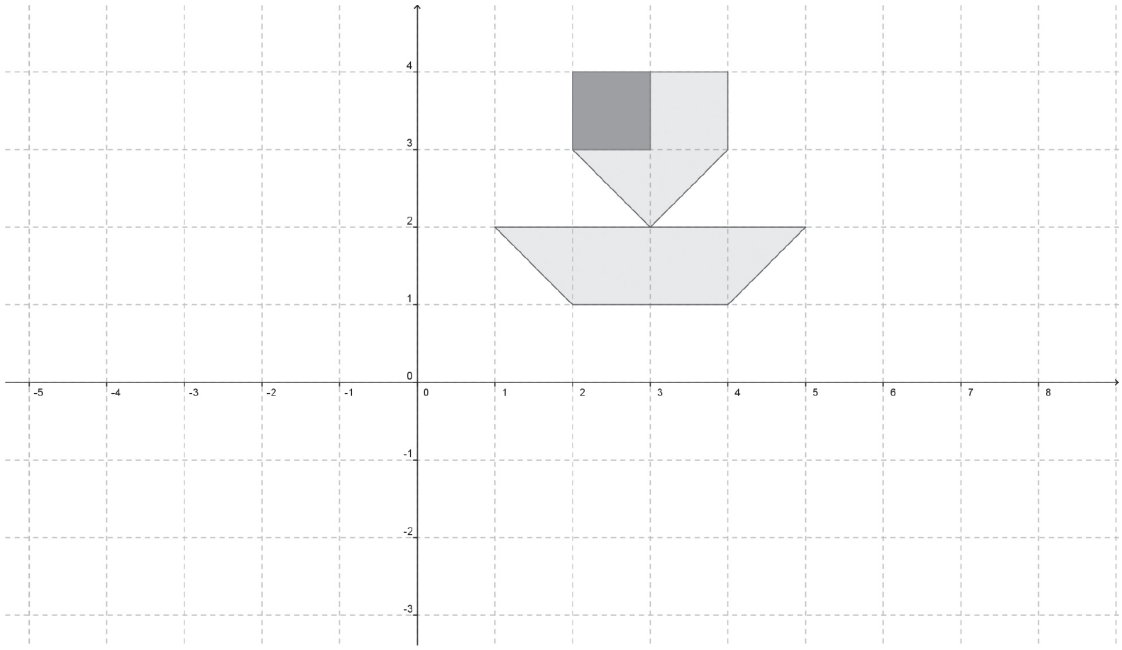


What can you say about the image and the object when the centre of symmetry is inside the shape?

When reflection takes place through a point outside of the object, as seen from the diagram below, we get a new image and we say that the image has been formed by central symmetry in the point (or by reflection in the point). Note that the point is the centre of symmetry for the combined shape (the object and its image).



Q. 1 Draw the image of the shape in the diagram by central symmetry through the origin (0,0) and then answer the questions that follow.



i. What happened to the object when it was reflected in the origin (0,0)? Explain this in one sentence.

ii. How would you describe the orientation of the image after reflection?

iii. If you reflected the image back through the origin, would you get the original shape?

Central symmetry can be done through any point on the plane. Pick a point which will reflect the object above

i. into the fourth quadrant

ii. into the second quadrant.

Briefly describe the image in each case.

GEOMETRY 3

SYLLABUS TOPIC: CO-ORDINATE GEOMETRY

LEARNING OUTCOME

As a result of completing the activities in this section you will be able to

- read and plot points
- find midpoint, distance, slope, and points of intersection of lines with the axes

At Higher level

- find the point of intersection of two lines
- find the slopes of parallel and perpendicular lines

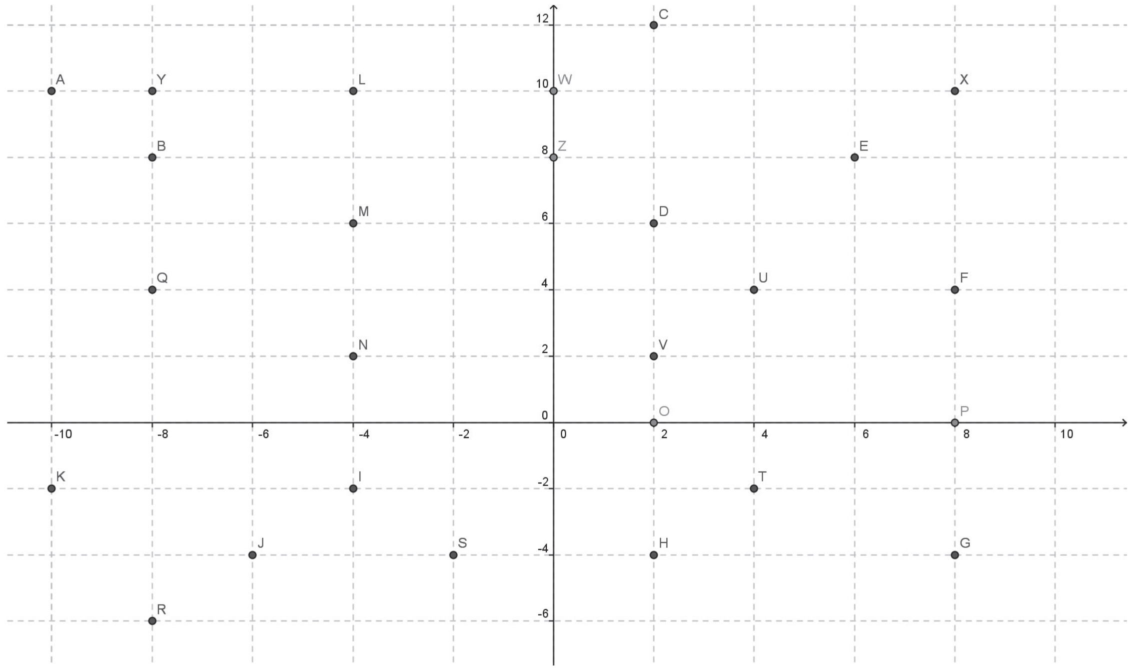
INTRODUCTION

The activities described below and the questions that follow allow you to deepen your understanding of concepts in co-ordinate geometry as well as solve problems using these concepts and their applications.

The plotting of points has allowed us to find locations on maps for generations. A global positioning system (GPS) has replaced 6 figure grid references for specifying locations these days. Our mobile phones have this function and many cars have in-built or mobile 'sat-nav' technology to help us navigate our way around big cities or to unfamiliar places. When we get there we can store the location as a 'way point' and find it again easily next time.

Activity 3.1

If you and your friends in school have a grid with each letter of the alphabet on it you can write secret messages to each other. You need to have the grid to compose and to read the messages. Use the grid below to answer the questions that follow.



i. What is your first name?

ii. Who is your favourite band or singer?

iii. What is your favourite TV programme

Here are my answers to those questions; can your figure them out?

i. (4,-2), (6,8), (-8, -6), (-8, -6), (-4,-2)

ii. (2,12), (2,0), (-4,10), (2,6) (8,0), (-4,10), (-10,10), (-8,10)

iii. (8,-4), (-8,-6), (6,8), (-8,10), (-2,-4) (-10,10),(-4,2), (-10,10), (4,-2), (2,0) (-4,6), (-8,10)

Q. 1 Make up five questions that could be answered in codes by other students.

Q. 2 Write a message of one sentence in code that you can give to another student to work out. The sentence should be written with pairs of co-ordinates in brackets for each letter and spaces between each of the words.

Extension activity

It is easy for anyone to read a message using the above grid.
Make your own grid with the letters in different places. Consider the following.

1. Can you confuse someone trying to break the code by having the same letter in two locations on your grid?
2. It is possible to include numbers or words in your grid?
3. Could you put the 'txt words' that you use on your mobile onto a grid?

For further exploration

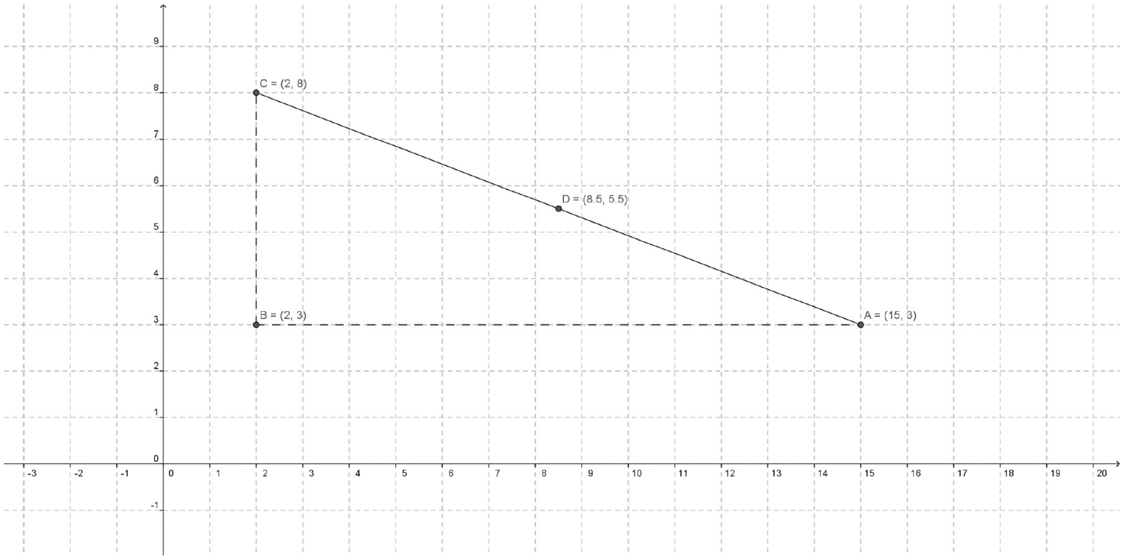
Rene Descartes was the mathematician famed for first devising the co-ordinated plane. Find out more about him and his life in your school library or on the internet if you have access (check out www.projectmaths.ie for information).

There are some famous codes that were used in the past to communicate or to encrypt messages. ENIGMA was one of these, and a movie was made about this code.

If you have access to the internet, search for information about Public Key Encryption (PKE).

Activity 3.2

You are familiar with finding the average or mean of a set of numbers. We add them and divide by the number of numbers. Using this idea, we can find the midpoint of a line segment in co-ordinate geometry.



In the example above we have $\dfrac{2+15}{2}=8.5$, and $\dfrac{8+3}{2}=5.5$

This gives us the result (8.5, 5.5). This seems to correspond with the midpoint of the line segment in our example.

Q. 1 **Find the midpoint of the line segments formed by the following pairs of points; remember to be careful with the minus signs.**

[E(1,1) and F(7,7)], [G(1,2) and H(3,6)], [J(4,7) and K(11,16)],
[P(0,4) and Q(−2,2)], [R(2,1) and S(4,3)], [T(2, −3) and U(− 2,5)]

The midpoint should be the average of the two end points. So, add the x co-ordinates and divide by two. Then repeat for the y co-ordinates. This can be summarised by the formula for the midpoint that you may be familiar with:

midpoint = $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Activity 3.3

The distance between two points is a length. We can physically measure a length with a ruler or tape measure, but it is not always possible to do this. So, we need to find a way of measuring in geometry without using instruments. The engineers who built the pyramids at Giza in Egypt figured out a way of finding lengths so they would not make errors; they based it on right angles. What became called the theorem of Pythagoras was known to the Egyptians long before Pythagoras was born. We can see how it helps us to devise a method for finding the length of a line segment, using the diagram from Activity 2 above.

There is a right angle at B (2,3). The length of the line segment [AB] can be found by subtracting the x values (15-2 = 13). The length of the line segment [BC] can be found by subtracting the y values (8-3 = 5). Since the angle at B is a right angle, the line segment [CA] is the hypotenuse. So, using what we know about the theorem of Pythagoras, we can say $13^2 + 8^2 = [CA]^2$.

i. Find the value of [CA] to the nearest whole number?
ii. Find the value of [CA] correct to the first decimal place?
iii. Find the value of [CA] correct to two decimal places?
iv. Using the co-ordinates P(-1,8), Q(-1,4) and R(2,4), draw another right angled triangle on the grid above. Find the length of the hypotenuse using the same method as before.

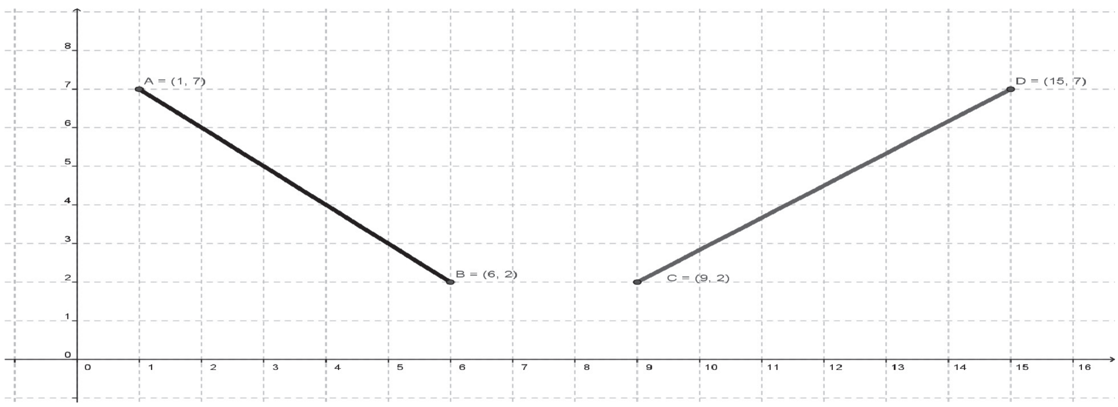
If this method works for these two triangles, will it work for all right angled triangles when we have the coordinates of the three vertices?
If it does work for all right angled triangles, can we work out a general rule or formula to calculate the length of the hypotenuse?

Activity 3.4

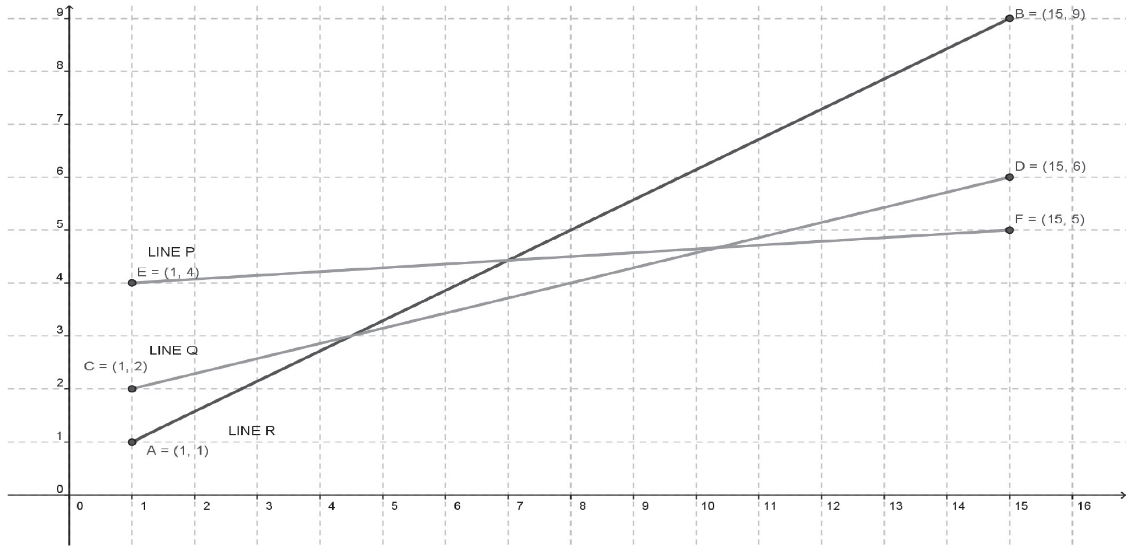
The slope or gradient of a line is the amount by which it goes up or down. In geography, where you have to do a cross-section on an ordinance survey map, you are making a profile of the slopes on the map – its topography. The slope of a hill tells us a lot about how suitable it can be for a road or for climbing. The slope of a road is often shown as a percentage or ratio on a road sign.

So, how do we explain how steep or shallow a slope is? We measure how much it goes up or down as we travel along it from left to right.

One of the following line segments has a positive slope and one has a negative slope. Write underneath which one is which and write a short explanation for your answer.



The slopes of the lines P, Q and R below are of one type. Are they positive or negative? How would you explain the difference between the slopes of these three lines?



Let's look at how the slope of line R changes between points A(1,1) and B(15,9). The line rises up from 1 to 9 in height and it goes forward from 1 to 15 along the horizontal. If we compare the change in height to the change in horizontal distance, we get an idea of how steep the slope is.

Change in height: 9–1 = 8; , change in horizontal: 15–1 = 14.

The comparison can be written down as a fraction or ratio: $\frac{8}{14} = \frac{4}{7}$

i. Compare the slopes of the other two lines P and Q. What can you conclude about the size of the slope and the size of the fraction?

ii. Is the slope of each line constant?

iii. Would it matter if we took different points on the lines to find the slope of?

iv. If this method works for calculating any slope, can we summarise it into a rule?

v. Slope is a property of a line; what does this mean?

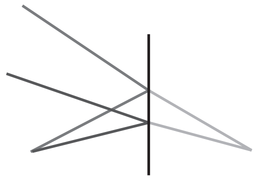
vi. It is easy to draw a line when you have two points on it. Draw the line which contains the points G(3,6) and H(9,–1). Work out the slope of this line. This line is different from other lines. What makes it different?

vii. We can use a point on a line and its slope to get its equation. How would you describe the equation of a line in your own words?

viii. We have seen above that it is possible to find the slope of a line when you have two points.Is it possible to find the equation of a line using the same two points?

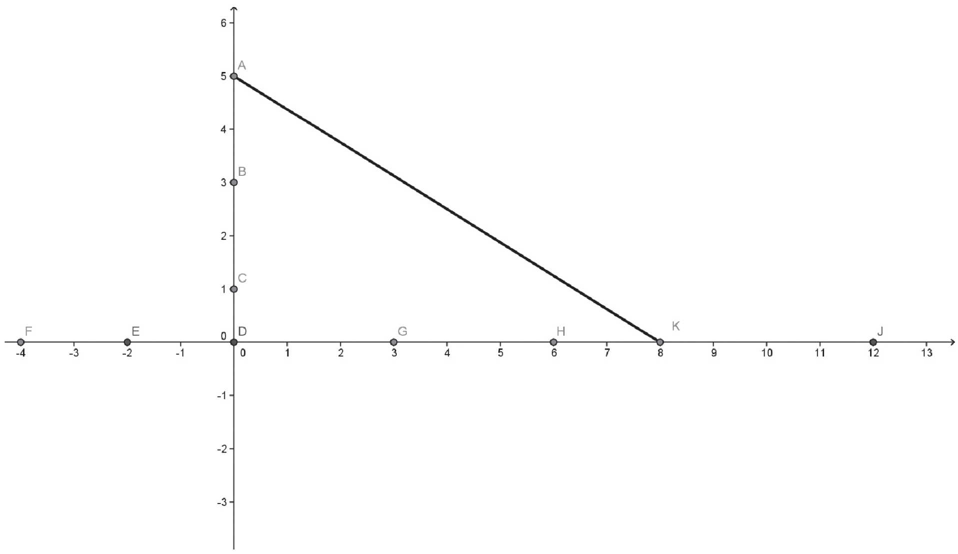
ix. If you were told about the slope of a line and were given a point on a line could you draw a picture of it?

Activity 3.5



Lines are long, and we draw parts of them on diagrams. These 'parts of lines' are called line segments. The more interesting parts of lines are when they come in contact with other lines or points or shapes. Then they have common elements or a shared location.

If we begin by looking at the X and Y axes and plotting the points along them, we can see that there is a common co-ordinate for points on the X-axis and also a common co-ordinate for points on the Y-axis. Give the co-ordinates of the points labelled on the diagram and write down the common co-ordinate for each axis.



Points on x-axis

What is the common coordinate for points on the x-axis?

Points on y-axis

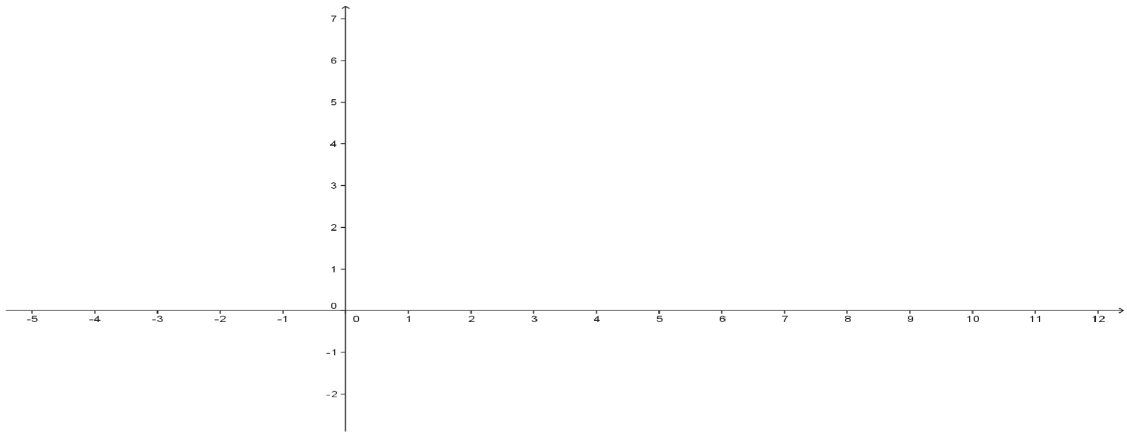
What is the common coordinate for points on the y-axis?

Find the equation of the line linking points A and K.

Activity 3.6

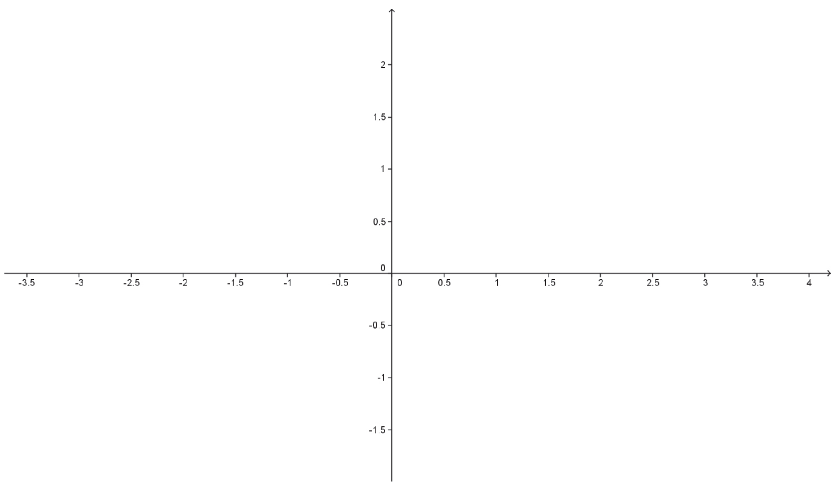
In algebra you may have studied simultaneous equations. If so, you have been able to find a unique value of x and one for y that satisfies two equations. If you were to draw a graph of each of those equations what would they look like?

$3x + 2y = 8$ and $2x - y = 3$ can be used as an example. Solve these simultaneous equations in the normal fashion to find the x and y values that satisfy each equation. Now consider what each of these equations represents when it is drawn as a graph. Plot both of the equations as lines on the axes provided below. Notice the co-ordinates of the point of intersection



Q.1 **Now try to find the solution to a similar problem, this time plotting the lines first and then solving the equations using algebra.**

$2x + 3y = -2$ and $3x + 7y = -6$ are the equations of two lines. Plot these lines on the grid below.

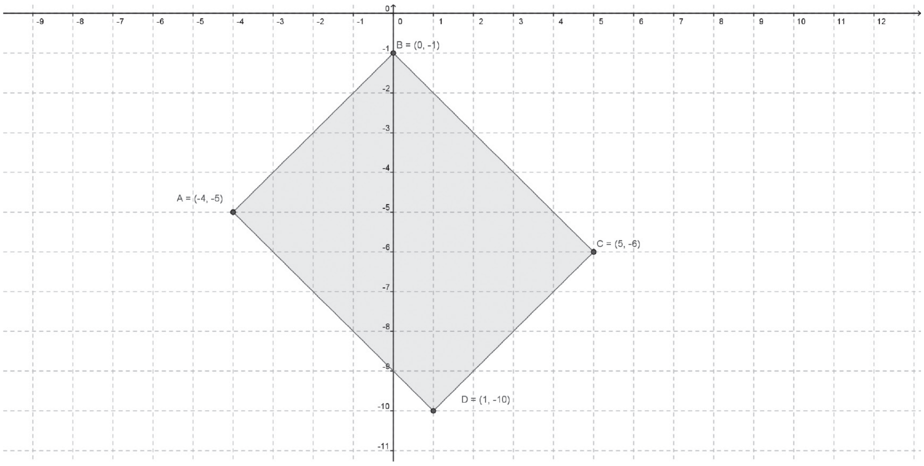


Although both methods give you an answer, comment on the two answers that you got. What conclusion did you reach about the accuracy of each method?

Activity 3.7

Describe parallel lines in one sentence.

What common property would you identify from the opposite sides [AB] and [CD] of the shape shown in the diagram below. How can you show that the property is the same in both sets of lines? Will this always be true for parallel lines?

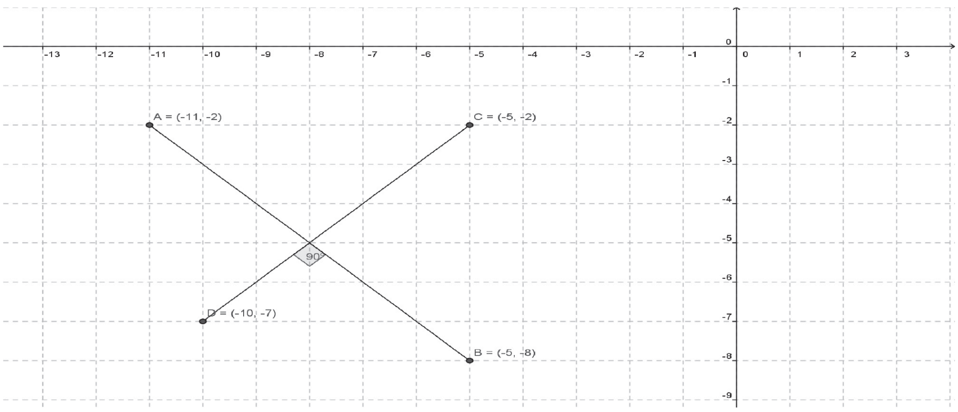


Generalise your findings into a rule that will apply to all parallel lines.

Other lines

Draw the line x = 4 and the line y = 4. Investigate the slopes of these lines. Write a couple of sentences to summarise your findings.

Q. 1 In the diagram below the lines are at right angles. Find the slope of each line and compare them. Write down any ideas you have about the slopes of perpendicular lines.



GEOMETRY 4

SYLLABUS TOPIC: TRIGONOMETRY

LEARNING OUTCOME

As a result of completing the activities in this section you will be able to

- apply the result of the theorem of Pythagoras
- use trigonometric ratios to solve problems(0° to 90°)

Higher level students will be able to

- solve problems involving surds
- solve problems involving right angled triangles
- manipulate measures in DMS and decimal forms

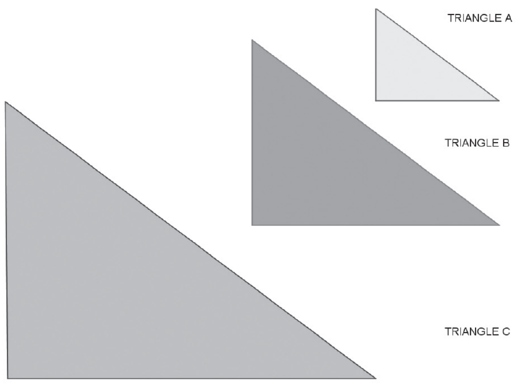
INTRODUCTION

The activities described below and the questions that follow allow you to deepen your understanding of concepts in trigonometry as well as solve problems using these concepts and their applications.

Activity 4.1

It is claimed that the ancient Egyptians used a rope with twelve equally spaced beads on it to check if their right angles were correct. Tie twelve equally spaced beads onto a length of wool or cord as illustrated below and see if you can form a right angle the way they did in ancient times.

Are all of the following triangles right angled? Triangle A has sides of 3, 4 and 5. Triangle B has sides of 6, 8 and 10. Triangle C has sides of 9, 12 and 15.



Activity 4.2

The amount of turning between the two rays (or arms) of an angle tells us how big the angle is. Using the table below, which has some values included already, look at how the values of sine, cosine and tangent change as the angle gets bigger. Give all answers correct to four decimal places.

Size of the angle in degrees	Sine (sin)	Cosine (cos)	Tangent (tan)
10°	0.1736	0.9848	0.1763
20°			
30°			
40°			
45°	0.7071	0.7071	1
50°			
60°			
70°			
80°			
90°	1	0	Error?

Q. 1 Write a single sentence as an answer to each of the following questions.

i. What have you noticed about the values of sin as the angle got bigger?

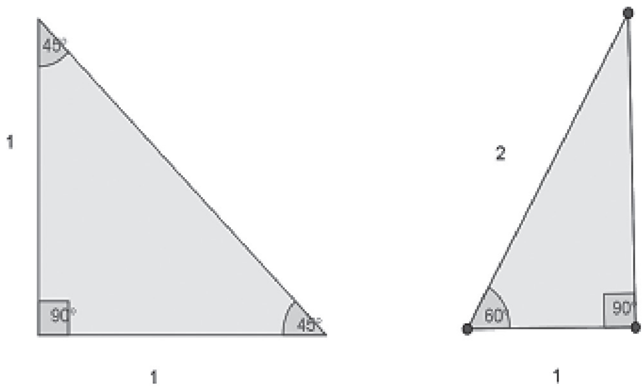
ii. What have you noticed about the values of cos as the angle got bigger?

iii. What have you noticed about the values of tan as the angle got bigger?

iv. Can you give a reason why a calculator displays 'ERROR' when you try to get the tan of 90°?

Activity 4.3

You have studied surds already in strand 3; they are numbers that can only be expressed exactly using the root sign. Surds occur quite often when we are trying to solve problems in triangles. Some of the best known surds are $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ and they occur in familiar triangles.



Find the missing sides and angles in the two triangles above.

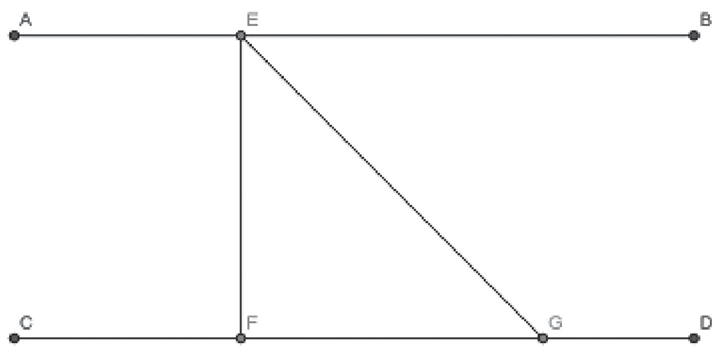
Complete the following table

Angle A	30°	60°	90°
cos A			
sin A			
tan A			

Activity 4.4

The work of surveyors, planners and engineers often involves solving real-life problems. Measurements can be made using instruments, and angles or distances easily found using trigonometry. In the diagram below the width of a river is being calculated. In order to make this calculation decide what information you require. Below the diagram is a series of measurements that were made and certain conditions are given. Is each piece of information necessary to solve the problem? Explain why. Is there another way of solving the problem without needing all of the conditions given?

Q. 1



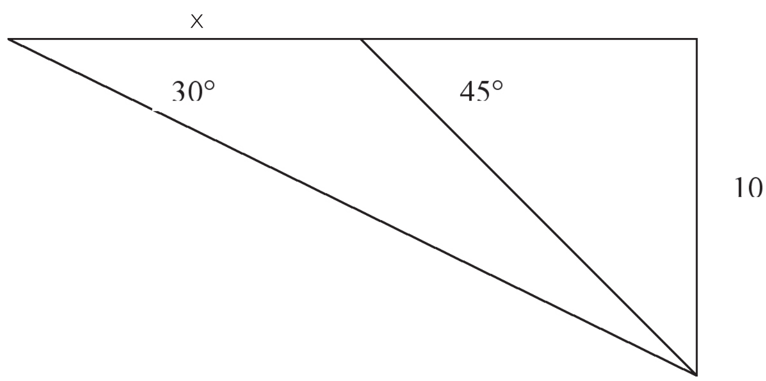
[AB] and [CD] represent river banks that are parallel.
[EF] makes a right angle with [CD]; |FG| is 100m and |∠EGF|is measured to be 52°

i. Find the width of the river.

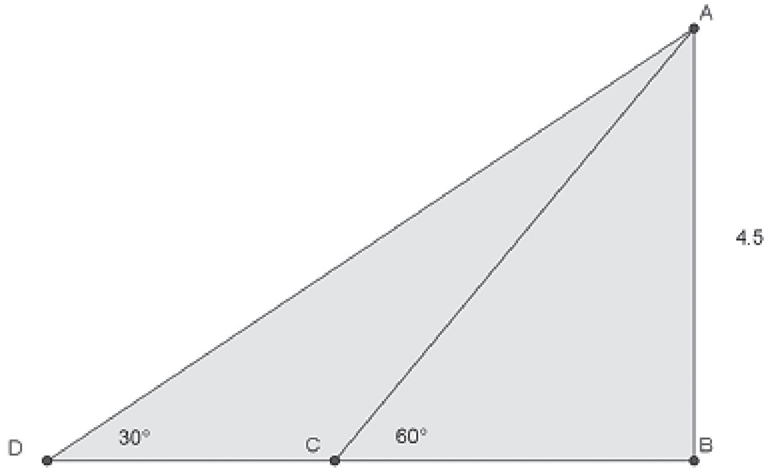
ii. Can you suggest another way to find the width of the river, without directly measuring it?

Q. 2 Try solving the following triangles

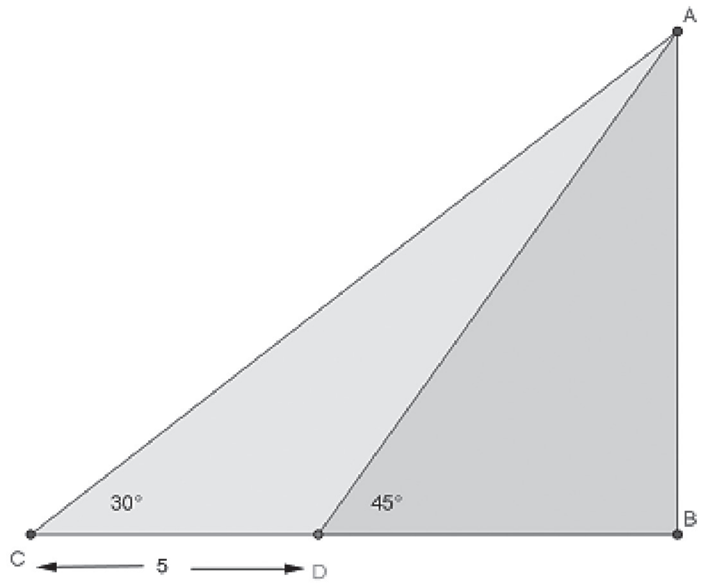
i. From the diagram find the length of x.



ii. From the diagram, find the value of |DC| in surd form.



iii. Find the lengths of [AD] and [AC] in surd form.



iv. Construct an angle B such that cos B = $\frac{\sqrt{3}}{5}$

Activity 4.5

If there are 360 degrees in a circle, 60 minutes in a degree, and 60 seconds in a minute how many seconds are there in a circle? [N.B. the usual notation is 360°, 60′ and 60″ respectively.]

In the right angled triangle XYZ, [YW] bisects ∠XYZ. |XW|= 4 and |XY|= 6. Calculate|∠XYW| (in degrees and minutes as well as in decimal form) and the length of [WZ].