Maths in Practice

Report and recommendations

June 2014

NCCA
An Chomhairle Náisiúnta Curaclaim agus Measúnachta
National Council for Curriculum and Assessment

Preface

This report comes at a time when the final syllabus revisions under the Project Maths initiative get under way in schools. The report contains a number of recommendations; work has already commenced on

- additional online resource material for teachers at www.ncca.ie/projectmaths and www.projectmaths.ie
- new composite guidelines by the Project Maths Development Team (PMDT) that link the resources available on www.projectmaths.ie with the syllabus
- planning additional PMDT support for 2014/2015.

The recommendations will also inform

- a planned review of Leaving Certificate Mathematics following the June 2015 examinations; this review will also be informed by a Chief Examiner's Report to be prepared by the State Examinations Commission in autumn 2015
- preparation of the new mathematics specification as part of the junior cycle developments; the new specification is scheduled to be published in autumn 2017, a year before it's introduced in schools.

You may email projectmaths@ncca.ie if you wish to comment on any aspect of this report.

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1. Introduction

This report presents a summary of the discussions and recommendations arising from a series of meetings of a Maths in Practice group convened to consider how teachers of mathematics could be further supported in engaging with the new mathematics syllabuses, particularly at Leaving Certificate.

Project Maths began on a phased basis1 in 24 schools in September 2008 and rolled out to all schools nationally two years later. In September 2012 a notable landmark was reached when phase 3 began in all schools. This will see all five strands of the mathematics syllabus examined at Leaving Certificate in June 2014, and at Junior Certificate in June 2015.

For an interim period—arising from the experience in the initial 24 schools—some Leaving Certificate Mathematics syllabus material in Strand 1 (statistics and probability) was deferred and an element of choice was allowed in the synthetic geometry section of Strand 2 (geometry and trigonometry). The final syllabus introduced in September 2013 marks the ending of these interim arrangements.

When the syllabuses were presented for approval to Council in June 2013, concern was expressed that some teachers were experiencing considerable difficulty interpreting the syllabus expressed in learning outcomes and that more support was needed in this area. It was agreed that a group of teachers should be established which would explore how such support might be provided; a similar arrangement had worked well in respect of issues which had arisen around Leaving Certificate foundation-level mathematics. The outcomes of these discussions would inform a review, following a complete cycle of implementation. The first examination of the revised syllabuses which include the previously deferred material will take place in June 2015.

[1] The *Project Maths* initiative involved the phased development and introduction of new syllabuses in Mathematics for both Junior Certificate and Leaving Certificate students. In each case, the 5 strands of the revised syllabuses were introduced in three phases. Phase 1 involved the introduction of Strand 1 (statistics and probability) and Strand 2 (geometry and trigonometry); Phase 2 involved the additional introduction of Strand 3 (number) and Strand 4 (algebra); and Phase 3 involved the additional introduction of Strand 5 (functions). The content of the previous syllabus was phased out as the new material was phased in and corresponding changes were made to the examinations at each level.

2. *Maths in Practice* group meetings

Following contact with teacher unions (ASTI and TUI), the Irish Mathematics Teachers' Association (IMTA) and the Project Maths Development Team (PMDT), a Maths in Practice group was convened, comprising nominees from these organisations as well as representation from the DES and the SEC. Appendix 1 lists the membership of the group.

Four meetings were held in the period September 2013 to January 2014 and preparatory or follow-up work was undertaken between meetings, and following the final meeting, based on points which arose at the meetings. A record was kept of the main points considered at each meeting and the suggested actions to follow, which facilitated collating the various points of discussion into a set of themes from which this report and recommendations emerged.

Overall, the group regarded the opportunities for engaging in discussion about the syllabus and its assessment to be beneficial. They acknowledged the wide range of supports for teachers which had been developed and sought to ensure that these were as accessible as possible in a manner which would best support teachers in planning and conducting their mathematics classes.

At the final meeting of the group, six themes were identified as a means of capturing the ideas and issues which had arisen for discussion over the course of the four group meetings:

1. Time
2. Syllabus and assessment
3. Support for teachers engaging with the syllabus
4. Teaching and learning
5. Professional development
6. External influences.

Each of these themes is considered in more detail in the next section of the report.

Following the final meeting of the group, an initial collation of the main points arising under each theme was prepared and circulated. Following feedback, a draft report was prepared for the group to comment on. The report was then finalised and re-circulated to the group.

3. Themes emerging from group discussions

3.1 Time

The time demand presented by the revised syllabus—with its emphasis on developing student understanding of concepts and their ability to apply knowledge and skills in solving problems—arose frequently in the discussions. This is particularly an issue at Leaving Certificate higher level. The design of the examination paper that requires candidates to answer all questions also contributes to the time demand inasmuch as it requires students and teachers to cover the course. While acknowledging that the need to fill in background knowledge and skills for students who had not experienced the new approaches and topics in junior cycle contributed to the time demand in the initial years, consideration may need to be given to a reduction in syllabus content if the aim of ensuring greater understanding of mathematics is to be achieved. Teachers have reported instances of additional class periods being required in order to 'cover the course', allied to the pressure to focus on exam preparation so students can cope with the changed style of questions, especially those of a problem-solving nature.

A timeline or timing guide would be helpful. In order to develop guidance of this nature, feedback from the schools is needed on their experience of how long different syllabus topics/sections take with different classes and levels. The prior learning that students bring to a lesson or set of lessons and their progress with learning is unique to each class group, which further complicates the task of providing a definitive guide on time allocation to topics. There is a tension between allowing time for student engagement in activities linked to the new approaches and emphasis, and the time needed to ensure that essential mathematical knowledge and skills are being developed. Possible ways to manage time include a 'flipped classroom' approach, where activities that have a high time demand could be undertaken as preparatory or follow-on homework, allowing class time to focus on the underlying concepts and principles. The use of technology to support such activities is also worth considering.

Recommendations

The Leaving Certificate syllabus should be reviewed in light of the experience in schools over a complete cycle. This opportunity will arise following the examinations of 2015.

Evidence-based timing guides, outlining the composition and prior learning experience of particular class groups and levels, should be made available to assist in lesson and course planning. Consideration should be given to providing examples of student work which illustrate how specified learning outcomes are being achieved.

3.2 Syllabus and assessment

The presentation of the syllabus in the form of learning outcomes has given rise to some uncertainties for teachers used to lists of content, as presented in the previous syllabus. The examination papers over a period of years provided exemplification of the standards of knowledge and processes required. In the revised syllabuses, there is a greater focus on developing student understanding of concepts as well as their ability to apply their mathematical knowledge and skills in both familiar and unfamiliar problems and contexts. The separation of foundation-level Leaving Certificate Mathematics from the other two syllabus levels and the inclusion of a description of topics, which arose from the work of the foundation level group established to consider this issue, is seen by teachers as particularly helpful.

Many of the queries about the syllabus reflect a general tendency to view the syllabus through the lens of the examinations. Participants reported concerns from teachers that they feel the examination papers are not fully aligned with the syllabus and, in some cases, questions seemed to extend what the syllabus requires. This perception is related to the uncertainties mentioned previously and to the inclusion of problem-solving questions of an unfamiliar nature in the 'contexts and applications' section of the examination paper.

Concern was also expressed about what many teachers and students seemed to believe is a requirement to learn a lot of definitions, and that this perception was having a detrimental effect on teaching and learning, with students engaging in rote learning to an extent that is neither intended nor desirable. Since only one learning outcome in the syllabus requires students to give a definition, it would seem that an appropriate distinction is not being made between explaining and defining. Students should understand terminology that arises in the course of their study; asking them to explain in their own words the terms they have encountered and the concepts they have studied is one way in which such understanding can be tested appropriately. Nonetheless, the perception that there is a requirement to rote-learn many definitions or explanations is of itself a cause of concern and should be addressed.

Another area of concern expressed by teachers was the requirement in some examination questions to use a particular solution strategy, while the syllabus expects students to be able to choose suitable strategies to solve problems: numeric, algebraic, graphical or mental. They believe that the requirement to use a specific strategy does not reflect the variety of approaches and methods that the syllabus seems to suggest or allow, nor the ideas and processes which are contained in the resources which have been made available. As an example, the use of 'hence' in an examination question has

the effect of narrowing solutions to one required strategy, whereas the use of ‘hence, or otherwise’, would allow alternative strategies to be employed. While a directive to use a particular approach arises infrequently in examination questions, in some instances the task can be properly narrowed in order to assess particular skills or procedures specified in the syllabus or to require that previous work be continued. If the approach to be adopted is always left open to candidates, there is a risk that only one approach will be considered in class and used in the examination, thereby depriving students of experience with a range of approaches as required by the syllabus. Nonetheless, it is clear that teachers would prefer to see the choice of solution method left open in all cases.

Other points of discussion related to the format of the examination papers, including increased use of contexts, and the breakdown of marks allocation. There is still a degree of uncertainty about the length of the written answer required; some teachers considered that this is not always clear from the space allowed for answering and that more guidance was needed. It was pointed out that, generally, additional answer space is provided at the back of the booklet and extra sheets are also available in the examination centres, although in recent years candidates have used this extra space only when offering additional solutions.

Recommendations

While recognising that there will be instances where it is appropriate to direct students to carry out a task in a particular way, the examinations should seek to maximise the opportunities for candidates to select their own appropriate strategies for solving problems.

In light of the need for questions testing the new syllabus to be more text-heavy than was traditionally the case in mathematics examinations, care should continue to be taken to ensure that the text used in questions is as simple, straightforward and concise as possible in the context of the learning outcomes being tested, and that it is appropriate to the examination level.

Teachers should receive further guidance on the importance of developing in candidates a good understanding of concepts and terms through discussion and application, so that students will be confident that they can explain the terms they use and the concepts and processes they have studied. Teachers would benefit from supplementary material or examination-style questions exemplifying how such understanding can be assessed and the standard that is expected at the different syllabus levels.

3.3 Support for teachers engaging with the syllabus

Much of the discussion at each meeting centred on this main theme, with a number of practical proposals being explored, developed and adapted. Some of these related to the syllabus itself, some to resources which are currently available on the NCCA and the PMDT websites, while others are seen as being noted for consideration as part of the revised specification for mathematics which will arise under the Junior Cycle Developments.

Syllabus documents

While accepting that the current syllabus documents will apply for the next two years, an improvement in the presentation of the syllabus would lead to greater clarity for teachers and facilitate teacher planning of lessons. The links between different topics/strands in the syllabus are not always obvious. The progression from junior cycle to senior cycle and the lines of distinction between ordinary level and higher level—particularly where the same learning outcomes apply at both levels—are sometimes unclear.

In the discussions about the syllabuses, suggestions were made in relation to

- making more visible the connections within and across syllabus strands and topics
- drawing attention to the kinds of skills which are both developed by and support the study of these topics
- having the syllabuses in an accessible format which facilitates planning; for example, using a numbering/reference system for the different learning outcomes which is both consistent and convenient and making the syllabuses available in a usable text format which would allow teachers to develop their own documentation
- indicating through exemplification how specified learning outcomes are being achieved; this would also provide clarity for teachers about what achieving the same learning outcome represents at different syllabus levels.

As a short-term measure, the FAQ section of the NCCA website could provide information on the ‘Big Ideas’ in mathematics—such as algebra, functions, calculus—and also address specific concerns though illustrative questions and answers. An online version of the syllabus could facilitate the use of embedded links to highlight connections between topics as well as to tag relevant teaching and learning resources. Examples of lesson ideas and/or student work to illustrate the kinds of standards expected could also be referenced in an electronic version of the syllabus—something which is not possible with hard-copy documents due to their ‘flat’ nature.

A model of how the syllabus might be re-presented was developed and refined. This involves a referencing/numbering system which would allow teachers to navigate and access syllabus information more efficiently, as well as a layout which provides clearer grouping of similar or related statements and learning outcomes. Appendix 2 illustrates how this might look when applied to one section of Strand 3 (Number) at Leaving Certificate.

Development of a composite online resource

A further discussion took place around the potential for combining this syllabus re-formatting approach with generating a teacher's guide which would link particular topics in the syllabus with available resources, whether on the NCCA website or the PMDT website.

The PMDT are in the process of updating reference material on their website. A listing of resources on the NCCA website has been compiled, which includes reference to relevant syllabus sections. It is proposed to convene a small working group of NCCA/PMDT personnel to explore how a more comprehensive reference resource might be generated which would allow teachers, when planning lessons, to easily access specific learning outcomes in the syllabus and relevant support material such as Teaching and Learning Plans, examples of student work, or suggested/recommended activities to support learning.

Recommendations

Explore the feasibility of using the curriculumonline approach for the LC Maths syllabus, maximising the features of an online environment to identify the connections within and across the strands and to provide exemplification of standards. The NCCA should also consider developing a curriculumonline resource for JC Maths ahead of the proposed schedule for introduction of the specification under the Junior Cycle Developments.

Re-design and re-format the LC Maths syllabus to make referencing the learning outcomes more convenient and to provide a clear sequence and progression of sub-topics across ordinary and higher level.

Develop and publish additional 'Big Ideas' materials and expand the FAQs on the NCCA website to help clarify areas of syllabus uncertainty for teachers.

The NCCA and the PMDT should collaborate to develop a composite reference resource that associates the various support materials with the relevant syllabus strands and learning outcomes. Consideration should also be given to using one site for all support materials so that what has been developed remains available in the longer term.

3.4 Teaching and learning

While much of the initial professional development for teachers of mathematics focused on individual strands and topics, due mainly to the phased nature of the syllabus changes, later workshops drew attention to making connections across the strands. By engaging with the ‘Big Ideas’ about mathematics education, as mentioned above, teachers can be supported in maximising the linkage between syllabus strands and topics when adopting and refining changed approaches to teaching and learning in their classes. However, teachers still need support in developing and using appropriate tasks that exploit the connections between topics rather than planning for individual lessons that focus on isolated areas of mathematics in a linear fashion.

The changed approach and emphasis in teaching and learning requires more detailed planning for, and integration of, knowledge and skills development. Teachers would benefit from support in the form of teaching and learning suggestions for topics which illustrate how the connections across the syllabus strands can be managed. This could be a focus for the next phase of professional development for mathematics teachers. Ideas for tasks which are seen as effective in promoting student development of thinking and problem-solving skills would be welcome. Annotated examples of student work would assist teachers with both the initial planning of lessons and the evaluation of student progress.

The issue of the time demanded to ensure development of student understanding also has implications for planning the sequence and pace of learning. Evidence-based timing guides which reflect the make-up and prior learning experiences of differing class groups would assist a school maths department to plan more effectively. Encouragement for schools to adopt a flexible approach to timetabling would allow for opportunities to engage in a more in-depth treatment of topics where this is required or desirable.

Annotated examples of student work would help teachers to develop their capacity to assess the level of conceptual understanding being achieved by students as well as the opportunity to identify the kinds of misconceptions that can arise—which may impede the student’s progress and development in later topics. For example, algebra is seen as a significant topic in its own right, but also as underpinning many other topics and strands in mathematics. A firm foundation for developing both knowledge and skills in the area of algebra is laid down in learning about number, number operations, number patterns, etc. Teachers and students would benefit from knowing why they do a particular activity and how that learning fits with the continuum of conceptual development.

Recommendations

Develop ‘Big Ideas’ documents which outline the continuum of key concept development and the end point of this continuum for different levels in a post-primary context.

Provide support for teachers in using the ‘Big Ideas’ to maximise the linkage across syllabus strands and topics, and in developing teaching and learning strategies that enable students to understand and appreciate the interconnectedness of different areas of mathematical knowledge and skills.

Develop a range of topics and examples with approaches that will promote conceptual understanding, support the development of key knowledge and skills, and allow the teacher to gain insights into, and address, student misconceptions. These could also help to highlight connections between topics.

3.5 Professional development

The current programme of workshops for mathematics teachers is coming to a conclusion, but support will continue to be provided by the Project Maths Development Team. The Professional Diploma in Mathematics for Teaching affords opportunities for out-of-field mathematics teachers who successfully complete the diploma to meet the Teaching Council’s requirements for registration as a mathematics teacher. However, teachers who already hold a recognised qualification in mathematics would also benefit from further study in the area of mathematics education.

Design-based research and reflective practice offer opportunities for teachers to engage in continuing professional development and this should be encouraged. While the Professional Diploma in Mathematics for Teaching, which is currently funded by the DES and offered by UL, enables out-of-field teachers of mathematics to improve their mathematical and pedagogical content knowledge, the group felt that other teachers would benefit from a third level qualification in maths pedagogy. The use of design-based research as a focus for professional development could enable teachers to gain a third-level post-graduate qualification, ideally at Masters level. Such research would be extremely beneficial for mathematics teachers and could contribute significantly to improving maths pedagogy in Ireland. A post-graduate course along these lines could be run by one or more institutions through local education centres.

Recommendations

Explore the potential of design-based research as a focus of continuing professional development for mathematics teachers, including its potential for achieving a post-graduate qualification at Masters level.

The next phase of the NCCA research grant scheme might consider inclusion of teachers of mathematics, so that their research experiences could inform policy in relation to developing effective teaching and learning practice in mathematics.

3.6 External influences

The syllabus and assessment changes under *Project Maths* and associated developments have contributed to increased pressure on teachers. There is a tension between parents' and students' expectations on the one hand—mainly in relation to achievement in the examinations—and the expectations of the 'system' on the other hand, in terms of student development of knowledge and skills in mathematics. Media attention to mathematics, and particularly in relation to *Project Maths*, has added to the pressure on teachers to 'cover the course' and to ensure that students are well prepared for the examinations.

The change in emphasis which has accompanied the changes in the syllabus and its assessment was not sufficiently explained to students, parents and the wider public. Students still expect to be shown how to solve problems and then given additional work of this kind so that they can practise their skills. Despite the changed nature of the examinations, there remains an expectation that every possible type of problem can be experienced in class so that there is no 'surprise' in the examination papers. This has added to the time demand on teachers, in a situation where the syllabus has not been reduced to any significant extent. At the same time, there is criticism in relation to the inclusion of some new topics, such as in statistics, at the expense of others, such as parts of the calculus section, which have not been retained.

A reconceptualization of mathematics teaching and learning is needed so that there is a clear understanding of the changed emphasis and approach brought about by the *Project Maths* initiative and how this is reflected in the changed style of the examination papers. Students are no longer engaged in a teacher-centred endeavour where they cover a fixed body of knowledge in a linear fashion; instead, students are challenged to engage with an interconnected body of ideas and reasoning processes, and to work

collaboratively with their teacher and their peers in developing their mathematical knowledge and skills.

The awarding of bonus points for achieving a minimum of a D grade in higher-level Leaving Certificate Mathematics has indeed increased the numbers of students at this level, but it has also increased the challenge for teachers in terms of course completion and the pace of learning in class where there is a greater diversity of ability than was the case in the past.

4. Conclusion

In the course of the discussions by the Maths in Practice group, a number of other developments took place. The final report of the NFER research into the impact of *Project Maths* on student achievement, learning and motivation was published, as were the OECD and national reports on PISA 2012. A background paper on the review of the Applied Mathematics syllabus at Leaving Certificate was being prepared, which takes into consideration the mathematics syllabus changes already in place under *Project Maths*. The last of the ten scheduled professional development workshops for teachers of mathematics is currently under way.

The NFER research report drew attention to aspects of the initiative which have not yet come to full fruition:

- Although there is evidence that students are engaging in activities associated with the revised syllabuses, more traditional approaches continue to be widespread.
- It is possible that teachers are currently emphasising the content of the revised syllabuses rather than the processes promoted within it.
- Students are demonstrating mastery of mathematical procedures, but there is less evidence of problem solving and making connections between mathematical topics.

While the overall performance of Irish students in PISA 2012 was significantly above the OECD average, the scores of higher-achieving students did not differ significantly from the corresponding OECD average. In the four content area subscales, Irish students obtained significantly higher mean scores on Change and Relationships, Quantity, and Uncertainty and Data subscales compared to the OECD average scores; however, they performed significantly less well on the Space and Shape subscale.

In order to progress the recommendations contained in this report, the NCCA, the PMDT, the DES Inspectorate and the SEC should collaborate in planning the steps and resources necessary to achieve them. A design/planning team, along the lines of the group which was established for the development of the programme of workshops, may be an effective means of ensuring a coordinated approach.

As the revised mathematics syllabuses bed in and students progress to Leaving Certificate having experienced the changed syllabus in the junior cycle, relevant recommendations contained in this report can be taken into consideration as part of a planned review at Leaving Certificate and also in the development of the mathematics specification under the Junior Cycle Developments.

Appendix 1: Membership of *Maths in Practice* group

Mr	Robert	Chaney	ASTI
Ms	Ann	Piggott	ASTI
Mr	Jerry	McCarthy	ASTI
Mr	Paddy	Flood	IMTA
Ms	Catherine	Kierans	IMTA
Ms	Pauline	Gallery	TUI
Mr	Ciaran	Duffy	TUI
Ms	Imelda	Moloney	TUI
Mr	Tom	O'Connor	SEC
Mr	Seamus	Knox	DES
Ms	Sheelagh	Clowry	PMDT
Ms	Aoife	Kelly	NCCA
Ms	Rachel	Linney	NCCA
Mr	Bill	Lynch	NCCA

Appendix 2: Illustration of a more useful syllabus layout

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	– recognise irrational numbers and appreciate that **R** ≠ **Q** – work with irrational numbers – revisit the operations of addition, multiplication, subtraction and division in the following domains: • **N** of natural numbers • **Z** of integers • **Q** of rational numbers • **R** of real numbers and represent these numbers on a number line – investigate the operations of addition, multiplication, subtraction and division with complex numbers **C** in rectangular form a+ib – illustrate complex numbers on an Argand diagram – interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place-value understanding – consolidate their understanding of factors, multiples, prime numbers in **N** – express numbers in terms of their prime factors – appreciate the order of operations, including brackets – express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \le a < 10$ and perform arithmetic operations on numbers in this form	– geometrically construct √2 and √3 – prove that √2 is not rational – calculate conjugates of sums and products of complex numbers – verify and justify formulae from number patterns – investigate geometric sequences and series – prove by induction • simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series • simple inequalities such as $n! > 2^n$, $2^n \ge n^2$ $(n \ge 4)$ $(1+x)^n \ge 1+nx$ $(x > -1)$ • factorisation results such as 3 is a factor of $4^n - 1$ – apply the rules for sums, products, quotients of limits – find by inspection the limits of sequences such as $\lim_{n\to\infty} \frac{n}{n+1}$; $\lim_{n\to\infty} r^n$, $\lvert r\rvert<1$ – solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment – derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

Strand 3: Number – Ordinary level and Higher level

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems (continued)	– appreciate that processes can generate sequences of numbers or objects – investigate patterns among these sequences – use patterns to continue the sequence – generalise and explain patterns and relationships in algebraic form – recognise whether a sequence is arithmetic, geometric or neither – find the sum to *n* terms of an arithmetic series	

This example is based on the current Leaving Certificate Maths syllabus. The illustration shows a section of the syllabus as it is currently set out, while that on the opposite page shows how the same text might appear in a re-formatted and number-referenced layout which facilitates grouping of related learning outcomes at ordinary level and higher level, as well as internal reference to other sections of the syllabus.

Strand 3: Number [ORDINARY LEVEL AND HIGHER LEVEL]

3.1 Number Systems

Students working at OL should be able to:	*In addition, students working at HL should be able to:*
a) recognise irrational numbers and appreciate that $R \neq Q$	
b) work with irrational numbers	
	c) geometrically construct $\sqrt{2}$ and $\sqrt{3}$
	d) prove that $\sqrt{2}$ is not rational
e) revisit the operations of addition, multiplication, subtraction and division in the following domains: a. N of natural numbers b. Z of integers c. Q of rational numbers d. R of real numbers and represent these numbers on a number line	
f) investigate the operations of addition, multiplication, subtraction and division with complex numbers C in rectangular form $a + ib$	
g) illustrate complex numbers on an Argand diagram	
h) interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate *- see also Strand 4, section 4.4*	
	i) calculate conjugates of sums and products of complex numbers *- see also Strand 4, section 4.4*
j) develop decimals as special equivalent fractions, strengthening the connection between these numbers and fraction and place-value understanding	
k) consolidate their understanding of factors, multiples, prime numbers in N	
l) express numbers in terms of their prime factors	
m) appreciate the order of operations, including brackets	
n) express non-zero positive rational numbers in the form: $a \times 10^n$, where $n \in Z$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form	

3.1 Number Systems (continued)

Students working at OL should be able to:	*In addition, students working at HL should be able to:*
o) appreciate that processes can generate sequences of numbers or objects	
p) investigate patterns among these sequences	
q) use patterns to continue the sequence	
r) generalise and explain patterns and relationships in algebraic form	
	s) verify and justify formulae from number patterns
t) recognise whether a sequence is arithmetic, geometric or neither	
u) find the sum to n terms of an arithmetic series	
	v) investigate geometric sequences and series
	w) prove by induction: ▪ simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series ▪ simple inequalities such as: $! > 2^n$ $2^n \geq {}^2 \quad (n \geq 4)$ $(1+x)^n \geq 1 + nx \quad (x > -1)$ ▪ factorisation results such as 3 is a factor of $4^n - 1$
	x) apply the rules for sums, products, quotients of limits
	y) find by inspection the limits of sequences such as: $\lim_{n\to\infty}\left(\frac{n}{n+1}\right)$; $\lim_{n\to\infty} r^n$, $\lvert r\rvert < 1$
	z) solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment *- see also section 3.3*
	aa) derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums