"Opening Our Ideas": How a detracked mathematics approach promoted respect, responsibility, and high achievement.

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Abstract

This article describes the ways in which the mathematics department of an urban, ethnically diverse school, brought about high and equitable mathematics achievement. The teachers employed heterogeneous grouping and *complex instruction*, an approach designed to counter status differences in classrooms. As part of this approach teachers encouraged multi-dimensional classrooms, valued the perspectives of different students, and encouraged students to be responsible for each another. The work of students and teachers at Railside was equitable partly because students achieved more equitable outcomes on tests, but also because students learned to act in more equitable ways in their classrooms. Students learned to appreciate the contributions of students from different cultural groups, genders and attainment levels, a behavior that I have termed *relational equity*. This article describes the teaching practices that enabled the department to bring about such important achievements.

"What makes the class good is that everybody's at different levels so everybody's constantly teaching each other and helping each other out." (Zane, Railside school)

Introduction.

One of the most difficult challenges facing teachers of mathematics, and other subjects, is the wide range of students they teach. Mathematics classes often include students with low motivation and weak knowledge alongside others with advanced understanding and high motivation. Not surprisingly many teachers support the practice of ability grouping so that they may narrow the range and teach more effectively. In two different research studies I have conducted, in England and the US, I have followed students through high schools, investigating the impact of different teaching and grouping methods upon learning. In both studies the schools that used mixed ability approaches resulted in higher overall attainment and more equitable outcomes (Boaler, 2002, 2004). But in both cases the mathematics departments that brought about higher and more equitable attainment employed particular methods to make the heterogeneous teaching effective. In this article I will describe the approach of Railside school, an urban high school in California. At Railside the students not only scored at high levels on tests, with differences in attainment between students of different cultural groups diminishing or disappearing while they were at the school, but the students learned to treat each other with

respect. They learned to appreciate the contributions of students from different cultural groups, social classes, genders and attainment levels and develop extremely positive intellectual relations. I have termed this behavior relational equity (see also Boaler, *in press*), and this article will explain how it was achieved. It is commonly believed that students will learn respect for people from different cultures and circumstances if they learn through culturally relevant examples, or consider the history of different cultures. At Railside, the respectful relationships that students developed came about through a collaborative problem solving approach in which students worked together and learned to appreciate the different insights, methods, and perspectives that different students offered in the collective solving of problems.

Our study of Railside school was conducted as part of a larger, four-year study of three US high schools. At Railside, the department employed a mixed-ability reform-oriented approach, the other two mathematics departments employed tracking and traditional teaching methods. During the four-year study we collected a range of data, including approximately 600 hours of classroom observations, assessments given to the students each year, questionnaires and interviews. Railside school was more urban than the other two schools, with more English language learners and higher levels of cultural diversity (approximately 38% of students were Latino/a, 23% African American, 20% White, 16% Asian or Pacific Islanders. 3% were from other groups). On tests given to the students each year, the Railside students started at significantly lower levels than students at the other two schools but within two years they were achieving at significantly higher levels. Students at Railside were also more positive about mathematics and took more courses. In year 4, 41% of seniors were enrolled in calculus, compared with approximately 27% in the other two schools. Importantly, inequities between students of different ethnic groups disappeared or were reduced in all cases at Railside whereas they remained at the other schools that employed tracking (for more detail see Boaler, 2004).

Some mathematics departments employ group work with limited success, particularly because groups do not always function well, with some students doing more of the work than others, and some students being excluded or choosing to opt out. At Railside the teachers employed additional strategies to make group work successful. They adopted an approach called complex instruction, designed by Liz Cohen and Rachel Lotan (Cohen, 1994; Cohen & Lotan, 1997) for use in all subject areas. The approach aims to counter social and academic status differences in classrooms, starting from the premise that status differences do not emerge because of particular students but because of group *interactions*. The approach includes a number of recommended practices that the mathematics department employed and refined for use in their subject area. In the next section I will review seven of the practices that the teachers employed and that our long term observations, interviews with students, and detailed analyses, showed to be important in the promotion of equity. The first four (multidimensional classrooms, student roles, assigning competence, and student responsibility) are recommended in the complex instruction approach, the last three (high expectations, effort over ability, and learning practices) were consonant with the approach and they were important to the high and equitable results that were achieved.

Equitable Teaching Practices.

Multidimensionality

In many mathematics classrooms there is one practice that is valued above all others – that of executing procedures correctly and quickly. The narrowness by which success is judged means that some students rise to the top of classes, gaining good grades and teacher praise, while others sink to the bottom with most students knowing where they are in the hierarchy created. Such classrooms are uni-dimensional – the dimensions along which success is presented are singular. A central tenet of the complex instruction approach is what the authors refer to as *multiple ability treatment*. This approach is based upon the idea that expectations of success and failure can be modified by the provision of a more open set of task requirements that value many different abilities. Teachers should explain to students that no one student will be "good on all these abilities" and that each student will be "good on at least one" (Cohen & Lotan, 1977, p. 78).

At Railside the teachers created multidimensional classes by valuing many dimensions of mathematical work. This was achieved – in part – by giving students what the teachers referred to as *group-worthy problems* – – open-ended problems that illustrated important mathematical concepts, allowed for multiple representations, and had several possible solution paths (Horn, 2005). The teachers had created the algebra curriculum themselves, adapting problems from different curriculum to make them group-worthy. This enabled more students to contribute ideas and feel valued. When we interviewed the students and asked them "What does it take to be successful in mathematics class?" they offered many different practices such as: asking good questions, rephrasing problems, explaining well, being logical, justifying work, considering answers, and using manipulatives. When we asked students in the traditional classes in the other two schools in our study what they needed to do in order to be successful, they talked in much more narrow ways, saying that they needed to concentrate and pay careful attention. The different dimensions that students believed to be an important part of mathematical work at Railside were valued in the teachers' interactions and the grading system.

The multidimensional nature of the classes at Railside was an extremely important part of the increased success of students. Put simply, when there are many ways to be successful, many more students are successful. Students are aware of the different practices that are valued and they feel successful because they are able to excel at some of them. The following comments given by students in interviews give an indication of the multidimensionality of classes -

> *With math you have to interact with everybody and talk to them and answer their questions. You can't be just like "oh here's the book, look at the numbers and figure it out"*
> *Int: Why is that different for math?*

It's not just one way to do it (…) It's more interpretive. It's not just one answer. There's more than one way to get it. And then it's like: "Why does it work"? (Jasmine, Y1)

It is rare to hear students describe mathematics as more broad and more *interpretive* than other subjects. This breadth was important to the wide rates of success and participation achieved.

Roles

When students were placed into groups they were also given a particular role to play, such as *facilitator, team captain, recorder/reporter* or *resource manager* (Cohen & Lotan, 1997). The premise behind this approach is that all students have important work to do in groups, without which the group cannot function. At Railside the teachers emphasized the different roles at frequent intervals, stopping, for example, at the start of class to remind facilitators to help people check answers or show their work or to ask the group "What did you get for number 1?" Students changed roles at the end of each unit of work. The teachers reinforced the status of the different roles and the important part they played in the mathematical work that was being undertaken. The roles contributed to the complex interconnected system that operated in each classroom, a system in which everyone had something important to do and all students learned to rely upon each other.

Assigning Competence

An interesting and subtle approach that is recommended within the complex instruction literature is that of *assigning competence*. This is a practice that involves teachers raising the status of students that may be of a lower status in a group, by, for example, praising something they have said or done that has intellectual value, and bringing it to the group's attention; asking a student to present an idea; or publicly praising a student's work in a whole class setting. This practice was one that I could not fully imagine until I saw it enacted. My first awareness of it came about when a quiet Eastern European boy muttered something in a group that was dominated by two happy and excited Latina girls. The teacher who was visiting the table immediately picked up on it saying "Good Ivan, that is important". Later when the girls offered a response to one of the teacher's questions he said, "Oh that is like Ivan's idea, you're building on that". He raised the status of Ivan's contribution, which would almost certainly have been lost without such an intervention. Ivan visibly straightened up and leaned forward as the teacher reminded the girls of his idea. Cohen (1994) recommends that if student feedback is to address status issues, it must be public, intellectual, specific and relevant to the group task (Cohen, 1994, p. 132). The public dimension is important as other students learn about the broad dimensions that are valued; the intellectual dimension ensures that the feedback is an aspect of mathematical work, and the specific dimension means that students know exactly what the teacher is praising.

Teaching Students to be Responsible for Each Other's Learning

A major part of the equitable results attained at Railside was the serious way in which teachers expected students to be responsible for each other's learning. Many schools employ group work which, by its nature, brings with it an element of interdependence, but Railside teachers went beyond this to ensure that students took their responsibility to each other very seriously. One way in which teachers nurtured a feeling of responsibility was through the assessment system. For example, teachers occasionally graded the work of a group by rating the quality of the conversations groups had. In addition, the teachers occasionally gave group tests, which took several formats. In one version, students worked through a test together, but the teachers graded only one of the individual papers and that grade stood as the grade for all the students in the group. A third way in which responsibility was encouraged was through the practice of asking one student in a group to answer a follow-up question after a group had worked on something. If the student could not answer the question, the teacher would leave the group to further discussion before returning to ask the same student again. In the intervening time, it was the group's responsibility to help the student learn the mathematics they needed to answer the question.

The teaching strategy of asking one member of a group to give an answer and an explanation, without help from their group-mates, was a subtle practice that had major implications for the classroom environment. This practice meant that students were responsible to everyone in their group. In the following interview extract the students talk about this particular practice and the implications it held:

> *Int: Is learning math an individual or a social thing?*
> *G: It's like both, because if you get it, then you have to explain it to everyone else. And then sometimes you just might have a group problem and we all have to get it. So I guess both.*
> *B: I think both - because individually you have to know the stuff yourself so that you can help others in your group work and stuff like that. You have to know it so you can explain it to them. Because you never know which one of the four people she's going to pick. And it depends on that one person that she picks to get the right answer. (Gisella & Bianca, Y2)*

The students in the extract above made the explicit link between teachers asking any group member to answer a question, and being responsible for their group members. They also communicate an interesting social orientation that becomes instantiated through the mathematics approach, saying that the purpose in knowing individually is not to be better than others but so "you can help others in your group."

Two of the practices that I have come to regard as being particularly important in the promotion of equity, and that are central to the responsibility students show for each other, are justification and reasoning. At Railside students were required to justify their answers, giving reasons for their methods, at almost all times. There are many good reasons for this – justification and reasoning are intrinsically mathematical practices (RAND, 2002; Martino & Maher, 1999) – but these practices also serve an interesting and particular role in the

promotion of equity. The following boy was not one of the highest achievers in the class, and it is interesting to hear him talk about the ways he was supported by the practices of justification and reasoning:

> *Most of them, they just like know what to do and everything. First you're like "why you put this?" and then like if I do my work and compare it to theirs. Theirs is like super different 'cos they know, like what to do. I will be like – let me copy, I will be like "why you did this? And then I'd be like: "I don't get it why you got that." And then like, sometimes the answer's just like, they be like "yeah, he's right and you're wrong" But like – why?" (Juan, Y2)*

Juan made it clear that he was helped by the practice of justification and that he felt comfortable pushing other students to go beyond answers and explain *why* their answers were given. At Railside, the teachers carefully prioritized the message that each student had two important responsibilities – both to help someone who asked for help, but also to ask if they needed help. Both were important in the pursuit of equity, and justification and reasoning emerged as helpful practices in the learning of a wide range of students.

High Expectations

There were many other, related aspects of the teachers' approach that I can only briefly review in this short paper. For example, it was critical to the success of the students that teachers kept the demand of lessons intellectually high, both by providing complex problems and by following up with high-level questions. When students could not complete questions the teachers would leave groups to work through their understanding rather than providing them with small structured questions that led them to the correct answer. In interviews with the students, it became clear that they appreciated the high demands placed upon them. The students' appreciation was also demonstrated through questionnaires. For example, one of the questions started with the stem: "When I get stuck on a math problem, it is most helpful when my teacher…" This was followed by answers such as "tells me the answer" "leads me through the problem step by step" and "helps me without giving away the answer". Students could respond to each on a four-point scale (strongly agree, agree, disagree, strongly disagree). Almost half of the Railside students (47%) *strongly* agreed with the response: "Helps me *without* giving away the answer," compared with 27% of students in the 'traditional' classes at the other two schools.

Effort Over Ability.

In addition to the actions in which teachers engaged, the teachers also gave frequent and strong messages to students about the nature of high achievement in mathematics, continually emphasizing that it was a product of hard work and not of innate ability. I have already described the multidimensionality of classrooms and the fact that teachers took every opportunity to value something students could do, but they also kept reassuring

students that they could achieve anything if they put in the effort. This message was heard by students and they communicated it to us in interviews, with absolute sincerity. For example:

> *To be successful in math you really have to just like, put your mind to it and keep on trying – because math is all about trying. It's kind of a hard subject because it involves many things. (…) but as long as you keep on trying and don't give up then you know that you can do it. (Sara, Y1)*

In the year 3 questionnaires, we offered the statement "Anyone can be really good at math if they try" 84% of Railside students agreed with this, compared with 52% of students in the traditional classes.

Learning Practices.

The final aspect of the teachers' practice that I will highlight also relates to the expectations they offered the students. In addition to stressing the importance of effort the teachers were very clear about the particular ways of working in which students needed to engage. Cohen and Ball (2001) describe ways of working that are needed for learning as *learning practices*. For example, the teachers would stop the students as they were working and talking and point out valuable ways in which they were working. In one videotaped example of this, Guillermo, the department co-chair, helped a boy named Arturo. Arturo said he was confused, so Guillermo told him to ask a specific question; as Arturo framed a question he realized what he needed to do and continued with his thinking. Arturo decided the answer to the question he was working on was "550 pennies" but then stopped himself saying "No, wait, that's not very much." At that point Guillermo interrupted him saying:

> Wait, hold on a second, two things just happened there. Number one is, when I said "what is the exact question?" you stopped to ask yourself the exact question and then suddenly you had ideas. That happens to a lot of students. If they're confused, the thing you have to do is say, "OK what am I trying to figure out? Like exactly", and, like, say it. So say it out loud or say it in your head but say it as a sentence. That's number one and number two, then you checked out the answer and you realized the answer wasn't reasonable and that is *excellent* because a lot of people would have just left it there and not said, "what, 500 pennies? That's not very much." (Guillermo, Math department co-chair)

Prior to the beginning of new work teachers set out the valued ways of working, encouraging students to, for example, pick "tricky" examples when writing a book (one of the projects they completed) as they would "show off" the mathematics that they knew; they also encouraged students individually as shown in the example above. The teachers communicated very clearly to students which learning practices would help them achieve. This was also true of the teachers in the school in England that I studied (Boaler, 1997, 2002) who also brought about more equitable outcomes.

Relational Equity

It would be hard to spend years in the classrooms at Railside without noticing that the students were learning to treat each other in more respectful ways than is typically seen in schools and that ethnic cliques were less evident in the mathematics classrooms than they are in most schools. Further, such behavior did not just *happen* to take place in a mathematics classroom; it was fundamentally related to the students' conceptions of and work within mathematics. Thus, the work of students and teachers at Railside was equitable partly because they achieved more equitable outcomes on tests, with few achievement differences aligned with cultural differences, but also because they learned to act in more equitable ways in their classrooms. Students learned to appreciate the contributions of different students, from many different cultural groups and with many different characteristics and perspectives. It seemed to me that the students learned something extremely important, that would serve them and others well in their future interactions in society, which is not captured in conceptions of equity that deal only with test scores or treatment in schools. I propose that such behavior is a form of equity, and I have termed it *relational* equity (see also Boaler, in press).

It is commonly believed that students will learn respect for different people and cultures if they have discussions about such issues or read diverse forms of literature in English or social studies classes. I propose that all subjects have something to contribute in the promotion of equity and that mathematics, often regarded as the most abstract subject removed from responsibilities of cultural or social awareness, has an important contribution to make. For the respectful relationships that Railside students developed across cultures and genders that they took into their lives were only made possible by a mathematics approach that valued different insights, methods and perspectives in the collective solving of particular problems.

Conclusion.

I have focused upon Railside school in this paper because it is an important case of an urban, low-income high school that brought about high and equitable achievement. Our four-year, longitudinal study, in which we monitored students at this and two other schools, revealed the importance of the approach that the school employed in supporting mixed ability teaching and providing high level learning opportunities for a wide range of students. Railside school is not a perfect place - the teachers would like to achieve more in terms of student achievement and the elimination of inequities, and they rarely feel satisfied with the achievements they have made to date, despite the vast amounts of time they spend planning and working. But research on urban schools, and the experiences of mathematics students in particular, tells us that the achievements at Railside are extremely unusual. In this paper, I have attempted to convey the work of the teachers in bringing about the reduction in inequalities as well as general high achievement. In doing so, I hope also to have given a sense of

the complexity of the relational and equitable system that they have in place. Teachers who have heard about the achievements of Railside's math department have asked for their curriculum so that they may use it, but while the curriculum plays a part in what is achieved at the school, it is only one part of a complex, interconnected system. At the heart of this system is the work of the teachers, and the many different equitable practices in which they engage.

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Additional Resources for Classroom Use.

www.stanford.edu/~joboaler/

The website above includes a downloadable paper, entitled: 'Promoting Equity in Mathematics Classrooms – Important Teaching Practices and their impact on Student Learning' which is a longer version of this paper with more evidence and details on the approach described.

www.complexinstruction.org

More information on the complex instruction approach can be found at the website above.

Additional Resources for Classroom Use.

Effective Mathematics Teaching Approaches.

Website: http://www.stanford.edu/~joboaler/

Boaler, J. (2002) *Experiencing school mathematics: traditional and reform approaches to teaching and their impact on student learning.* (Mahwah, NJ, Lawrence Erlbaum Association).

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Complex Instruction:

Website:

http://cgi.stanford.edu/group/pci/cgi-bin/site.cgi

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