Read an interview with a teacher and find out how she helped her students develop the concept of partitioning by engaging them in a rich task that required them to use representation to help reason and justify their ideas.

Task

Using only the coloured card provided make $3\frac{3}{4}$. Write $3\frac{3}{4}$ in as many ways as you can and justify your naming using your poster.

What mathematics did you want your students to learn from engaging with this task?

Well ultimately I want my students to be able to operate efficiently on rational numbers and to understand the algorithms we commonly use, in order to do this they need a well-developed concept of partitioning. This task gives me an opportunity to assess this conceptual understanding in order to progress the learning whilst at the same time helping students make connections with their previous mathematical experiences. I'm hoping for a few ah ha moments. All the students have dealt with fractions in Primary school but I'm not sure about their conceptual understanding, judging from their errors I suspect it's quite poor

The syllabus learning outcomes the students will be working on are

- investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers
- consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value
- engage with the idea of mathematical proof
- use the equivalence of fractions, decimals and percentages to compare proportions
- explore patterns and formulate conjectures
- explain findings
- justify conclusions
- communicate mathematics verbally and in written form
- apply their knowledge and skills to solve problems in familiar and unfamiliar contexts

How did you manage the task in your class? I divided the class into groups of 4 and gave each group a task. The tasks were similar in so much as each group was required to make a rational number greater than 1 from a piece of A4 coloured card and then rename the rational number in as many ways as they could. The tasks differed in difficulty as each group was given a different rational number to represent.

What did you find interesting about the students' approach to the task? I was surprised at how difficult the students found the task. They really grappled with the concept of 'the unit' I heard comments such as *But we only have one piece of card how can we show* $2\frac{3}{10}$ *with only I piece of card?.*

Many students' work displayed evidence of the fact that they did not understand the concept of equal sizes and students' work lacked precision.

How did you help the students' get over their initial difficulties so that they could access the task?

Some groups were really not in a position to engage with this task and I simplified it for these students by changing the focus fraction to one less than 1. You can read about this task here.

The following is an extract of a discussion with the group of students who were concerned that they needed more sheets of card

Me: Well how many pieces do you need then Josh?

Josh: ehm well 10 ..

Me: Why 10?

Josh: Because then 3 of them would be $\frac{3}{10}$

Me: But you are to make a poster of $2\frac{3}{10}$ So what about the 2

Josh: Then just 2 more

Me: So what would your poster look like then?

Josh: ehm 5 sheets

Me: [To the group] Would you agree with Josh?

When there was no real commitment to an answer from the group I engaged the whole class. I asked Josh to explain his thinking to the class.

Erica: I don't think that is right …..cos that means 3 Sheets are $\frac{3}{10}$ and then 2 sheets are 2 wholes

Me: So what do you think the 5 sheets would be?

Erica: ehm $\frac{5}{10}$

Sorcha: That's $\frac{1}{2}$

There was much discussion about the fact that 5 sheets of paper would be $\frac{1}{2}$ and eventually Josh said

Josh: Ye so 5 is a half and 10 is a whole so I would need 20 sheets to make 2 so I need 23 sheets to make $2\frac{3}{10}$

Me: So what would your poster look like?

Josh: 23 sheets of card

I drew 23 squares on the board each representing a piece of card and said to the whole class

Is it clear that this is a 'picture' of $2\frac{3}{10}$?

The majority of students looked confused and said no. Josh was prepared to defend his work, I was pleased about this.

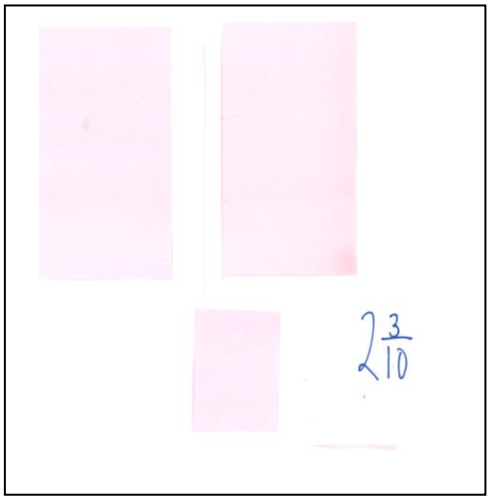
Josh: It is if you know that 10 sheets is 1 whole..you could do a key.

Me: That is true but could we make $2\frac{3}{10}$ in a way that everyone could see it was $2\frac{3}{10}$ without the need for a key?

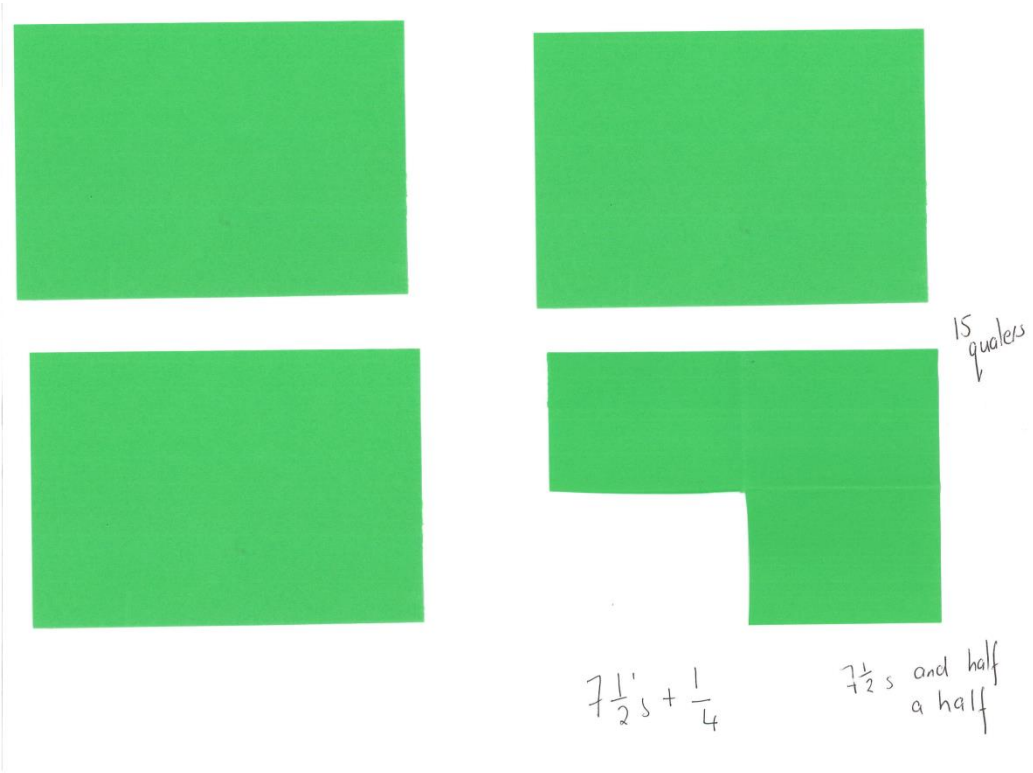
I left the groups working on this for a while it was interesting to see the struggles , they just could not see how to deal with the 2, they could make $\frac{3}{10}$ and a breakthrough came when Erica spoke.

Erica: Think about what the one is first then make it like this [she folds the paper in four and says this is one and this is one and now I need $\frac{3}{10}$. I'm going to fold to get that.

Here is Erica's poster



I found this piece of work interesting



There was plenty of opportunity for follow on work from this piece of work. I asked the group to write arithmetic sentences to describe the statements written on the poster, this gave a great opportunity to think about the concept of equality. I would use this poster in a later lesson to help students make sense for the algorithm we use for addition and multiplication.

Points for teacher discussion:

- Erica's comment *I don't think that is right …..cos that means 3 Sheets are* $\frac{3}{10}$ *and then 2 sheets are 2 wholes* was a turning point in this lesson. What would you do if your students did not provide this level of understanding? How would you progress the learning?
- Erica provided another turning point when she said *Think about what the one is first then make it.* How would you have progressed the lesson if your students were not thinking like this?
- How could you use the student's poster to help your class make sense of the algorithms we use for addition and subtraction