Leaving Certificate

Strand 3 and 4 Tasks

This set of tasks promotes understanding and gives you an opportunity to provide evidence of your learning in Strand 3 (Number) and Strand 4 (Algebra).

It is broken up into sections

- **Section A:** Working with number and Applied measure
- **Section B:** Complex numbers, solving equations
- **Section C:** Generating arithmetic expressions, geometric expressions, sequences and series.

Remember, if you are following the HL syllabus you need to be able to display evidence of understanding of the learning outcomes on the three syllabus levels; FL, OL and HL. In the same way, if you are following the OL syllabus you need to be able to display evidence of understanding of the learning outcomes on the two syllabus levels; FL and OL. Sometimes you will see similar tasks at two different levels, if you are taking the higher of the two levels it is useful for you to see how the same learning outcomes can be assessed at different levels. There is more help provided in the lower of the two levels, this help in a question is called **scaffolding**. If you are following HL expect to be given very little help in the question, students following HL are expected to display a sophisticated problem solving ability. When you look at scaffolded FL or OL tasks think about how the task might look if the scaffolding was removed altogether, this is likely to be the way it will be presented to you.

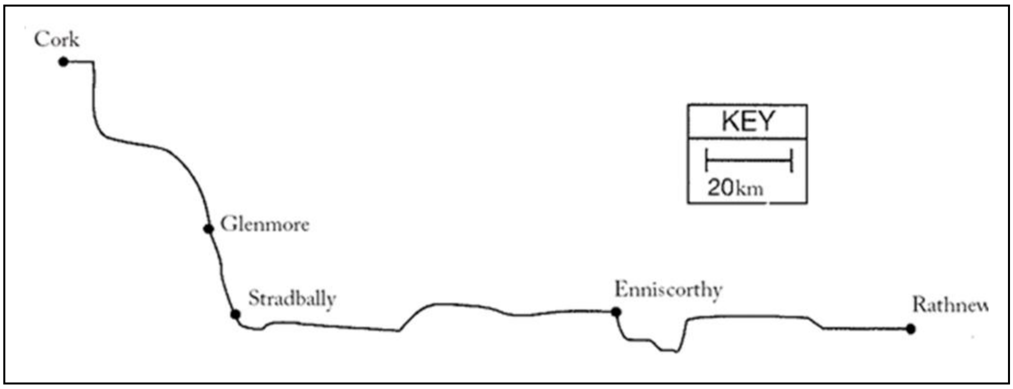
Examples of student work are included for a selection of the tasks. Try the tasks yourself before you look at other students' work. We invite you to **Compare, Examine, Discuss and Evaluate** the solution strategies provided.

Section A

TASK 1 and 2	Exploration, investigation and discussion Strand 3
Level	LCFL/OL
Learning outcome	This material provides you with the opportunity to display evidence that you can – interpret scaled diagrams – solve problems that involve calculating averages, speed, distance and time

Task 1 LCFL

Sean and Amy are travelling from Cork to Rathnew; the route they are taking is shown below.



(a) Using the key, complete the table showing the distances between the towns.

Stage of Journey	Distance travelled
Cork to Glenmore	
Glenmore to Stradbally	
Stradbally to Enniscorthy	
Enniscorthy to Rathnew	

(b) Sean and Amy will drive at an average speed of 60km/hr. They will stop for lunch along the way for half an hour. How long will it take for Sean and Amy to complete the journey?

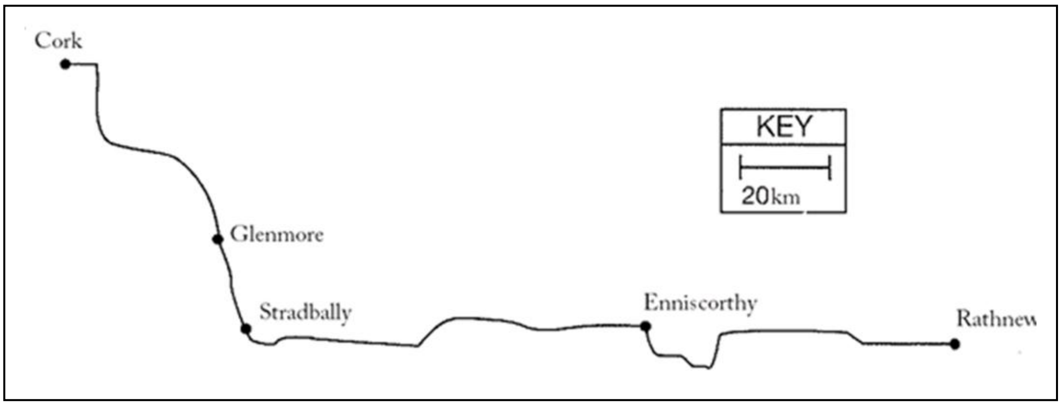
Explain your thinking.

(c) Sketch a distance time graph showing their journey. Mark each of the 5 towns on the graph.

One of the challenges of this question is using the key to estimate the distances between towns. If you estimate the distance badly how will this affect your answer to part (b)? How will if affect your graph in part (c)? If you underestimate the distance what effect would this have on your answers to parts (b) and (c)? If you overestimated the distance what effect would this have on your answers to parts (b) and (c)? What **precautions** could you take to make sure your estimate is as accurate as possible? That means how can you make sure that you don't make any mistakes when you are estimating the distance.

Task 2 LCOL

Sean and Amy are travelling from Cork to Rathnew; the route they are taking is shown below.

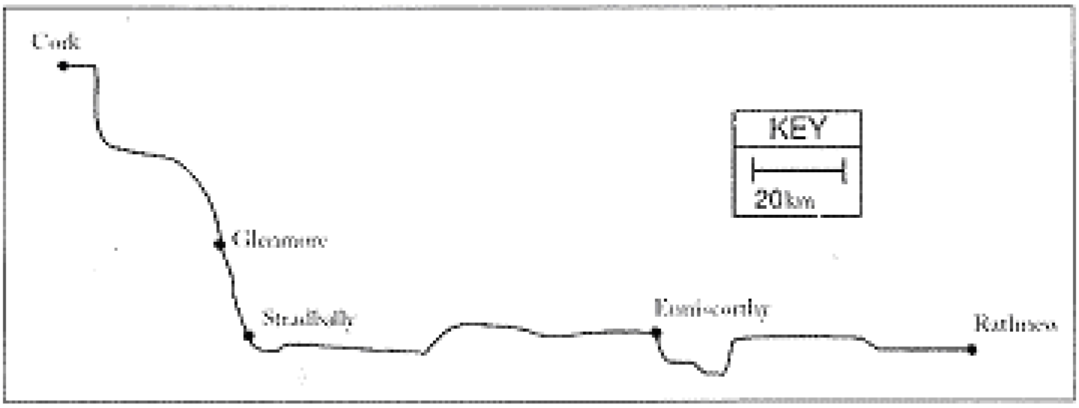


(a) Sean and Amy will drive at an average speed of 60km/hr. They will stop for lunch along the way for half an hour. How long will it take for Sean and Amy to complete the journey?
Explain your thinking.

(b) Sketch a distance time graph showing their journey. Mark each of the 5 towns on the graph.

Compare, Examine, Discuss and Evaluate

Sean and Amy are travelling from Cork to Rathnew the route they are taking is shown below



a) Sean and Amy will drive at an average speed of 60km/hr. They will stop for lunch along the way for half an hour.
How long will it take for Sean and Amy to complete the journey?.
Explain your thinking

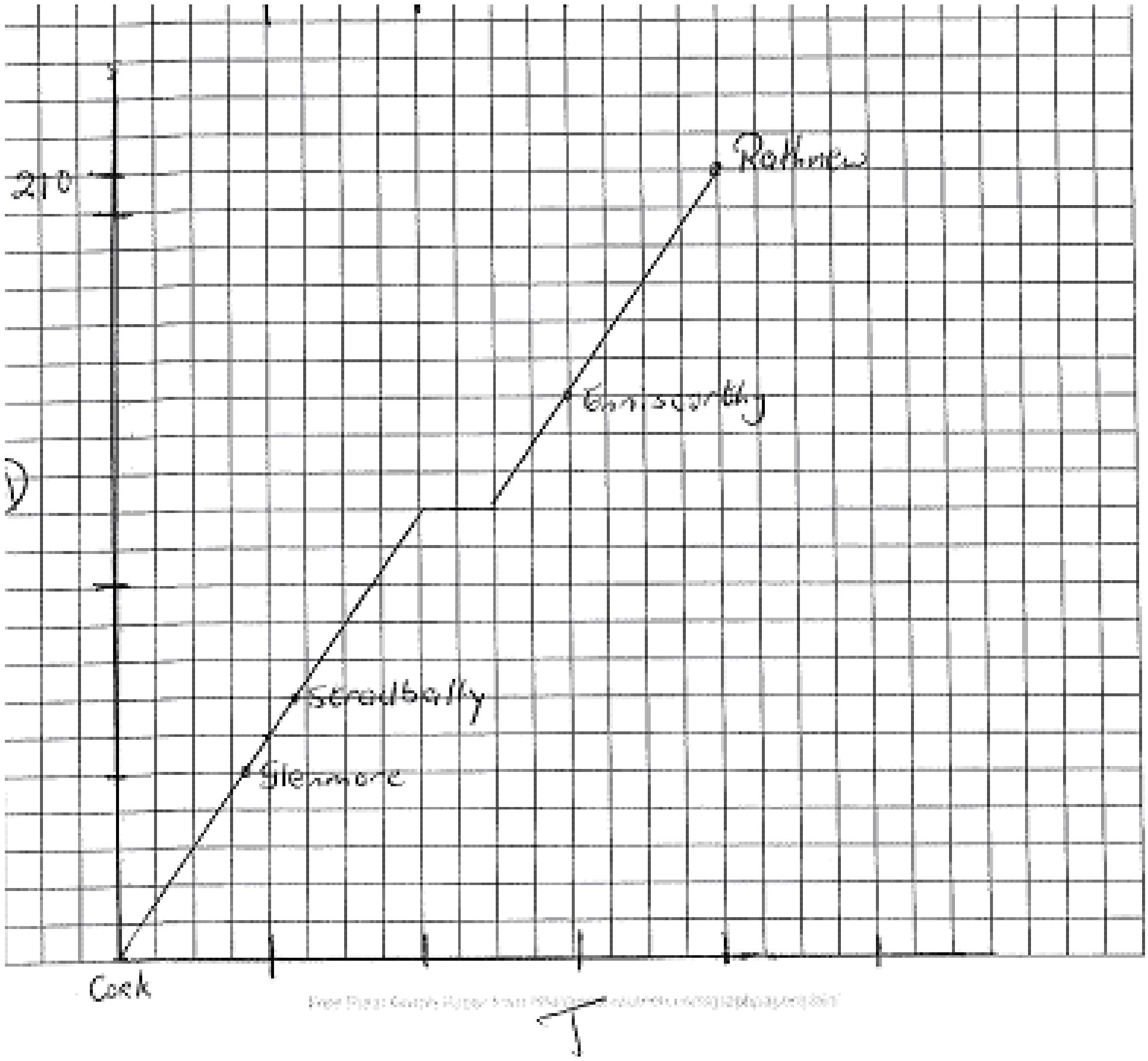
b) Sketch a distance time graph showing their journey. Mark each of the 5 towns on the graph.

a) I worked out how for it is from Cork to Glenmore to Stradbally to Eniscuelty to Rathnew

Cork to Glenmore 50km
Glenmore to Stradbally 20km
Stradbally to Eniscorthy 80km
Eniscorth to Rathne 60km

Total Distance = 210km

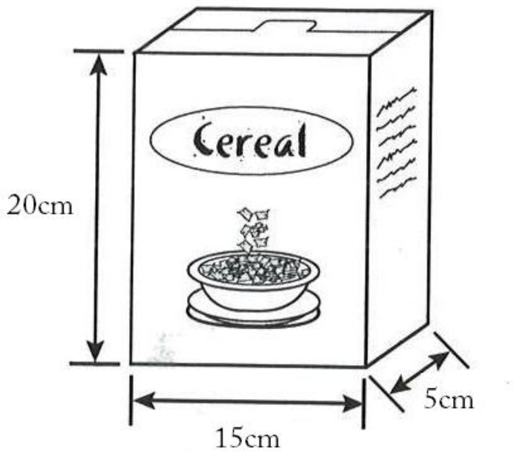
$T = \frac{D}{S} = \frac{210}{60} = 3.5$hrs and .5hrs for Lunch 4hrs

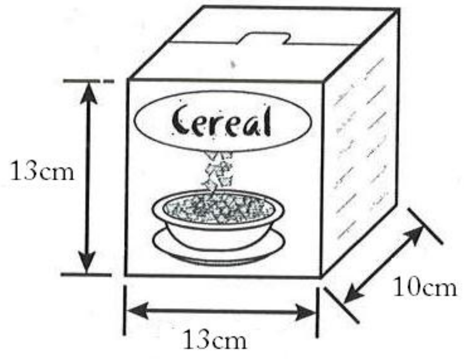

210
D
Rathnew
Enniscorthy
Stradbally
Glenmore
Cork
T

Tasks 3 - 18	Exploration, investigation and discussion Strand 3
Level	LCFL/OL/HL
Learning outcome	This material provides you with the opportunity to display evidence that you can – investigate the nets of rectangular solids – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, parallelogram, trapezium, and figures made from combinations of these – select and use suitable strategies to find surface area and volume of the following solid figures: rectangular block, cylinder, right cone, right prism and sphere – perform calculations involving surface area and volume of the following solid figures: rectangular block, cylinder, right cone, triangular-based prism (right angle, isosceles and equilateral), sphere, hemisphere, and solids made from combinations of these.

Task 3: LCFL

A company introducing a new cereal wants to use one of the boxes shown.





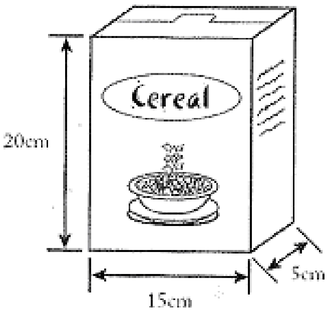
(a) Which box will hold the greater volume of cereal?

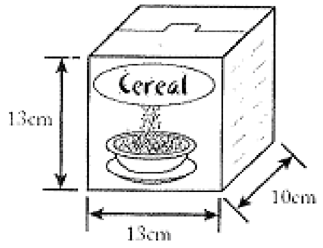
(b) Draw nets of each box (ignore the tabs for joining the sides). Clearly label the dimensions and use these to help you calculate the area of cardboard needed to make each box.

Compare, Examine, Discuss and Evaluate

LCFL

A company introducing a new cereal wants to use one of the boxes shown.





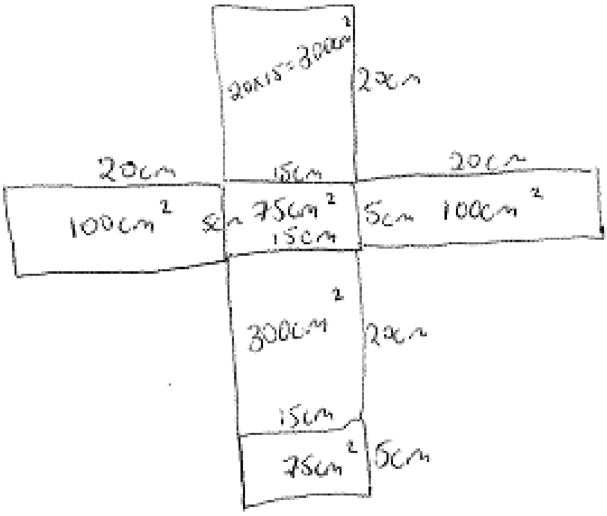
a) Which box will hold the greatest volume of cereal?

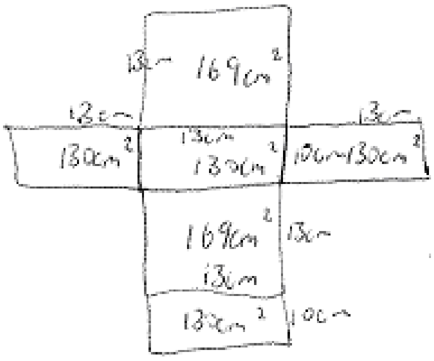
V= l x w x h
= 20 x 15 x 5
= 1500cm^3

V = l x w x h
= 13 x 13 x 10
= 1690cm^3

The one that is 13cm X 13cm X 10cm

b) Draw nets of each box. Clearly label the dimensions and use these to help you calculate the are of cardboard needed to make each box.



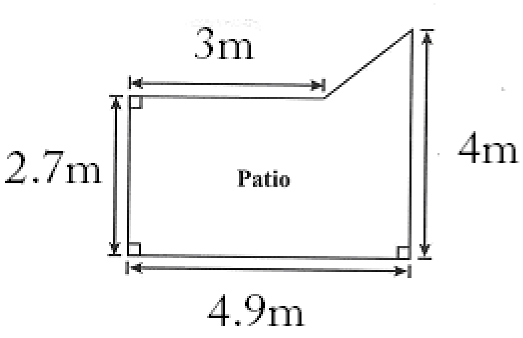


Area = 300+100 + 75+100 + 300 + 75
= 950 cm^2

Area = 520 + 338
= 858 cm^2

Task 4 LCFL

The Baker family patio is shown below.



Mrs Baker wants to pave the patio and fence it off from the rest of the garden.

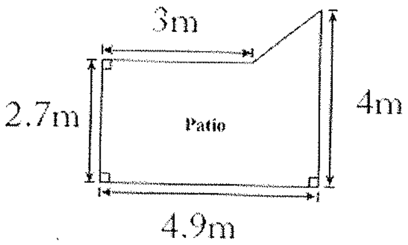
Calculate:

(a) the area that is to be paved

(b) the length of fencing required

LCFL

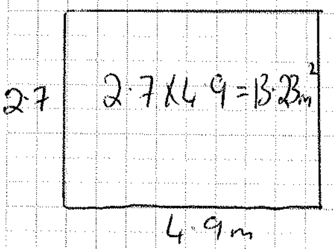
The Baker family patio is shown below.



Mrs Baker wants to pave the patio and fence it off from the rest of the garden.

Calculate

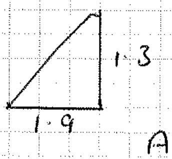
a) the area that is to be paved.



$2\cdot7 \times 4\cdot9 = 13\cdot23m^2$

$4\cdot9 - 3 = 1\cdot9$

$4 - 2\cdot7 = 1\cdot3$



$A = \frac{1}{2}(1\cdot9 \times 1\cdot3)$

$= \frac{1}{2}(2\cdot47)$

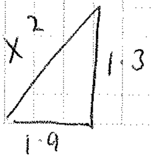
$= 1\cdot235\ m^2$

$Area = 13\cdot23 + 1\cdot235 = 14\cdot465\ m^2$

b) the length of fencing required

$2\cdot7m + 4\cdot9m + 4m + Xm + 3m$

$= 14\cdot6m + 2\cdot3\ m$



$X^2 = 1\cdot3^2 + 1\cdot9^2$

$= 1\cdot69 + 3\cdot61$

$= 5\cdot30$

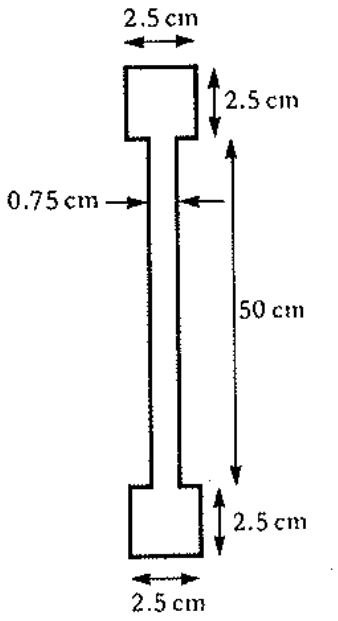
$X = \sqrt{5\cdot3}$

$= 2\cdot3\ m$

$= 16\cdot9m$

Task 5 LCOL

The cross-section through a domestic central heating radiator is shown below. The centre section is 50cm high and 0.75cm thick; at the top and bottom there are squares of side 2.5cm. Calculate the area of cross-section and hence find the volume of water in litres, inside a radiator 2m long.



Compare, Examine, Discuss and Evaluate

LCOL

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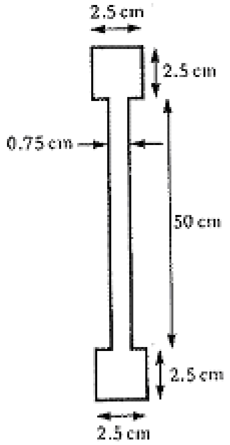


Area $= (2.5 \times 2.5) + (50 \times .75) + (2.5 \times 2.5)$

$= 6.25 + 37.5 + 6.25$

$= 50 cm^2$

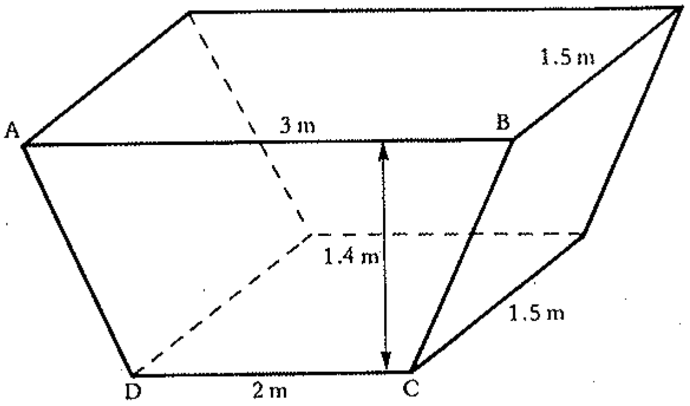
Volume $= 50 \times 200$

$= 10,000 cm^3$

$=$ 10 litres

Task 6 LCOL

The diagram shows a builder's skip. The skip has a **rectangular** base measuring 2m by 1.5m, a **rectangular** open top measuring 3m by 1.5m, and is 1.4m deep. The vertical sides are **trapeziums** and the sloping sides are **rectangles.**



(a) Find the area of the vertical side ABCD

(b) Find the volume of waste in the skip when it is filled level with the top

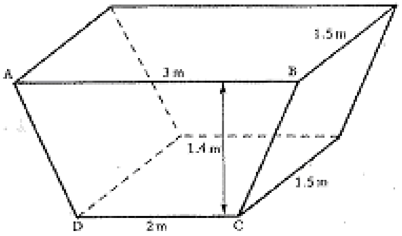
(c) By heaping it above the top a further 20% of waste material may be carried in the skip. How much does the skip hold when the waste is carried in this way?

(d) If $1m^3$ of waste has a mass of 600kg, find the mass of waste in the skip when it is filled level with the top. Give your answer in tonnes.

Compare, Examine, Discuss and Evaluate

LCOL

The diagram shows a builder's skip. The skip has a **rectangular** base measuring 2m by 1.5m, a **rectangular** open top measuring 3m by 1.5m, and is 1.4m deep. The vertical sides are **trapeziums** and the sloping sides are **rectangles.**



a) Find the area of the vertical side ABCD

$$\text{Area} = \frac{1}{2}(2+3) \times 1\cdot 4$$
$$= 3\cdot 5\,m^2$$

b) Find the volume of waste in the skip when it is filled level with the top

$$\text{Volume} = \text{Area} \times 1\cdot 5$$
$$= 3\cdot 5 \times 1\cdot 5$$
$$= 5\cdot 25\,m^3$$

By heaping it above the top a further 20% of waste material may be carried in the skip. How much does the skip hold when the waste is carried in this way?

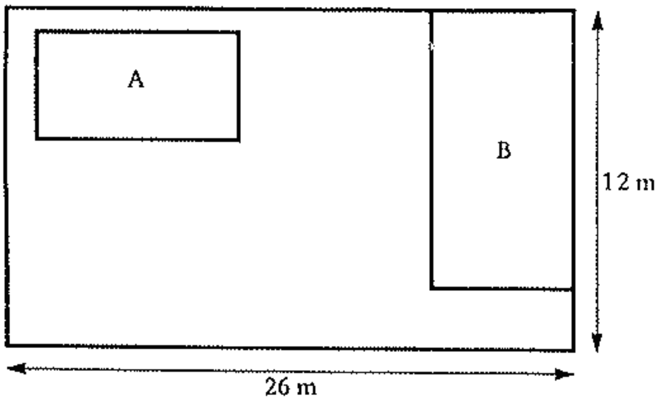
$$5\cdot 25 + \frac{20}{100} \times 5\cdot 25$$
$$5\cdot 25 + 1\cdot 05$$
$$6\cdot 30\,m^3$$

c) If $1m^3$ of waste has a mass of 600kg, find the mass of waste in the skip when it is filled level with the top. Give your answer in tonnes.

$$6\cdot 30 \times 600 = 3{,}780\text{ kg}$$
$$= 3\cdot 78\text{ tonnes}$$

Task 7 LCFL

Ben Carey is moving into his new house. A plan of the rectangular garden is shown in the diagram. He wants to have a swimming pool 1.5m deep measuring 8m by 4m at A. He also wants to have a workshop at B, measuring 10m by 6m, for which the ground must be excavated to a depth of 50cm in order to lay the foundations.



(a) Find the total area of the plot before any soil is removed.

(b) Find the volume of soil removed in order to build the pool.

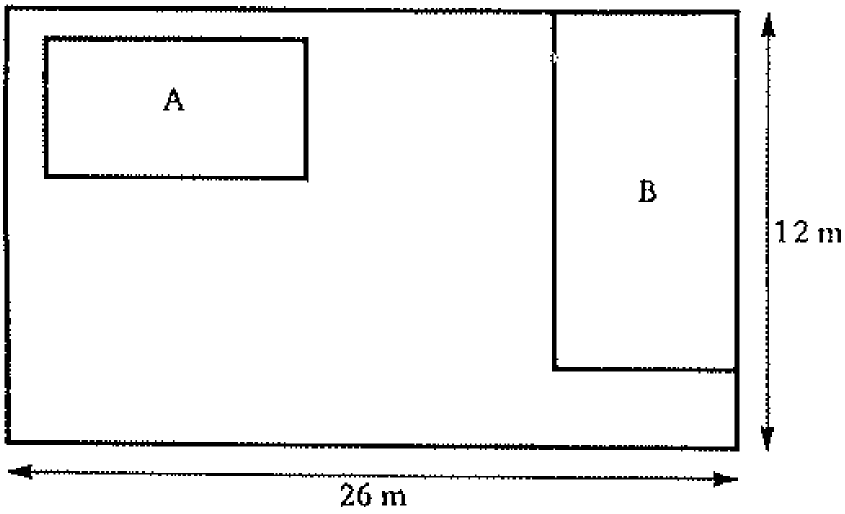
(c) Find the volume of soil excavated to lay the foundation of the workshop.

(d) Find the ground area of A and B.

(e) Find the area of ground that is untouched

Task 8 LCOL

Ben Carey is moving into his new house. A plan of the rectangular garden is shown in the diagram. He wants to have a swimming pool 1.5m deep measuring 8m by 4m at A. He also wants to have a workshop at B, measuring 10m by 6m, for which the ground must excavated to a depth of 50cm in order to lay the foundations.

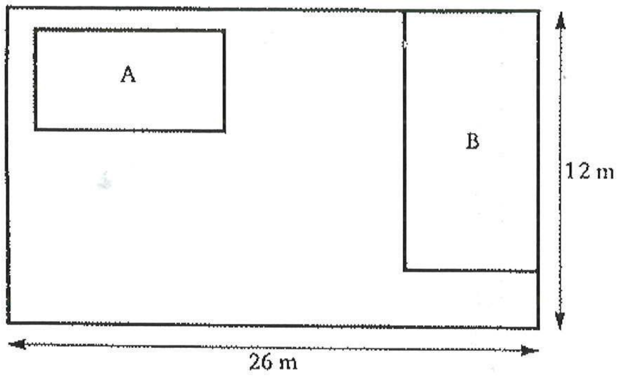


The earth that is excavated from A and B is now laid evenly over the remainder of the plot. By how much will the level of this area rise? Show how you worked out your answer.

Compare, Examine, Discuss and Evaluate

LCOL

Ben Carey is moving into his new house. A plan of the rectangular garden is shown in the diagram. He wants to have a swimming pool 1.5m deep measuring 8m by 4m at A. He also wants to have a workshop at B, measuring 10m by 6m, for which the ground must excavated to a depth of 50cm in order to lay the foundations.



The Earth that is excavated from A and B is now laid evenly over the remainder of the plot. By how much will the level of this area rise?. Show how you worked out your amswer.

Area of garden = 12x26
= $312m^2$

Area of Swimming pool = 8x4
= $32m^2$

Area of Workshop = 10x6
= $60m^2$

Area left = 312 - (92)
= $230m^2$

Vol of Earth from A = 8x4x1.5
= $48m^3$

Vol of Eart from B = 10x6x0.5
= $30cm^3$

Vol of Earth = $78cm^3$

$V = (l \times w) \times h$
$78 = 230 \times x$
$x = \frac{78}{230} = .339m$
$= 33.9cm$ Deep

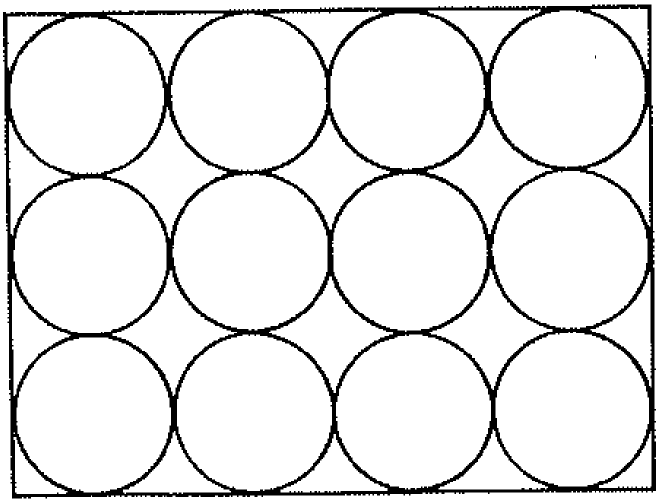
Task 9 : LCFL

Freshfruit is a company that manufactures canned fruit.

The fruit is packed into cylindrical cans **10cm** high with a base **6cm** wide**.**

For shipping purposes the cans are packed into open-ended, rectangular cardboard boxes.

The diagram below shows one possible arrangement, when the cans are packed 12 at a time.



Find

(a) The length and width of the box

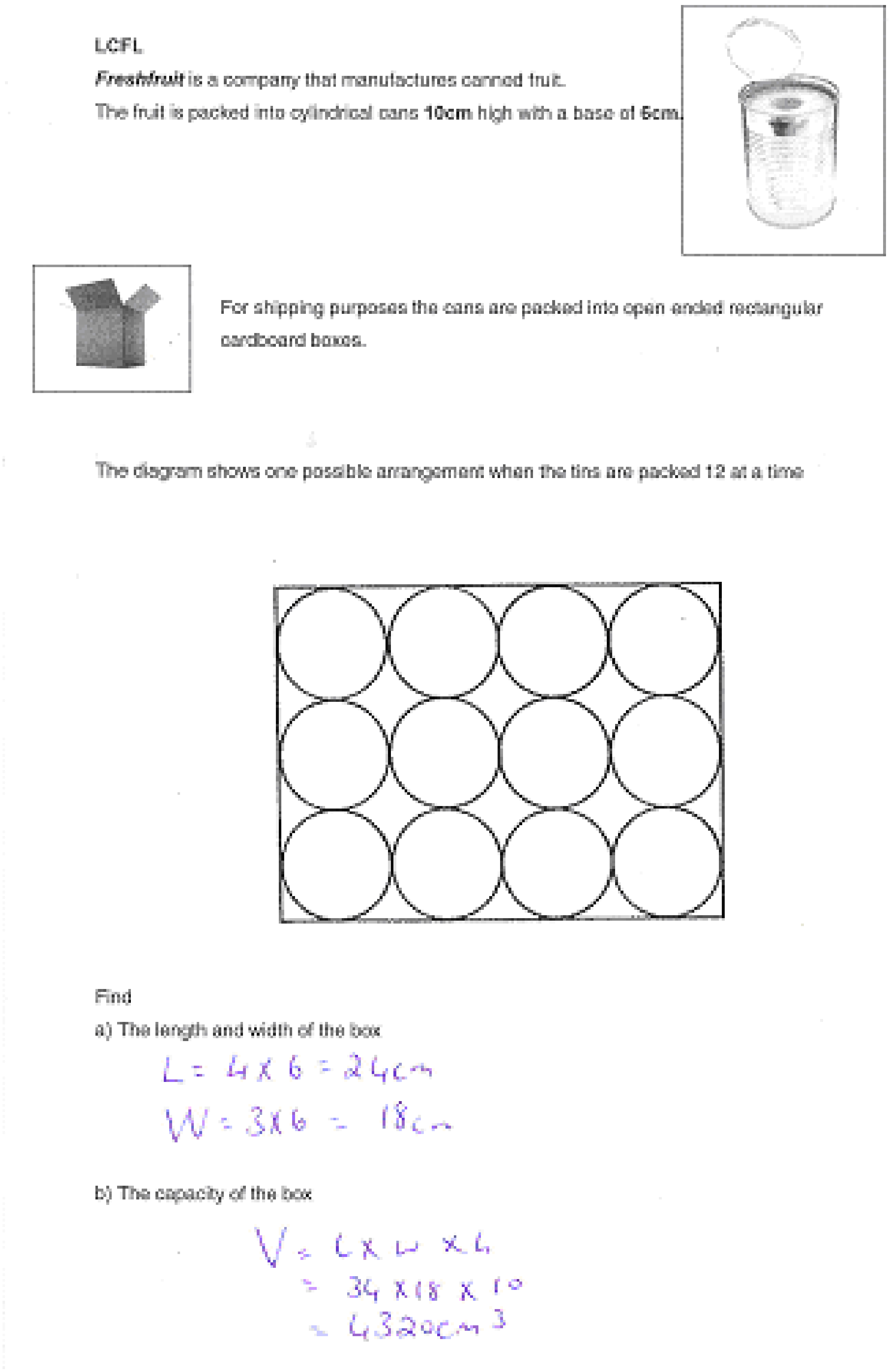
(b) The capacity of the box

(c) The volume of one of the cans

(d) The amount of space in the box that is unused

(e) The area of card used to make the box (ignore overlaps)

Compare, Examine, Discuss and Evaluate



LCFL

Freshfruit is a company that manufactures canned fruit.

The fruit is packed into cylindrical cans **10cm** high with a base of **6cm**.

For shipping purposes the cans are packed into open ended rectangular cardboard boxes.

The diagram shows one possible arrangement when the tins are packed 12 at a time

Find

a) The length and width of the box

L = 4 x 6 = 24cm

W = 3x6 = 18cm

b) The capacity of the box

V = L x w x h

= 34 x 18 x 10

= 4320cm^3

Task 10: LCOL

Freshfruit is a company that manufactures canned fruit.

The fruit is packed into cylindrical cans **10cm** high with a base of **6cm.**

For shipping purposes the cans are packed into open-ended, rectangular cardboard boxes.

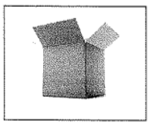
(a) If the cans are to be packed into boxes in one layer, 12 at a time, what are the dimensions of the box? How much spare space is in the box?

(b) If cardboard costs €2.50/m^2 what is the cost of one packaging box?

An employee suggests that to cut costs they should pack the cans in two layers with 6 cans on the base, arranged 2X3, and six cans placed on top of these, since less cardboard would be needed to make the boxes.

Do you agree with this suggestion? Explain why or why not? Illustrate your answer with a labelled diagram.

Compare, Examine, Discuss and Evaluate



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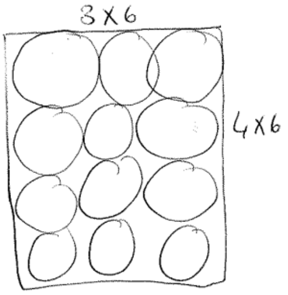


Length = 24 cm
Width = 18 cm
Height = 10cm

$V = 24 \times 18 \times 10$
$= 4320 cm^3$

$V = 6(\pi R^2 h)$
$\cancel{72}(\pi(3)^2(10))$
$= \cancel{12282.7} cm^3$
$= 3392.4 cm^3$

Spare Space = $4320 - \cancel{(12282.7)} = 927.6 cm^3$
3392.4
=

If cardboard costs €2.50/m² what is the cost of one packaging box?

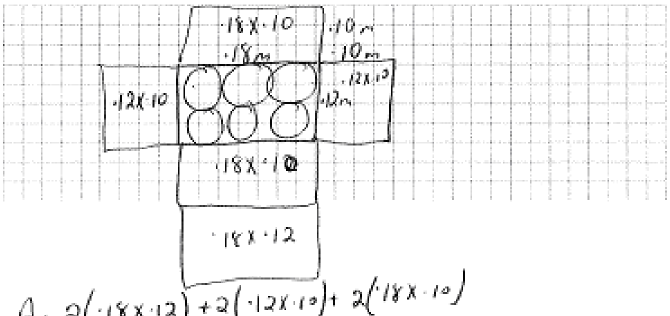
$A = 2(240) + 2(432) + 2(180)$
$= 1704 cm^2$
$= \cdot 1704 m^2$

Cost = $2 \cdot 50 \times \cdot 1704 = 42 \cdot 6c$

An employee suggests that to cut costs they should pack the cans in two layers with 6 cans on the base, arranged 2X3, and six cans placed on top of these as less cardboard would be needed to make the boxes.

Do you agree with this suggestion? Explain why or why not? Illustrate your answer with a labelled diagram.

Give reasons for your choice



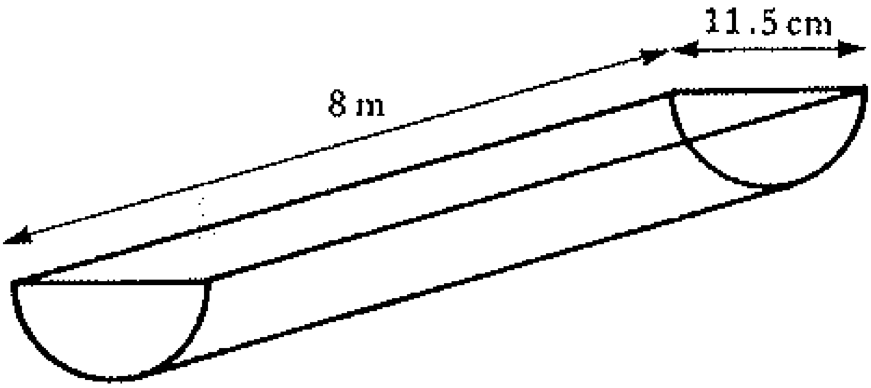
$A = 2(\cdot 18 \times \cdot 12) + 2(\cdot 12 \times \cdot 10) + 2(\cdot 18 \times \cdot 10)$
$= \cdot 0432 + \cdot 024 + \cdot 036$
$= \cdot 1032 m^2$

Yes I do cos if I arrange them like this I need less cardboard in fact ·0672m² less which saves me ·168c / box It all adds up . . .

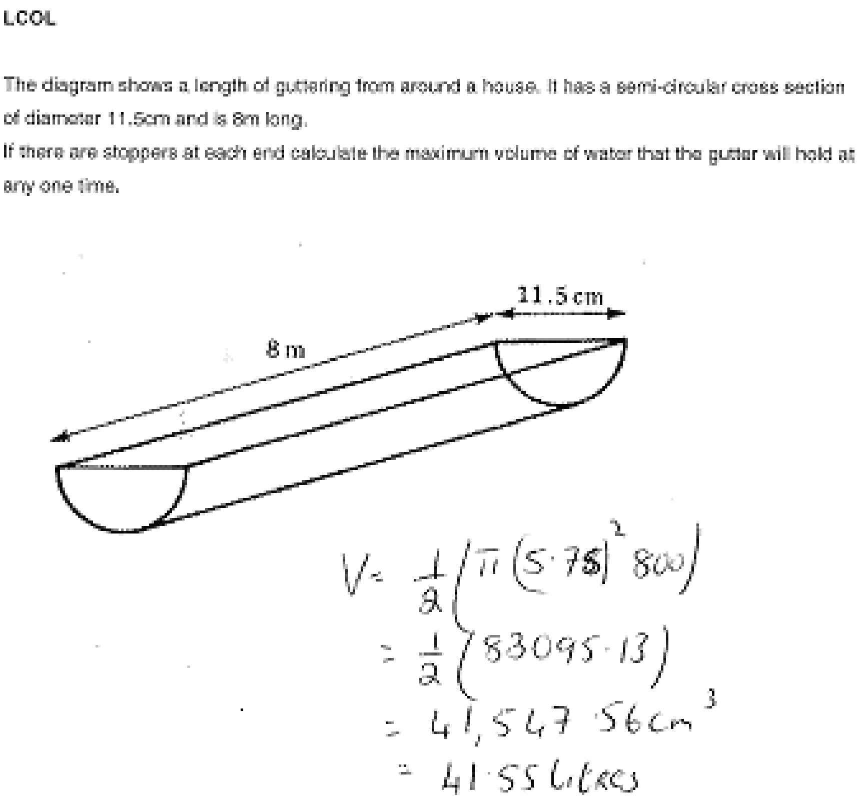
Task 11 LCOL

The diagram shows a length of guttering from around a house. It has a semi-circular cross section of diameter 11.5cm and is 8m long.

If there are stoppers at each end, calculate the maximum volume of water in litres that the gutter will hold at any one time**.**



Compare, Examine, Discuss and Evaluate



LCOL

The diagram shows a length of guttering from around a house. It has a semi-circular cross section of diameter 11.5cm and is 8m long.

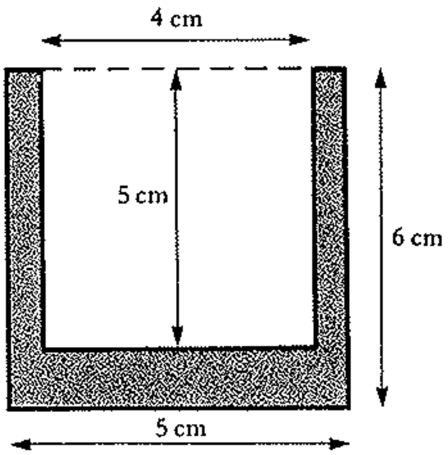
If there are stoppers at each end calculate the maximum volume of water that the gutter will hold at any one time.

$$V = \frac{1}{2}\left(\pi(5{\cdot}75)^2 800\right)$$
$$= \frac{1}{2}\left(83095{\cdot}13\right)$$
$$= 41{,}547\ 56\text{cm}^3$$
$$= 41{\cdot}55 \text{ Litres}$$

Task 12 LCFL

The diagram shows the vertical cross-section through a machine part.

It shows a solid metal cylinder of diameter 5cm and height 6cm from which a cylinder of diameter 4cm and depth 5cm has been removed.



Calculate

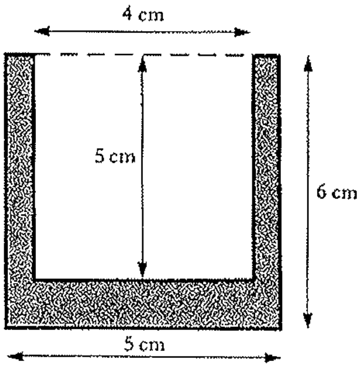
(a) the volume for the cylinder before the hole is bored

(b) the volume of the metal removed

(c) the mass of the finished machine part if the mass of 1cm^3 of the metal is 8.3g.

LCFL

The diagram shows the vertical cross section through a machine part.
It shows a solid metal cylinder of diameter 5cm and height 6cmfrom which a cylinder of diameter 4cm and depth 5cm has been removed.



a) Calculate the volume for the cylinder before the hole is bored

$V = \pi R^2 h$
$= \pi(2.5)^2 6 = 117.8 cm^3$

b) The volume of the metal removed.

$V = \pi R^2 h$
$= \pi(2)^2(5)$
$= 62.83 cm^3$

c) The mass of the finished machine part of the mass of 1cm^3 of the metal is 8.3g

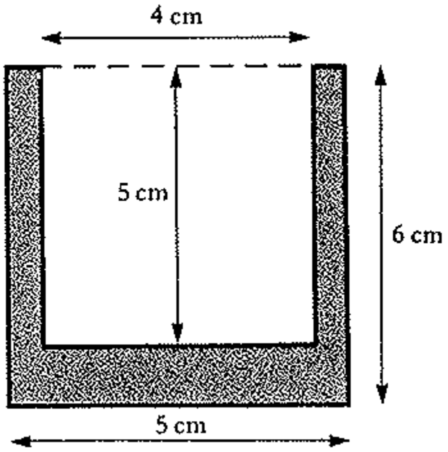
$V = 117.8 - 62.83$
$= 54.97 cm^3$

$Mass = 54.97 \times 8.3$
$= 456.24g$

Task 13 LCOL

The diagram shows the vertical cross-section through a machine part.

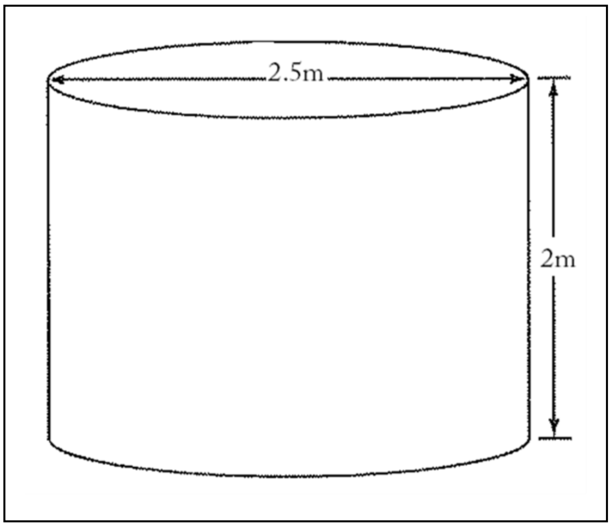
It shows a solid metal cylinder of diameter 5cm and height 6cm from which a cylinder of diameter 4cm and depth 5cm has been removed.



Calculate the mass of the finished machine part if the mass of 1cm^3 of the metal is 8.3g.

Task 14: LCOL

The diagram shows a water-storage tank proposed for the Byrne family home. It is in the shape of a cylinder of diameter 2.5m and height 2m.



The Byrnes want get a new tank to increase the volume of water they can store.

They can either increase the diameter by **1m** or increase the height by **1m**.

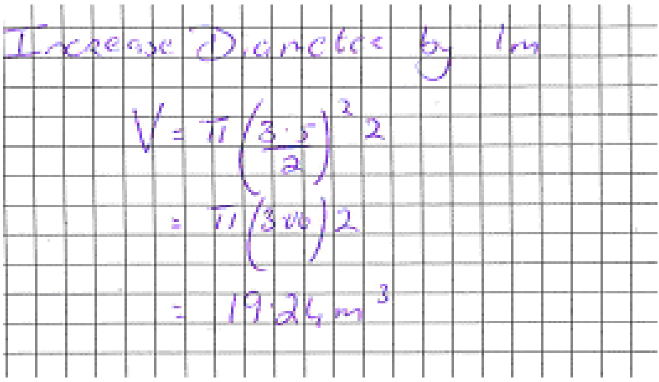
What is the difference, in litres, in the volume obtained by increasing the diameter by **1m** and the volume obtained by increasing the height by **1m**?

Show your calculations of the two volumes or give a complete explanation with numerical support to justify your answer.

The Byrnes decide to choose the tank that maximises the volume of water they can store. They want to buy insulation for their new water storage tank. Insulation comes in rolls 33m long and 2m wide.

How many layers of insulation can they wrap around the new tank if they use a complete roll? Explain your thinking

Compare, Examine, Discuss and Evaluate

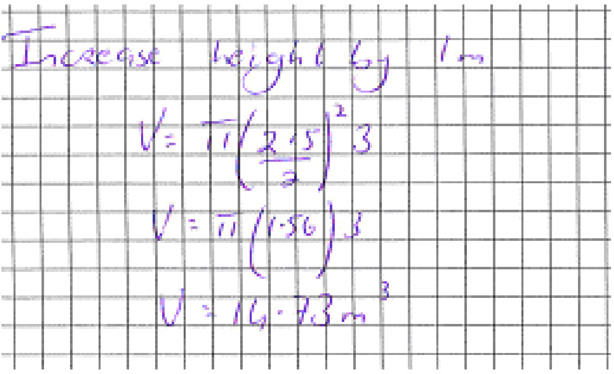


Increase Diameter by 1m

$$V = \pi\left(\frac{3 \cdot 5}{2}\right)^2 2$$

$$= \pi(3 \cdot 06)2$$

$$= 19 \cdot 24\,m^3$$

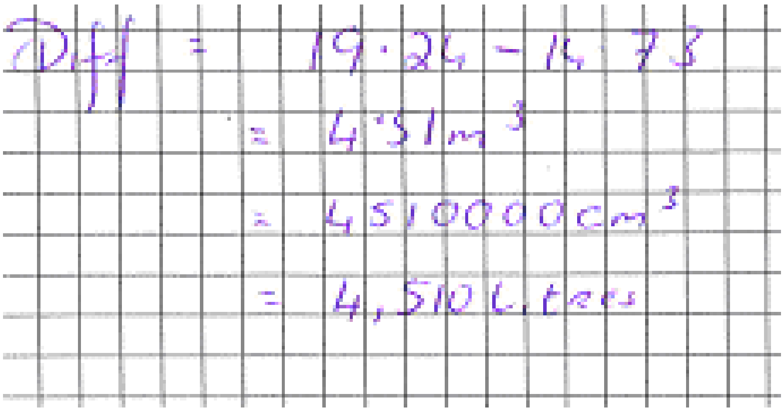


Increase height by 1m

$$V = \pi\left(\frac{2 \cdot 5}{2}\right)^2 3$$

$$V = \pi(1 \cdot 56)3$$

$$V = 14 \cdot 73\,m^3$$

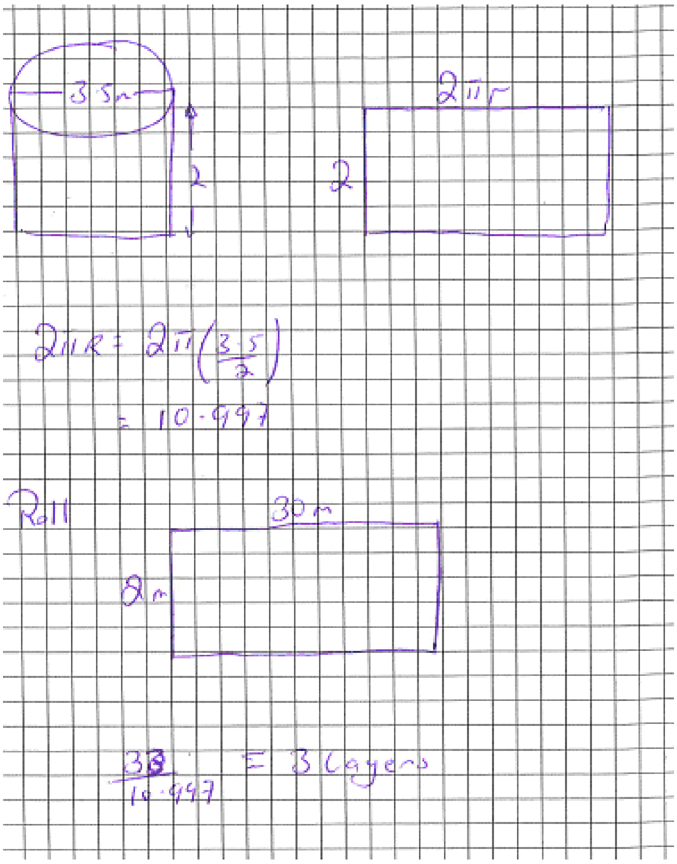


$$\text{Diff} = 19 \cdot 24 - 14 \cdot 73$$

$$= 4 \cdot 51\,m^3$$

$$= 4510000\,cm^3$$

= 4,510 Litres



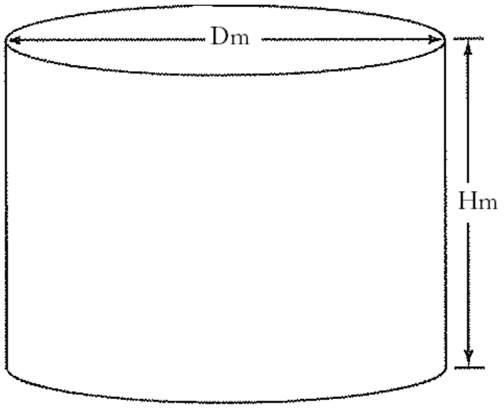
$2\pi R = 2\pi\left(\frac{3.5}{2}\right)$

$= 10.997$

$\frac{33}{10.997} = 3$ Layers

Task 15: LCHL

The diagram shows a water-storage tank proposed for the Byrne family home.



The Byrnes want to get a new tank to increase the volume of water they can store.

They can either increase the diameter by **1m** or increase the height by **1m**.

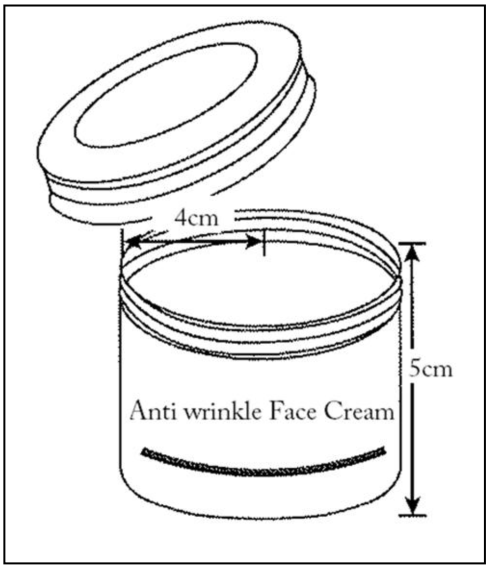
What is the difference, in litres, in the volume obtained by increasing the diameter by **1m** and the volume obtained by increasing the height by **1m**?

Task 16: LCOL

A cosmetic company manufacturing face cream wants to use larger jars.

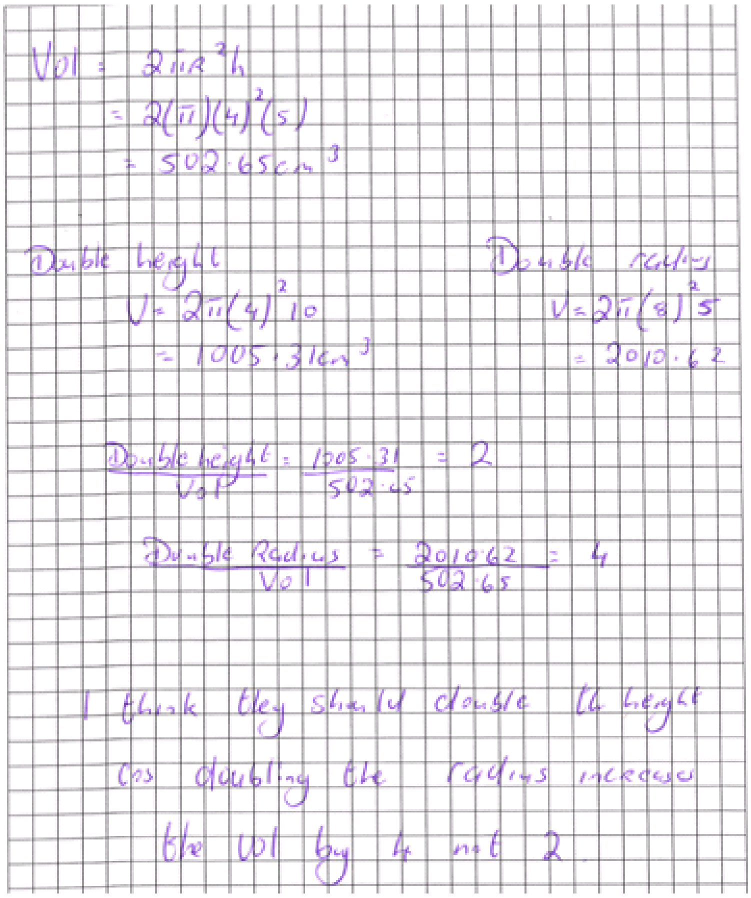
The new jar is to have exactly twice the volume of the current jar.

Is it best to double the height of the jar or double the radius of the jar?



Explain your thinking.

Compare, Examine, Discuss and Evaluate



$Vol = 2\pi R^2 h$

$= 2(\pi)(4)^2(5)$

$= 502{\cdot}65cm^3$

Double height

$V = 2\pi(4)^2 10$

$= 1005{\cdot}31cm^3$

Double radius

$V = 2\pi(8)^2 5$

$= 2010{\cdot}62$

$\frac{\text{Double height}}{\text{Vol}} = \frac{1005{\cdot}31}{502{\cdot}65} = 2$

$\frac{\text{Double Radius}}{\text{Vol}} = \frac{2010{\cdot}62}{502{\cdot}65} = 4$

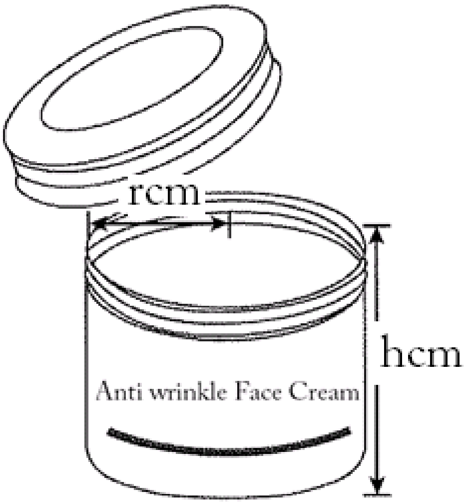
I think they should double the height cos doubling the radius increase the vol by 4 not 2.

Task 17: LCHL

A cosmetic company manufacturing face cream wants to use larger jars.

The new jar is to have exactly twice the volume of the current jar.

Is it best to double the height of the jar or double the radius of the jar?



Explain your thinking.

Compare, Examine, Discuss and Evaluate

$V = \pi r^2 h$

Double r $\quad V = \pi (2r)^2 h$

$= 4\pi R^2 h$

$= 4V$

Double h : $V = \pi r^2 (2h)$

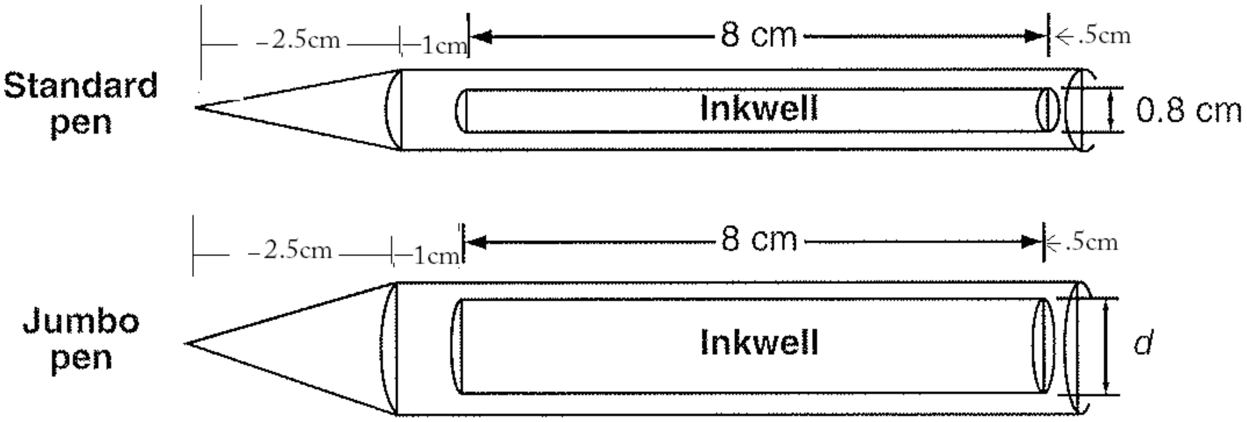
$= 2\pi r^2 h$

$= 2V$

So best to double height.

Task 18 LCHL

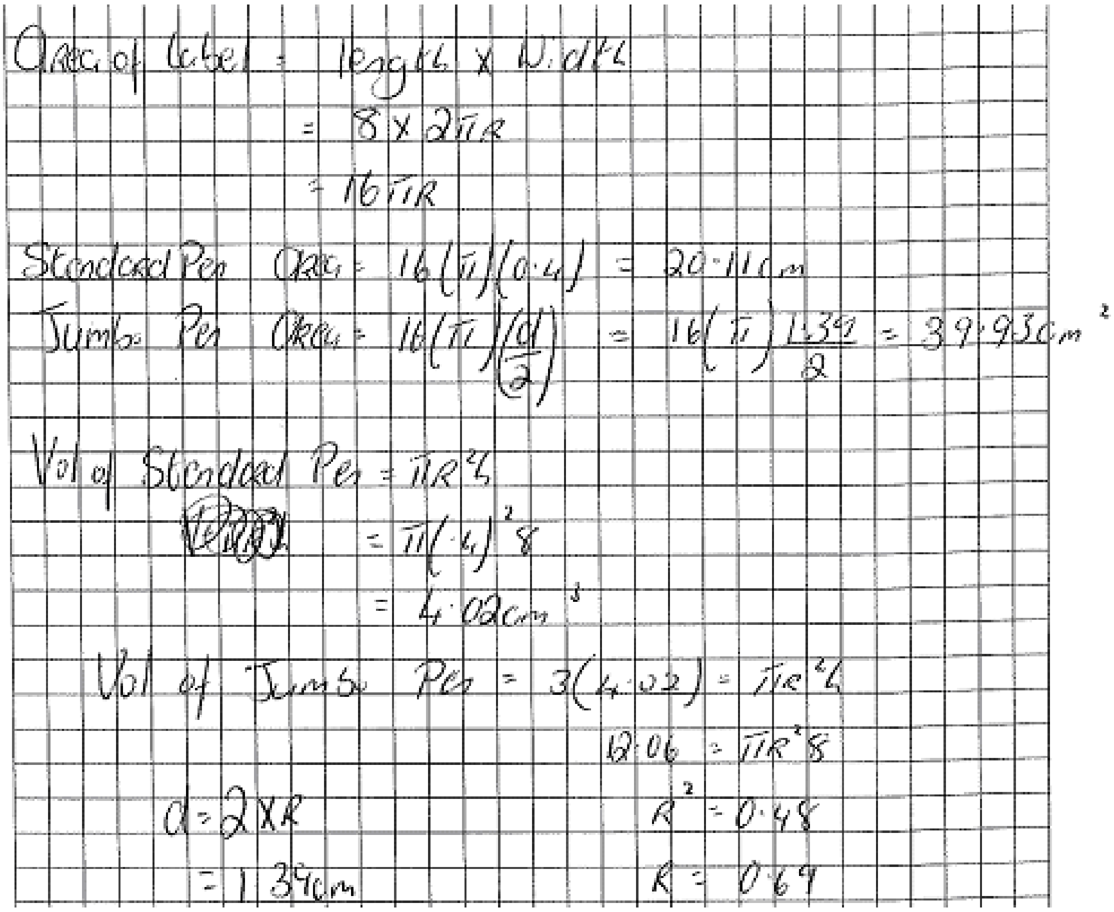
A company makes two sizes of pens similar to the ones shown below.



The **Jumbo pen** holds three times as much ink as the **Standard pen** and the company claims that it lasts three times longer.

A paper label surrounds the inkwell of each pen. What is the area of each label?

Compare, Examine, Discuss and Evaluate



Area of label = length x Width

$= 8 \times 2\pi R$

$= 16\pi R$

Standard Pen Area = $16(\pi)(0.4)$ = $20.11cm$

Jumbo Pen Area = $16(\pi)\left(\frac{d}{2}\right)$ = $16(\pi)\frac{1.39}{2}$ = $39.93cm^2$

Vol of Standard Pen = $\pi R^2 h$

$= \pi(.4)^2 8$

$= 4.02cm^3$

Vol of Jumbo Pen = $3(4.02) = \pi R^2 h$

$12.06 = \pi R^2 8$

$R^2 = 0.48$

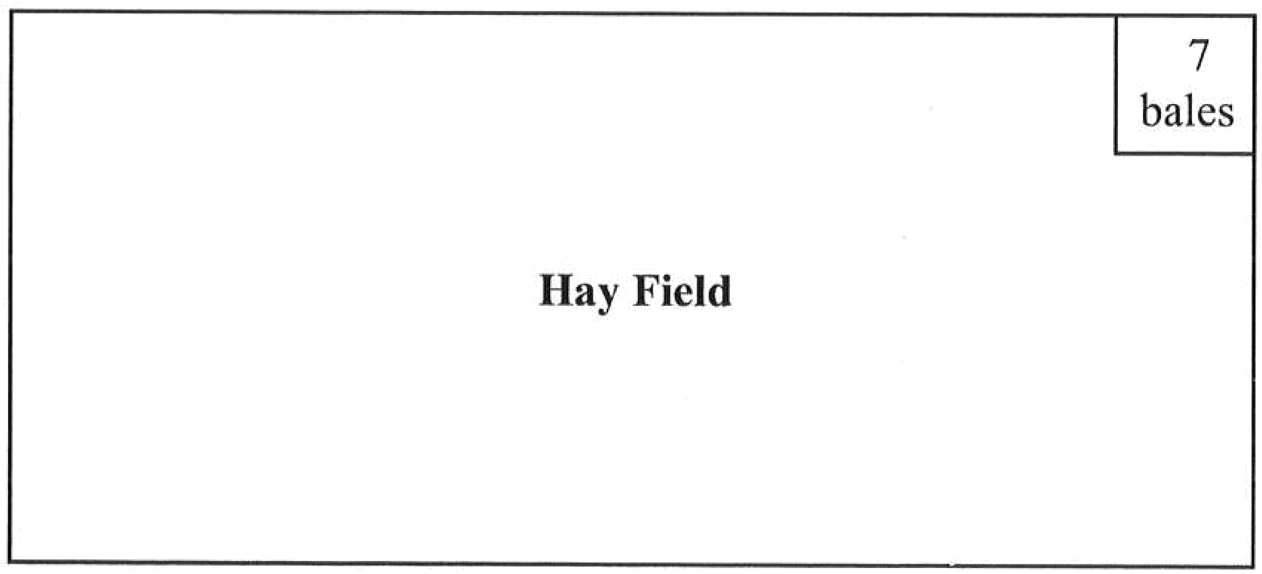
$R = 0.69$

$d = 2 \times R$

$= 1.39cm$

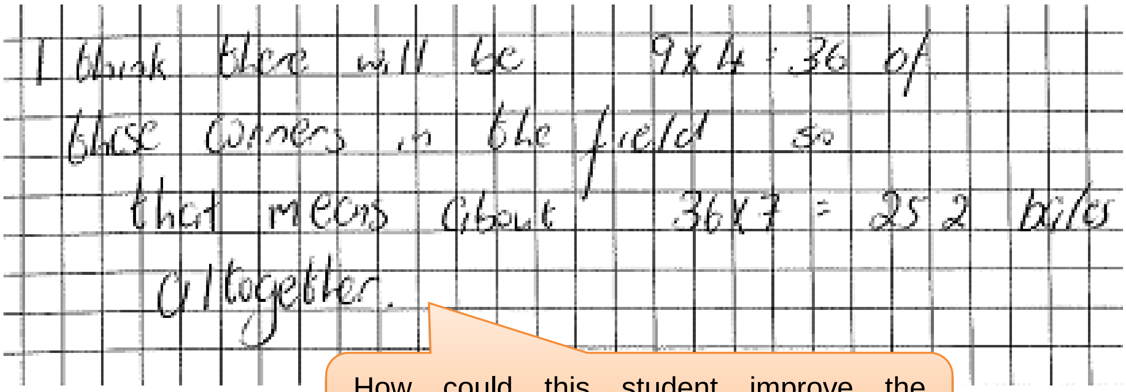
Task 19	Exploration, investigation and discussion Strand 3
Level	**LCFL**
Learning outcome	This material provides you with the opportunity to display evidence that you can – estimate the world around you

A farmer is baling hay; he got 7 bales from the corner of the field shown.



Estimate the number of bales in the whole field. Explain your thinking.

Compare, Examine, Discuss and Evaluate



I think there will be 9x4 = 36 of these corners in the field so that means about 36x7 = 252 bales altogether.

How could this student improve the accuracy of their estimate?

Task 20	Exploration , investigation and discussion Strand 3
Level	**LCFL**
Learning outcome	This material provides you with the opportunity to display evidence that you can – develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place value understanding

Students were sorting glassware in their science laboratory. The measures of the thickness of the glass are shown in the table below

Glass item	**Glass thickness (cm)**
A	1/8
B	0.25
C	3/16
D	0.5

Sort the glassware in order from the thinnest to the thickest. Explain your working.

Section B

Task 1	Exploration , investigation and discussion Strand 4
Level	LCHL
Learning outcome	This question gives you the opportunity to display **evidence** that you can – work with complex numbers in rectangular and polar form to solve quadratic and other equations including those in the form $z^n = a$, where $n \in \mathbf{Z}$, and $z = r \text{ Cos } \theta + i\text{Sin } \theta$ – use De Moivre's Theorem – use applications such as n^{th} roots of unity, $n \in \mathbf{N}$

Task 1 LCHL

With w denoting either of the two complex cube roots of unity, find

$$\frac{2w + 1}{5 + 3w + w^2} + \frac{2w^2 + 1}{5 + w + 3w^2}$$

giving your answer as a fraction a/b, where a, b are integers with no factor in common.

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how they solved the problem: Complex 1

Well, when I looked at this first I thought… right.. what I have here is two fractions so I should get the common denominator and simplify them so that was what I was going to do and then I looked at them and I thought this is going to be awful because I have two quadratic equations as the denominators and I'm going to have powers of 4 and all sorts....so I looked again and I thought how can the fact that w is one of the two complex cube roots of unity help me here?....

I worked out the cube roots of unity are 1, $-\frac{1}{2}$ minus root 3 over 2i, and $-\frac{1}{2}$plus root 3 over 2i, that is 1, w and w^2.

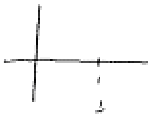
I know w cubed = 1 and $1 + w + w^2 = 0$ because the three roots of unity add to 1

So that really helped me solve the problem.

$z = 1 + 0i$

In Polar form

$$z = 1(\text{Cos}\,2\pi m + i\,\text{Sin}\,2\pi m)$$



$$z^{\frac{1}{3}} = (1+0i)^{\frac{1}{3}} = \text{Cos}\,\frac{2\pi m}{3} + i\,\text{Sin}\,\frac{2\pi m}{3} \qquad m = 0, 1, 2$$

$m = 0 \quad z^{\frac{1}{3}} = \text{Cos}\,0 + i\,\text{Sin}\,0 = 1$

$m = 1 \quad z^{\frac{1}{3}} = \text{Cos}\frac{2\pi}{3} + i\,\text{Sin}\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$m = 2 \quad z^{\frac{1}{3}} = \text{Cos}\frac{4\pi}{3} + i\,\text{Sin}\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

So there are 3 cubed roots of unity

$$\underset{1}{1}, \quad \underset{\omega}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}, \quad \underset{\omega^2}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$$

So $\omega^3 = 1$ and $\omega^2 + \omega + 1 = 0$

$$\frac{2\omega+1}{5+3\omega+\omega^2} + \frac{2\omega^2+1}{5+\omega+3\omega^2}$$

$$\frac{2\omega+1}{(\omega^2+\omega+1)+2\omega+4} + \frac{2\omega^2+2\omega+2-2\omega-1}{3\omega^2+3\omega+3-2\omega+2}$$

$$\frac{2\omega+1}{2\omega+4} + \frac{2(\omega^2+\omega+1)-2\omega-1}{3(\omega^2+\omega+1)-2\omega+2}$$

$$\frac{2\omega+1(2-2\omega) \cancel{-} (2\omega+1)(2\omega+4)}{(2\omega+4)(2-2\omega)}$$

$$\frac{4\omega-4\omega^2+2-2\omega-\left[4\omega^2+8\omega+2\omega+4\right]}{4\omega-4\omega^2+8-8\omega}$$

$$\frac{-4\omega^2+2\omega+2-4\omega^2-8\omega-2\omega-4}{-4\omega^2-4\omega+8}$$

$$\frac{-8\omega^2-8\omega-2}{-4\omega^2-4\omega+8}$$

$$\frac{-8\omega^2-8\omega-8+6}{-4\omega^2-4\omega-4+12}$$

$$\frac{-8(\omega^2+\omega+1)+6}{-4(\omega^2+\omega+1)+12} = \frac{6}{12} = \frac{1}{2}$$

Task 2	Exploration, investigation and discussion Strand 4
Level	**LCHL**
Learning outcome	This question gives you the opportunity to display **evidence** that you can – use the factor theorem for polynomials

It is important that you are able to interpret this theorem both geometrically and algebraically.

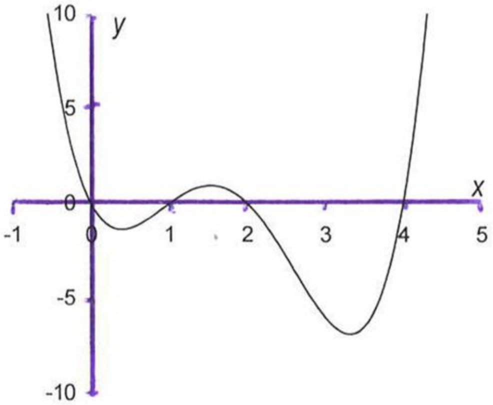
The **geometric** interpretation: *if f(x) is a polynomial whose graph crosses the x-axis at x = a, then (x-a) is a factor of f(x).*

The **algebraic** interpretation: *if f(x) is a polynomial and f(a) = 0, then (x-a) is a factor of f(x)*

Try the tasks below

Task 2:

The graph of the polynomial $y = x(x–1)(x–2)(x–4)$ is shown below.



Argue that the algebraic formula given in the form of factors allows you to see right away where the graph is above the *x*-axis, where it is below the *x*-axis and where it crosses the *x*-axis.

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how they completed the task 2

Well, first I thought where does the graph cross the x-axis? I know from the algebraic equation that it crosses at the points where $y = 0$. In the algebraic formula y is given as a product of factors, this is nice and handy for me because I know this will happen exactly when one of the factors is zero. Now the first factor is zero only at $x = 0$, and the second factor is zero only at $x = 1$, and the third and fourth factors are zero only at $x = 2$ and $x = 4$. So I can say that the graph crosses the x-axis at the points $x = 0$, 1, 2 and 4, and nowhere else. And that's exactly what I see in the diagram.

Now I need to know where y is positive and where it's negative.

Since y is a product, I can determine this if I know the signs of all the factors. It can be tedious working out the sign of each of the factors every time I have a new point but I can do the whole real line in one continuous swoop.

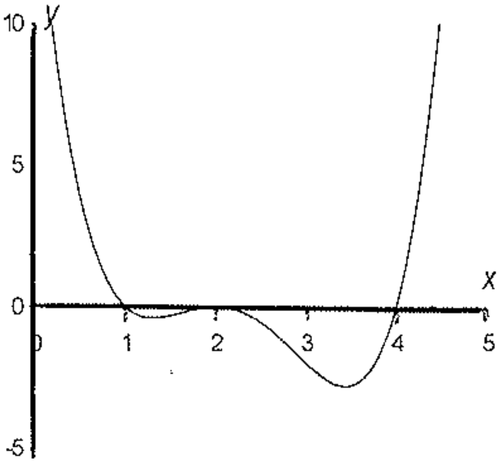
I am going to start with the value of x that is above 4 and let it decrease—sort of going from right to left along the x-axis.

- At the beginning (x above 4) all the terms are positive so y must be POSITIVE.
- As x passes below 4, the $(x–4)$ factor changes sign, but nothing happens to the other factors, so the whole expression changes sign and y becomes NEGATIVE.
- As x passes below 2, the $(x–2)$ factor changes sign, but nothing happens to the other factors, so the whole expression changes sign again and y becomes POSITIVE.
- As x passes below 1, the $(x–1)$ factor changes sign, but nothing happens to the other factors, so the whole expression changes sign once more and y becomes NEGATIVE.
- Finally as x passes below 0, the (x) factor changes sign, but nothing happens to the other factors, so the whole expression changes sign yet again and y becomes POSITIVE.

If I was going to summarise my method I would say that I passed through the zeros one by one and recorded not the sign of y, but the change in the sign and I used that to update the sign.

Note: Sometimes the graph does not cross the x axis at a zero or a root, i.e. when y = 0, but rather it hits the axis and bounces back on the same side of the axis.

Example 2B: At x = 2 y= 0 so 2 is a "zero" or "root" of the equation but the graph doesn't cross the x axis. It hits the axis and bounces back down again.



At the other zeros or roots 1 and 4, the graph of the polynomial crosses the x axis.
So what is the difference?

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how they explained the difference: Example 2B
I'm just looking at the graph and I am thinking.. OK..I see 3 roots or zeros: 1, 2 and 4. I prefer to call these zeros because they are x values that make y zero. Starting at the biggest one 4 I think: OK, if 4 is a root of the polynomial or a zero; Then (x-4) must be a factor of the polynomial ..when x is above 4 the factor is positive and so are the values of y and that is why the graph is above the x axis and Positive…then as x falls below 4 ..that's when I 'm going back from 4 towards zero the factor (x-4) will become negative and the product of factors which is y will be negative and the graph goes negative below the x axis. Then x gets to 2 and hits the axis because it is a zero. This means that (x-2) is a factor and as x goes below 2 the factor (x-2) becomes negative and y should become positive but it doesn't; it stays negative. So the only way that can be is if the factor is not (x-2) but $(x-2)^2$, then when x drops below 2 the factor (x-2) becomes negative but the factor $(x-2)^2$ becomes positive, and there is no change in the sign of y. It stays negative so the graph bounces back down again. Then when x gets to 1 y is zero so (x-1) is a factor and when x drops below 1 the factor (x-1) becomes negative and the sign of y changes again to be positive.

So it's like if you see the graph hitting the x axis but not crossing it, it means one of the factors must be squared (i.e. of degree 2)…I think it would probably be a bit like that if it hit and bounced back and sort of did a loop without hitting again it would mean the factor was cubed or degree 3.
I think I could write out the polynomial now. It would be $f(x) = (x-1)\,(x-2)^2(x-4)$.
Now if I was given the polynomial factorised I could sketch the graph.

Task 3:

Below are the algebraic equations of a number of polynomials in factored form. In each case make a rough sketch of the graph and explain the basis of your decisions. Do not plot points; simply identify the roots or zeros and observe whether the graph crosses or does not cross the axis at each of these.

Note: without plotting points, you won't really know how "high" the graph gets between the zeros, but that is not the focus of this exercise.

(a) $y = x\,(x-1)\,(x-2)\,(x-3)\,(x-5)$

(b) $y = (x+1)\,(x-1)\,(x-3)\,(x-4)^2$

(c) $y = x\,(x-1)^2\,(x-2)^2$

(d) $y = (x-1)\,(x-2)^2\,(x-3)\,(x-4)^3$

(e) $y = x^2\,(x^2-1)\,(x-4)^2$

(f) $y = (x^2-x)\,(x^2-1)\,(x+1)$

(g) $y = x\,(x-1)\,(x-2)^2\,(x-3)^3\,(x-4)^4$

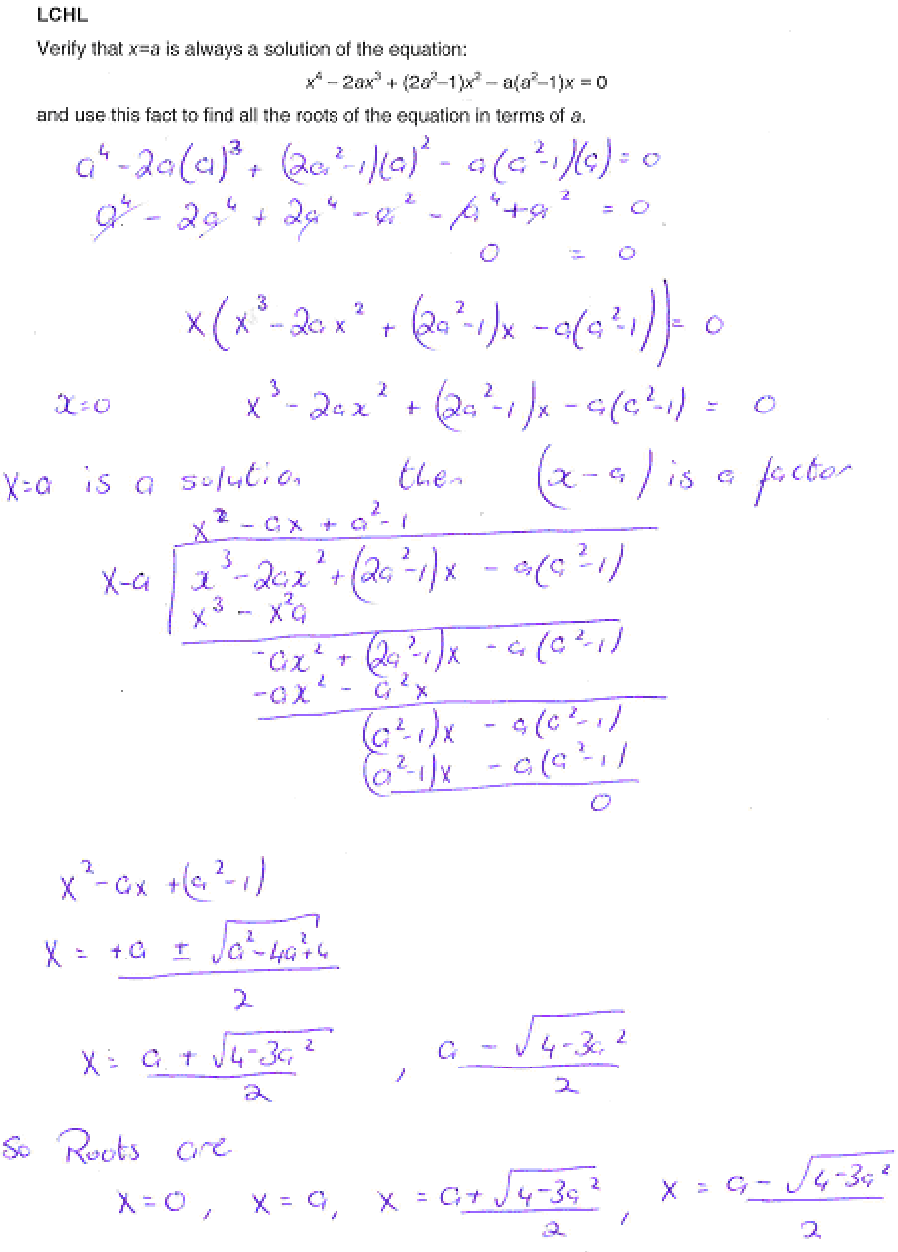
Task 4 LCHL

Verify that $x = a$ is always a solution of the equation:

$$x^4 - 2ax^3 + (2a^2-1)x^2 - a(a^2-1)x = 0$$

and use this fact to find all the roots of the equation in terms of a.

Compare, Examine, Discuss and Evaluate



LCHL

Verify that $x=a$ is always a solution of the equation:

$$x^4 - 2ax^3 + (2a^2-1)x^2 - a(a^2-1)x = 0$$

and use this fact to find all the roots of the equation in terms of a.

$$a^4-2a(a)^3+(2a^2-1)(a)^2-a(a^2-1)(a)=0$$

$$\cancel{a^4}-2a^4+2a^4-a^2-\cancel{a^4}+a^2=0$$

$$0=0$$

$$x\left(x^3-2ax^2+(2a^2-1)x-a(a^2-1)\right)=0$$

$x=0$ $\qquad x^3-2ax^2+(2a^2-1)x-a(a^2-1)=0$

$x=a$ is a solution then $(x-a)$ is a factor

$$\begin{array}{r|l}
 & x^2-ax+a^2-1 \\
\hline
x-a & x^3-2ax^2+(2a^2-1)x-a(a^2-1) \\
 & x^3-x^2a \\
\hline
 & -ax^2+(2a^2-1)x-a(a^2-1) \\
 & -ax^2-a^2x \\
\hline
 & (a^2-1)x-a(a^2-1) \\
 & (a^2-1)x-a(a^2-1) \\
\hline
 & 0
\end{array}$$

$$x^2-ax+(a^2-1)$$

$$x=\frac{+a\pm\sqrt{a^2-4a^2+4}}{2}$$

$$x=\frac{a+\sqrt{4-3a^2}}{2},\quad \frac{a-\sqrt{4-3a^2}}{2}$$

So Roots are

$$x=0,\quad x=a,\quad x=\frac{a+\sqrt{4-3a^2}}{2},\quad x=\frac{a-\sqrt{4-3a^2}}{2}$$

Task 5 LCHL

Explain where the graph of the polynomial $y= x(x-1)(x-2)(x-4)$

- crosses the x axis
- Is above the x axis
- Is below the x axis

Sketch the graph of the polynomial.

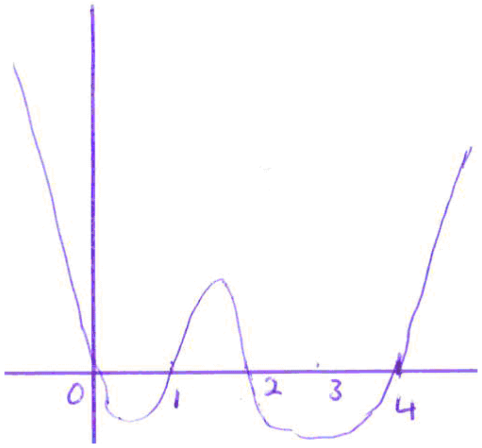
Compare, Examine, Discuss and Evaluate

It crosses the x axis at $X=0$, 1, 2 and 4

because when
$x=0$ $y=0(-1)(-2)-4)=0$
$x=1$ $y=1(0)(-1)(-3)=0$
$x=2$ $y=2(1)(0)(-2)=0$
$x=4$ $y=4(3)(2)(0)=0$

It is above the x axis when $x>4$ because the product of all the factors is positive so y is positive.

as x goes below 4 $(x-4)$ factor becomes negative so product of factors is neg and the graph drops below x axis and hits axis again at $x=2$ then as x goes below 2 $(x-2)$ factor becomes negative and the product becomes positive so y is positive and graph goes above the x axis again until $x=1$ and as x goes below 1 $(x-1)$ factor becomes neg and so does the product so graph goes below x-axis again till $x=0$ below 0 x is neg so product will be +ve and graph will go above the x axis


0
1
2
3
4

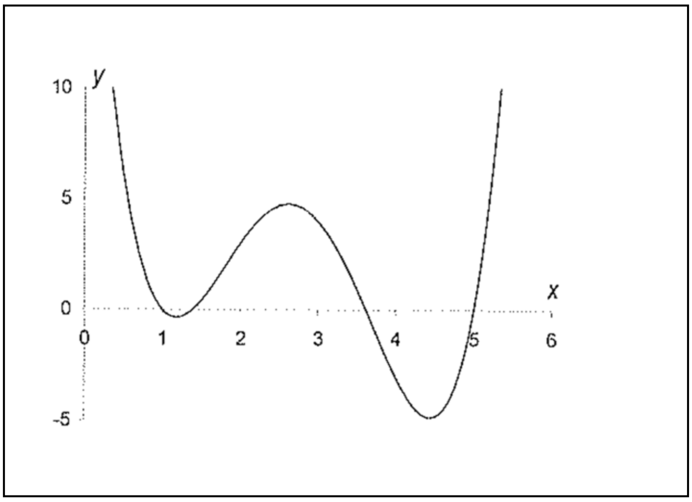
Task 6: LCHL

The graph of the polynomial is shown

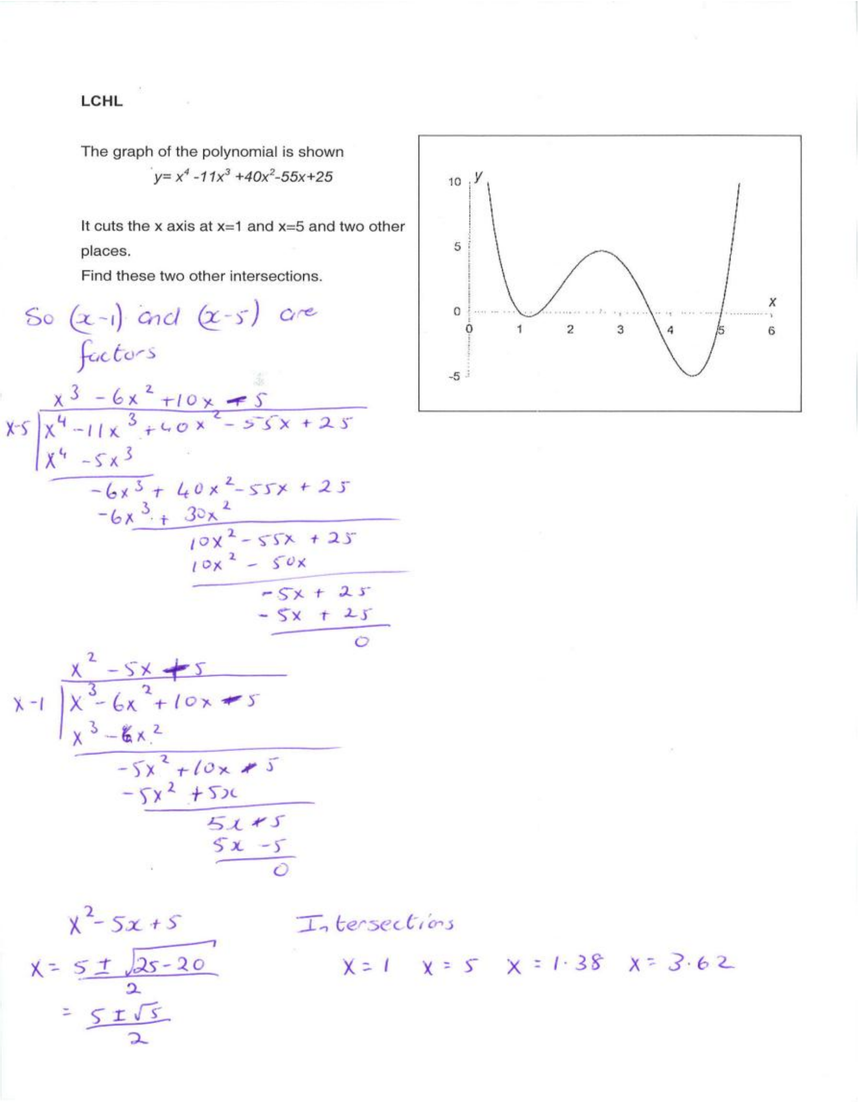
$y= x^4 -11x^3 +40x^2-55x+25$

It cuts the x axis at x=1 and x=5 and two other places.

Find these two other intersections.



Compare, Examine, Discuss and Evaluate



LCHL

The graph of the polynomial is shown

$y= x^4 -11x^3 +40x^2-55x+25$

It cuts the x axis at x=1 and x=5 and two other places.

Find these two other intersections.

So $(x-1)$ and $(x-5)$ are factors

$$x-5 \,\overline{)\, x^4-11x^3+40x^2-55x+25} \quad = x^3-6x^2+10x-5$$

x^4-5x^3

$-6x^3+40x^2-55x+25$

$-6x^3+30x^2$

$10x^2-55x+25$

$10x^2-50x$

$-5x+25$

$-5x+25$

0

$$x-1 \,\overline{)\, x^3-6x^2+10x-5} \quad = x^2-5x+5$$

x^3-6x^2

$-5x^2+10x-5$

$-5x^2+5x$

$5x-5$

$5x-5$

0

x^2-5x+5

$x=\frac{5\pm\sqrt{25-20}}{2}$

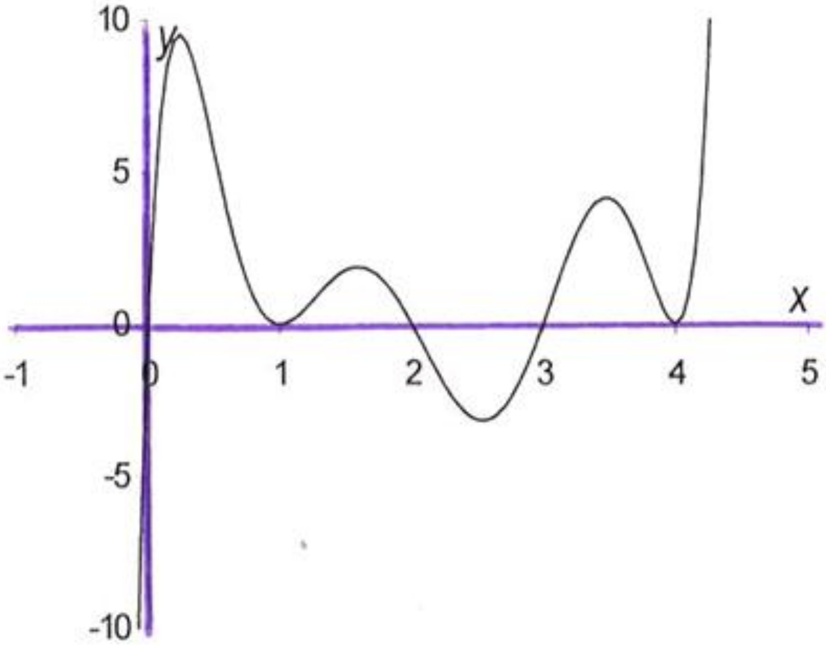
$=\frac{5\pm\sqrt{5}}{2}$

Intersections

$x=1 \quad x=5 \quad x=1.38 \quad x=3.62$

Task 7 LCHL

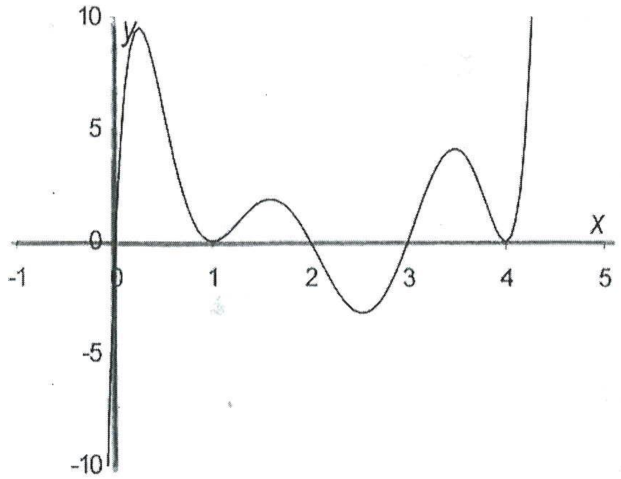
What is the algebraic equation of the polynomial whose graph is shown below? Explain your thinking.



Compare, Examine, Discuss and Evaluate

LCHL

What is the algebraic equation of the polynomial whose graph is shown below? Explain your thinking.



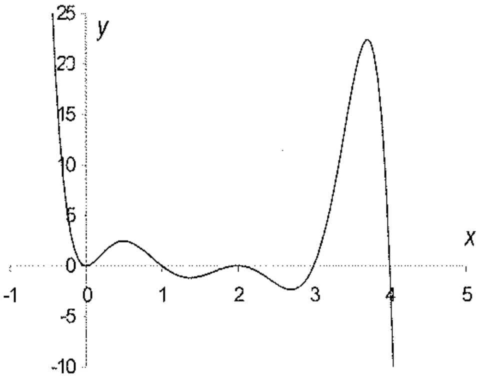
I can see from the graph that the roots or zeros of the polynomial are 0, 1, 2, 3 and 4 because this is where $y=0$ so that means x $(x-1)$ $(x-2)$ $(x-3)$ and $(x-4)$ are factors of the polynomial.

at $x=4$ the graph just hits the axis and doesn't cross it so this means the graph doesn't go from +ve to -ve it should if the factor is $(x-4)$ since this would be neg if $x<4$ so $(x-4)^2$ must be a factor and so must $(x-1)^2$ be a factor for the same reason that the graph just hits the x axis and doesn't cross it as x drops below 1

So polynomial is $y = x(x-1)^2(x-2)(x-3)(x-4)^2$

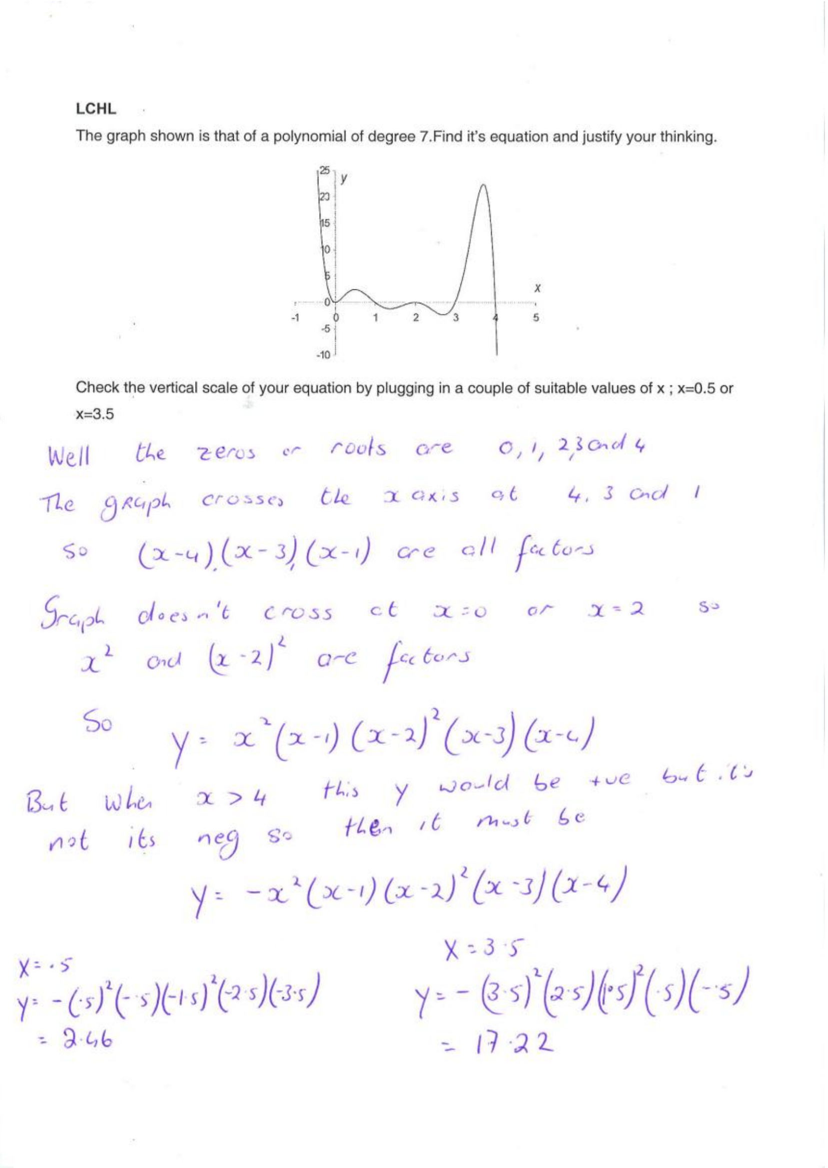
Task 8 LCHL

The graph shown is that of a polynomial of degree 7. Find its equation and justify your thinking.



Check the vertical scale of your equation by using a couple of suitable values of x: x=0.5 or x=3.5

Compare, Examine, Discuss and Evaluate



LCHL

The graph shown is that of a polynomial of degree 7.Find it's equation and justify your thinking.

Check the vertical scale of your equation by plugging in a couple of suitable values of x ; x=0.5 or x=3.5

Well the zeros or roots are 0, 1, 2, 3 and 4

The graph crosses the x axis at 4, 3 and 1

so $(x-4), (x-3), (x-1)$ are all factors

Graph doesn't cross at $x=0$ or $x=2$ so x^2 and $(x-2)^2$ are factors

So $y = x^2(x-1)(x-2)^2(x-3)(x-4)$

But when $x > 4$ this y would be +ve but it's not its neg so then it must be

$$y = -x^2(x-1)(x-2)^2(x-3)(x-4)$$

$X = .5$

$y = -(.5)^2(-.5)(-1.5)^2(-2.5)(-3.5)$

$= 2.46$

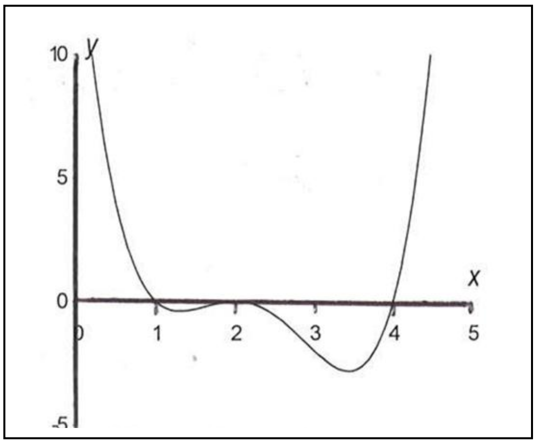
$X = 3.5$

$y = -(3.5)^2(2.5)(1.5)^2(.5)(-.5)$

$= 17.22$

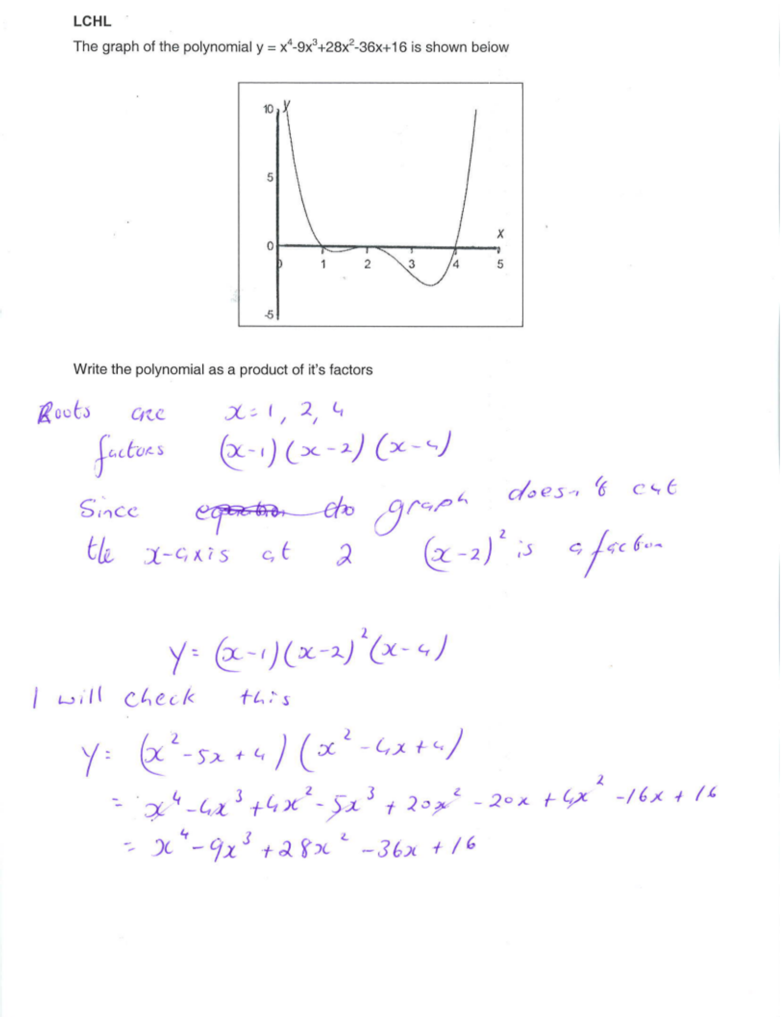
Task 9 LCHL

The graph of the polynomial $y = x^4-9x^3+28x^2-36x+16$ is shown below



Write the polynomial as a product of its factors. Explain your thinking

Compare, Examine, Discuss and Evaluate



LCHL

The graph of the polynomial $y = x^4-9x^3+28x^2-36x+16$ is shown below

Write the polynomial as a product of it's factors

Roots are $x = 1, 2, 4$

factors $(x-1)(x-2)(x-4)$

Since ~~equation~~ the graph doesn't cut the x-axis at 2 $(x-2)^2$ is a factor

$$y = (x-1)(x-2)^2(x-4)$$

I will check this

$$y = (x^2-5x+4)(x^2-4x+4)$$

$$= x^4-4x^3+4x^2-5x^3+20x^2-20x+4x^2-16x+16$$

$$= x^4-9x^3+28x^2-36x+16$$

Section C

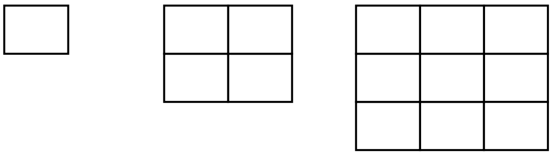
Exploration	Exploration, investigation and discussion. Strand 3, 4
Level	JCHL/ LCFL/OL
Learning outcome	Seeing patterns, making generalisations and giving explanations for them promotes a way of thinking that underpins school algebra. This activity provides you with the opportunity to develop your algebraic reasoning by engaging in problems that encourage you to – use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence – use tables, diagrams and graphs as tools for representing and analysing patterns and relations – develop and use their own generalising strategies and ideas and consider those of others Throughout the activity you will be examining non-constant rates of change and LC learners will have the opportunity to make explicit the link between **linear functions** and **arithmetic sequences** and **exponential functions** and **geometric sequences**. These activities provide an ideal segue to the investigation of **series** and **logarithms.** The slope presentation will also help you as you work through these problems.

Prior knowledge: This activity introduces you to non-linear relations and provides you with the opportunity to further examine relationships between the representation, the table and the graph. It is essential that you have engaged with linear relationships before embarking on this work where you look in detail at the concept of “rate of change” and what this means in the different representations. Try the slope presentation now, it will help you reinforce your understanding of linear relationships.

Note to Students: The focus of the first four questions is on making tables and graphs and not formulas. There is an opportunity to work on formulas when you get to problem 5.

Exploration 1

Growing Squares: Extend this table for the number of tiles needed to make squares with side lengths 1,2,3….up to 10. Make observations about the values in the table. What would a graph for this situation look like? Will it be linear? How do you know? Make a graph to check your prediction.



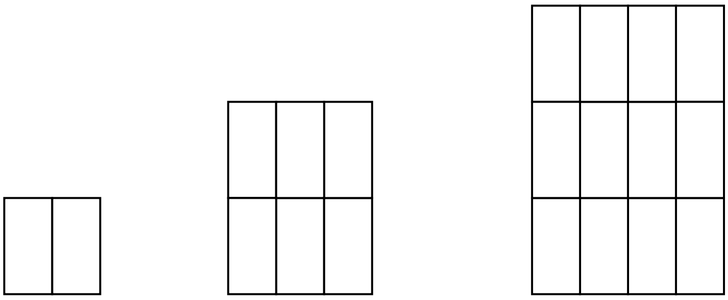
Side Length	Number of Tiles
1	1
2	4
3	9
4	16
5	25

Exploration 2

Growing Rectangles: Look at the pattern of growing rectangles below. Continue the table up to rectangles with a height of 10. Use drawings if you need to. Make observations about the values in the table

What would a graph for this table look like? Will it be linear? How do you know?

Make a graph to check your prediction.



Height	No of Tiles
1	2
2	6
3	12
4	?

Exploration 3

Pocket money story

I ask my Dad for pocket money. All I want, I say is for you to give me pocket money for this month. Give me 2 cent on the 1st day of the month, double that for the 2nd day, and double that for the 3rd day and so on. On the first day I will get 2 cents; on the 2nd day 4 cents; on the 3rd day 8 cents, etc .That is all I want.

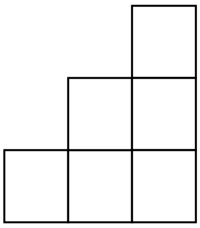
Make a table to show how much money I will get each day for the first 10 days of the month. Make observations about the values in the table. What would a graph look like? Would it be linear? How do you know? Make a graph to check your prediction.

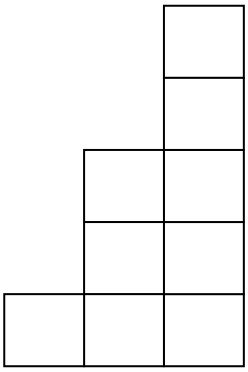
Exploration 4

Staircase Towers: Look at the staircases below. Make a table expressing the relationship between the total number of tiles and the number of towers. Make observations about the values in the table. What would a graph look like? Would it be linear? How do you know?

Make a graph to check your prediction.

Number of Towers	Number of Tiles
1	1
2	3
3	6





No of Towers	No of Tiles
1	1
2	4
3	9

Exploration 5

In the problems in **exploration 1 – 4** you have looked at tables and graphs for the situations. Now work to express the relationships by finding an expression for the 50th or 100th case and then by finding an expression for the nth case.

As you work through the questions, think about the properties of linear relationships compared with non-linear ones.

- How are the situations different?
- How are the tables different?
- How are the graphs different?
- How are the generalised expressions different?

A distinguishing feature of quadratic and exponential relations is the way the changes vary. These are key ***ideas to arrive at.***

In a group discussion, compare the tables and graphs for ***growing rectangles***, ***staircase towers*** and the ***pocket money problem***.

Think about the following

- what is the same about the graphs and tables?
- what is different about the graphs and tables?
- how is the non-linear nature of the graph related to the tables and the context?

Focus on how the numbers change in each column; for example, for ***growing rectangles*** and ***staircase towers.***

Here are some student comments

"*the numbers grow in a particular way in the right column, but it's different from the left. They go up in 1 in the left and in **growing rectangles** they go up by 4 and 6 and 8, it's not the same but in a pattern, 2 extra each time. In **staircase towers** they go up in 2 and 3 and 4 – again, a pattern, 1 extra each time*."

Compare the increase in the right column for ***growing rectangles*** and the first ***staircase towers.*** This is a very important observation***;*** the **change of the change** increase by 1 each time in the first staircase towers and the **change of the change** increase by 2 each time in growing rectangles.

The rate of change of the changes therefore for ***growing rectangles*** is twice as fast as the rate of change of the changes for ***staircase towers***. Now focus on how this observation manifests itself in the graphs.

Once the connection between the changes of the changes has been made, analyse the tables for each of the relations and see how each is manifested in the table.

Refer to the concept of slope presentation.

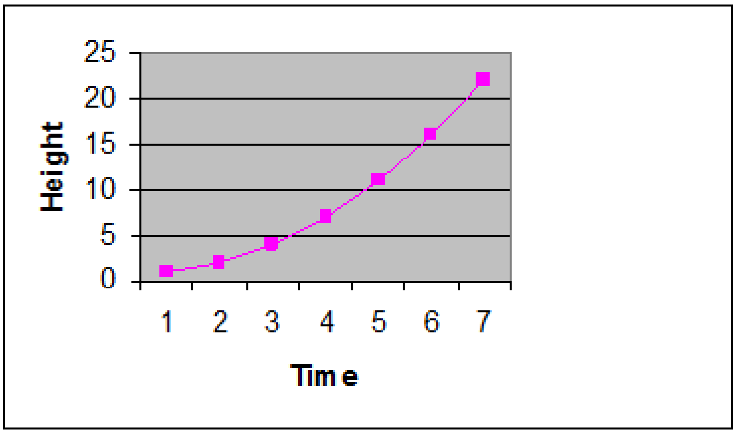
Exploration 6

Consider the case of a fantasy animal called Walkasaurus. The following table shows how the Walkasaurus's height changes with time.

Age (Years)	Height (cm)
0	1
1	2
2	4
3	7
4	11
5	16
6	22

Would a graph of this table be linear? How can you tell?

Now look at the graph.

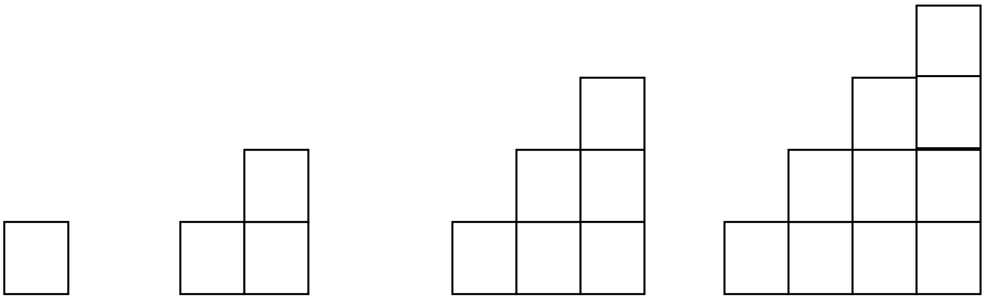


Explain why the graph looks like it does. Which of the problems in the previous *Growing Patterns* assignments is this similar to? How is it similar and how is it different?

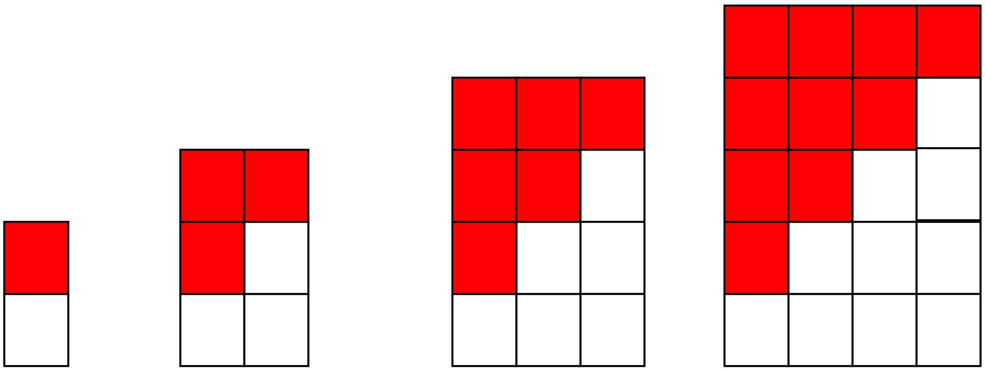
Note:

The contexts for the problems in this section have been carefully chosen to give relationships that can be compared. These comparisons can be used to help find the formula that define the relations.

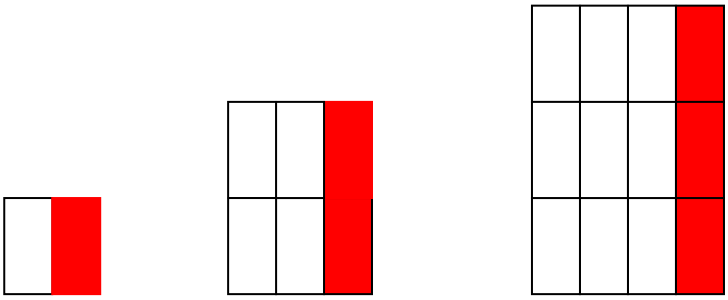
Consider looking for the formula for the staircase tower function.



Look at the following representation. It is double the original.



See that this is very similar to the growing rectangles situation.



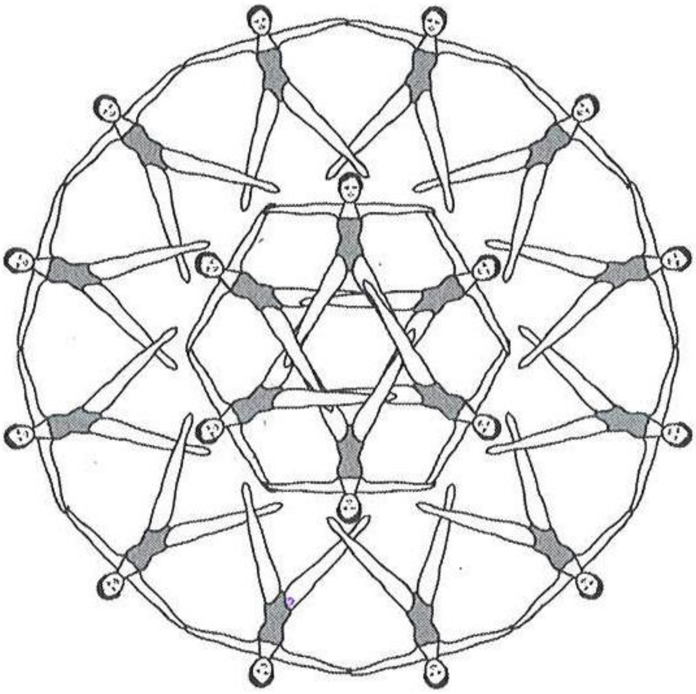
So, a connection can be made between the formulae.

Similarities can be seen between the table for Walkasaurus and that in the staircase tower problem in so far as the changes are similar. An important difference however is the fact that Walkasaurus' height begins at year 0 whereas the staircase tower table begins at 1. Try this!

Graphing these two on the same axis will make it easy for you to see that the curve is the same, just that one starts higher than the other. The formula for Walkasaurus therefore will be similar to that of the staircase tower but will need to be adjusted upwards by 1. Try this!

Task 1: LCFL

Sophie is planning for a synchronised swimming show. The diagram shows the pattern she wants to create.



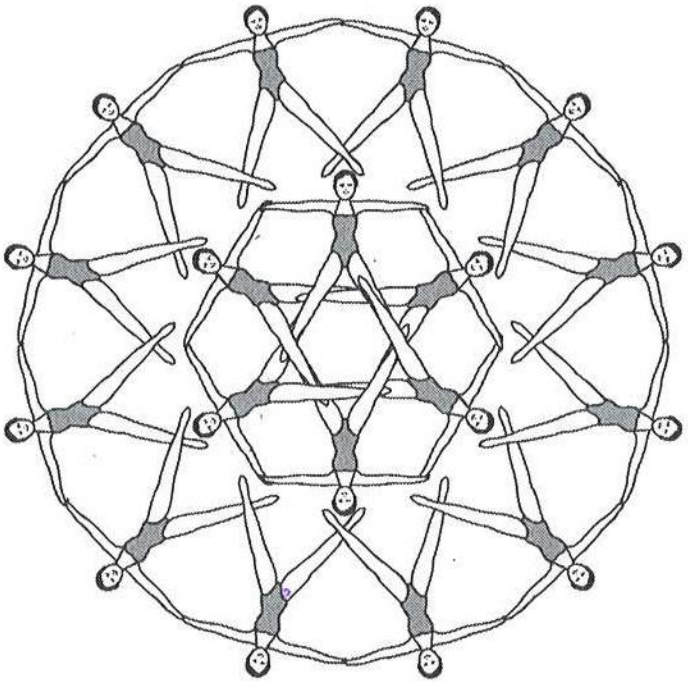
How many more swimmers would she need if she were to add an extra ring to the pattern? Explain your thinking.

If she were to add a 4th ring to the pattern how many swimmers would be in this ring?

Explain the pattern in words.

Task 2: LCHL

Sophie is planning for a synchronised swimming show. The diagram shows the pattern she wants to create**.**



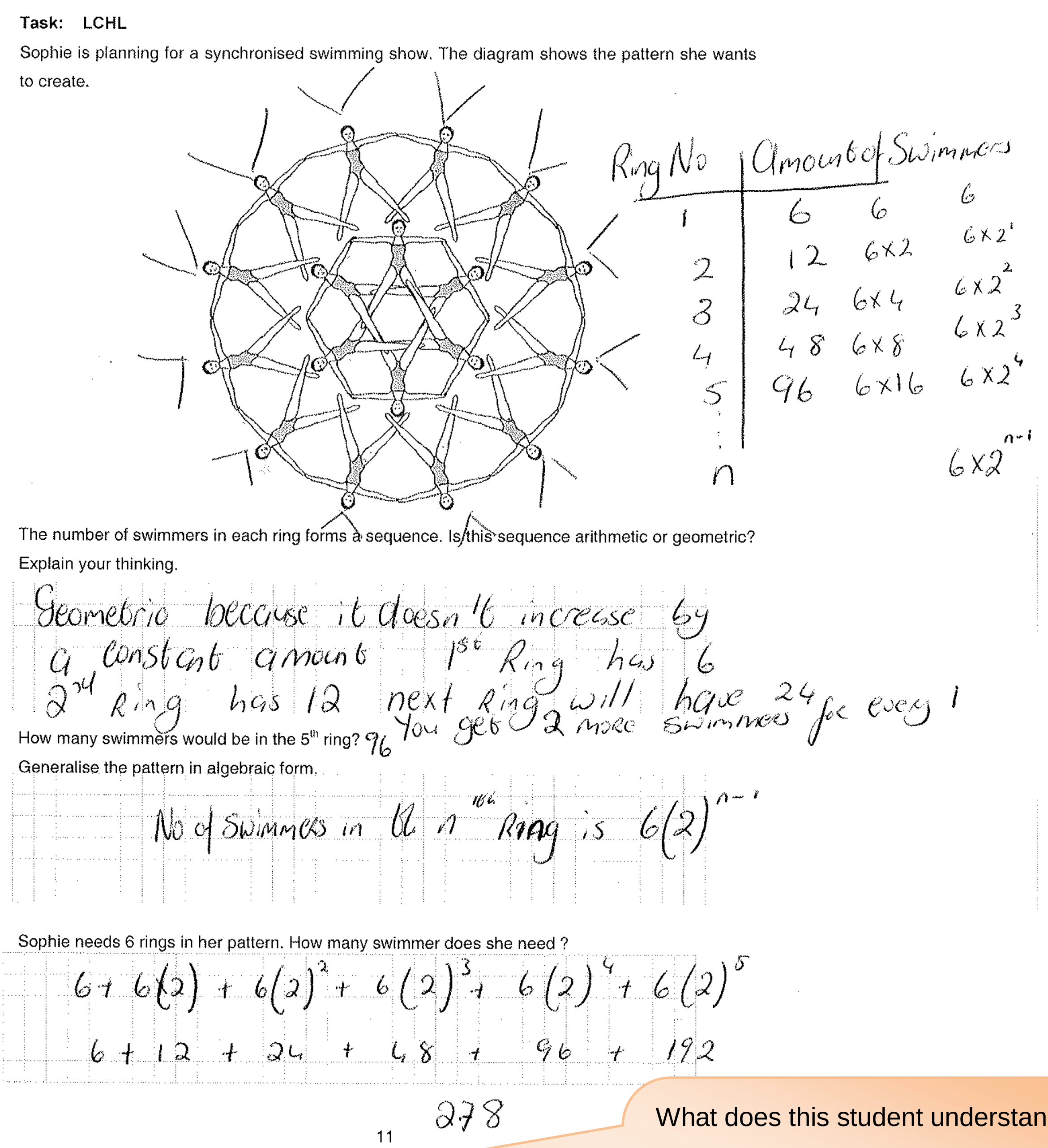
The number of swimmers in each ring forms a sequence. Is this sequence arithmetic or geometric? Explain your thinking.

How many swimmers would be in the 5th ring?

Generalise the pattern in algebraic form.

Sophie needs 6 rings in her pattern. How many swimmers does she need ?

Compare, Examine, Discuss and Evaluate



Task: LCHL

Sophie is planning for a synchronised swimming show. The diagram shows the pattern she wants to create.

Ring No	Amount of Swimmers		
1	6	6	6
2	12	6x2	6×2^1
3	24	6x4	6×2^2
4	48	6x8	6×2^3
5	96	6x16	6×2^4
⋮			
n			$6 \times 2^{n-1}$

The number of swimmers in each ring forms a sequence. Is this sequence arithmetic or geometric? Explain your thinking.

Geometric because it doesn't increase by a constant amount 1st Ring has 6 2nd Ring has 12 next Ring will have 24 for every 1 You get 2 more swimmers

How many swimmers would be in the 5th ring? 96

Generalise the pattern in algebraic form.

No of Swimmers in the n^{th} Ring is $6(2)^{n-1}$

Sophie needs 6 rings in her pattern. How many swimmer does she need ?

$6+6(2)+6(2)^2+6(2)^3+6(2)^4+6(2)^5$

$6+12+24+48+96+192$

278

11

What does this student understand? Evaluate the method. Are there any limitations? How would you know how many swimmers would be need for a pattern with 68 rings?

Task 3: LCOL

In a lecture theatre there are 28 seats in the first row, 29 seats in the 2nd row, 30 seats in the third row, and so on.

How many seats are in row 10 of the lecture theatre?

The pattern of seats forms a sequence. Is this an arithmetic sequence or a geometric one ?

Explain your thinking.

If there are 70 seats in the last row of the lecture theatre, how many rows are in the theatre?

How many seats are there in total in the first 20 rows?

Compare, Examine, Discuss and Evaluate

Task: **LCOL**

In a lecture theatre there are 28 seats in the first row, 29 seats in the 2^{nd} row, 30 seats in the third row, and so on.

How many seats are in row 10 of the lecture theatre?

Row No	Amount of Seats
1	28
2	29
3	30
4	31
5	32

$\overset{+1}{\frown}\ \overset{+1}{\frown}\ \overset{+1}{\frown}\ \overset{+1}{\frown}$

28, 29, 30, 31, 32 . . . 70

0 1 2 3

No of Seats = 28 + 1(Row No)

The pattern of seats forms a sequence. Is this an arithmetic sequence or a geometric one ?
Explain your thinking.

Arithmetic because it goes up by 1 each time

~~Amount of Seats = 28+1(No of Rows)~~

There are 70 seats in the last row of the lecture theatre. How many rows are there in the lecture theatre?

How many seats are there in total in the first 20 rows?

70 − 28 = 42

$$70,\ 69,\ 68,\ 67$$

$$\downarrow \qquad \searrow$$

28+1(No of Rows) 28+1(Row No)

421

28+1(42) 28+1(41)

28+29+30+31+32=2(60)+30
33+34+35+36+37=2(70)+35
38+39+40+41+42=2(80)+40
43+44+45+46+47=2(90)+45

120+140+160+180+150

500+250

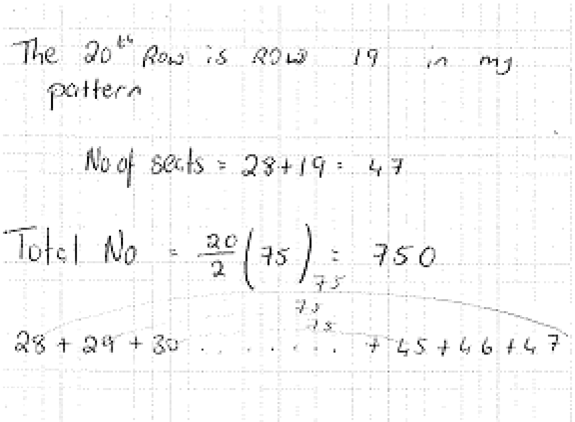
$\boxed{750}$

So 70 in Row No 42
means 43 Rows cos I
started at zero

What does this student mean by the statement …"I started at zero."?

Examine this strategy can you extend it for larger numbers? Can you generalise it for any numbers?

Examine the work of this student as they answer the question ‘*How many seats are there in the first 20 rows of the lecture theatre?’*



The 20th Row is ROW 19 in my pattern

No of seats = 28+19 = 47

Total No $= \frac{20}{2}(75) = 750$

75

75

75

28 + 29 + 30 . . . . + 45 + 46 + 47

Task 4: LCOL

Jason examined the total in his savings account over a number of months and recorded the amounts in the table below.

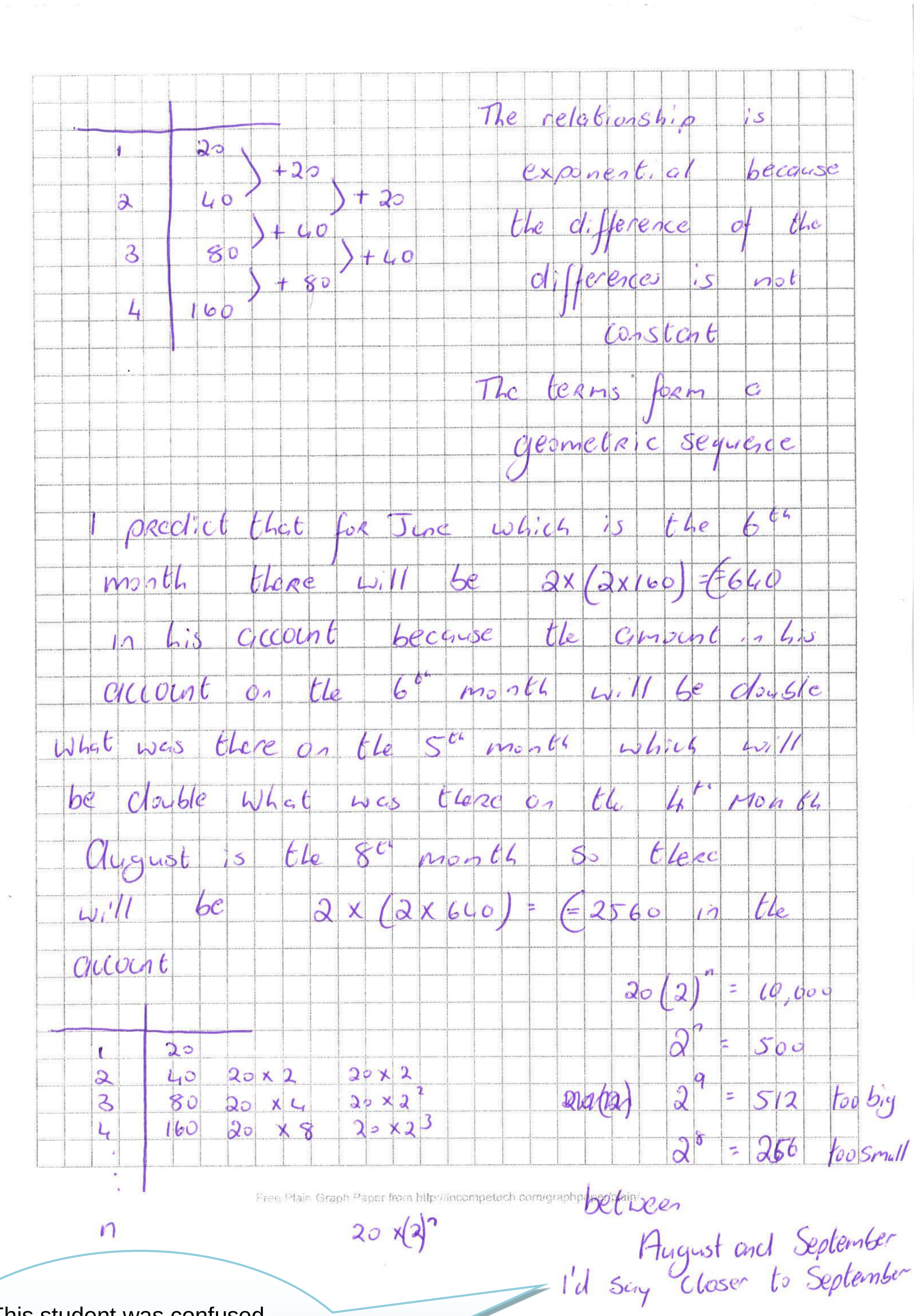
Month	**Amount (€)**
January	20
February	40
March	80
April	160

(a) Predict how much will be in his account in a) June, b) August? Explain your prediction.

(b) Write a general expression describing Jason's pattern of savings.

(c) If Jason continues to save in this way when will he have €10,000 in his account?

Compare, Examine, Discuss and Evaluate



1	20		
		) +20	
2	40		) + 20
		) + 40	
3	80		) + 40
		) + 80	
4	160		

The relationship is exponential because the difference of the differences is not constant

The terms form a geometric sequence

I predict that for June which is the 6th month there will be 2x(2x160) = €640 in his account because the amount in his account on the 6th month will be double what was there on the 5th month which will be double what was there on the 4th month

August is the 8th month so there will be 2 x (2 x 640) = €2560 in the account

$20(2)^n = 10,000$

$2^n = 500$

$2^9 = 512$ too big

$2^8 = 256$ too small

between August and September

I'd say closer to September

1	20		
2	40	20 x 2	20 x 2
3	80	20 x 4	20×2^2
4	160	20 x 8	20×2^3
⋮			
n		$20 \times (2)^n$	

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This student was confused. The generalised expression is incorrect. Can you see why? Correct the student's mistake.

Task 5: LCOL

You begin a biology experiment with 10 amoeba in a Petri dish and record the number of amoeba every minute in the table below

Time (mins)	No of amoeba
0	10
1	20
2	40
3	80

(a) How many amoeba will there be after 5 mins? Explain how you arrived at your answer.

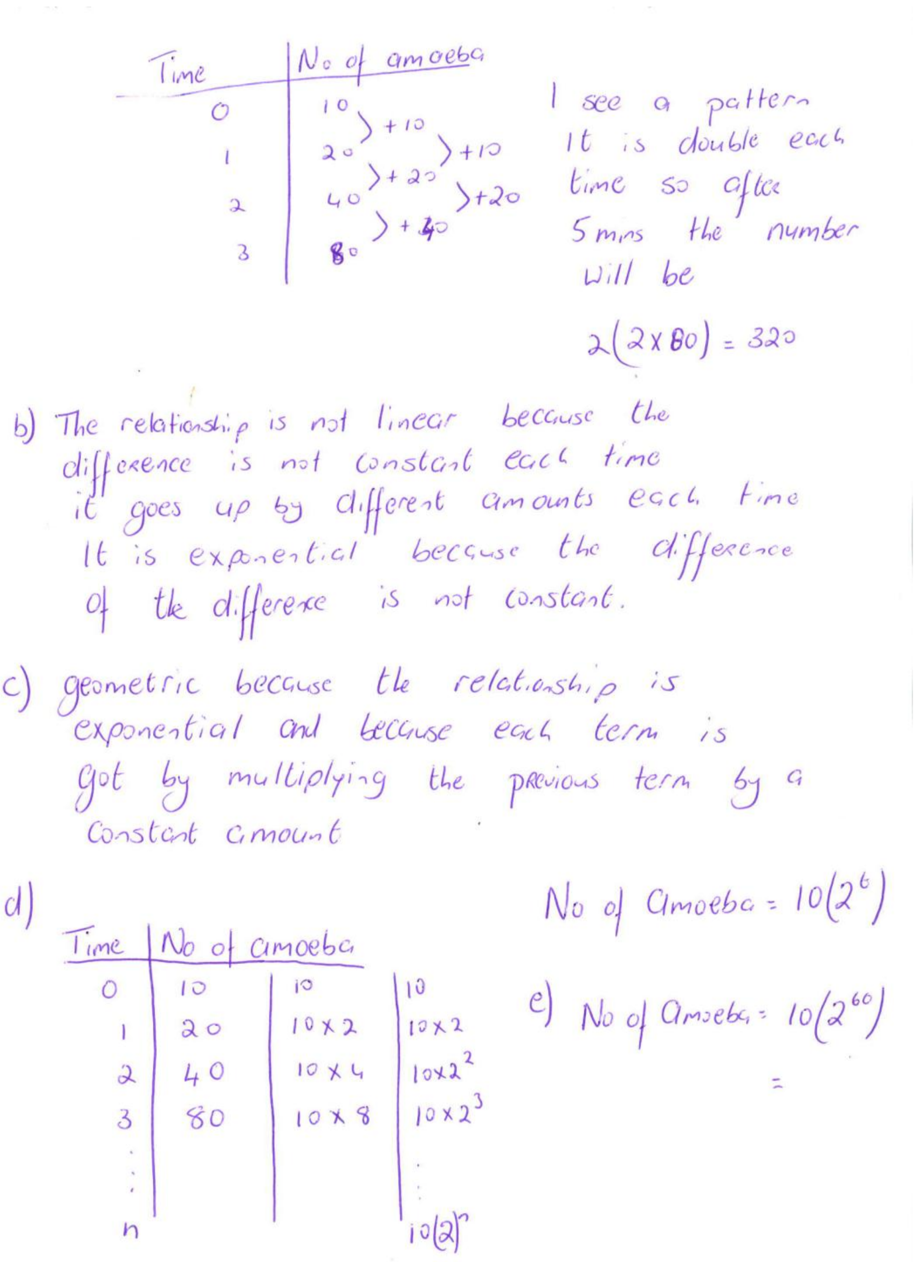
(b) What type of relationship is there between the time in minutes and the number of amoeba in the Petri dish? Is it linear? How do you know? Explain your thinking.

(c) The terms produce a sequence; is this sequence **arithmetic** or **geometric**? How do you know?

(d) Write a general expression showing how the number of amoeba varies with time.

(e) How many amoeba will be in the dish after 1 hour?

Compare, Examine, Discuss and Evaluate



Time	No of amoeba
0	10
1	20
2	40
3	80

$10 \xrightarrow{+10} 20 \xrightarrow{+20} 40 \xrightarrow{+40} 80$

$+10 \xrightarrow{+10} +20 \xrightarrow{+20} +40$

I see a pattern It is double each time so after 5 mins the number will be

$2(2 \times 80) = 320$

b) The relationship is not linear because the difference is not constant each time it goes up by different amounts each time It is exponential because the difference of the difference is not constant.

c) geometric because the relationship is exponential and because each term is got by multiplying the previous term by a constant amount

d)

Time	No of amoeba		
0	10	10	10
1	20	10 × 2	10 × 2
2	40	10 × 4	10×2^2
3	80	10 × 8	10×2^3
⋮			⋮
n			$10(2)^n$

No of Amoeba = $10(2^t)$

e) No of Amoeba = $10(2^{60})$

=

Task 6: LCOL

Look at a calendar for this month. Examine the column that represents all the Thursdays in this month.

(a) What are the dates?

(b) What kind of sequence do these numbers represent? Explain how you know.

(c) If it is arithmetic, what is d, the common difference? If geometric, what is r, the common ratio?

(d) If that sequence continued, what would be the 100th term?

Task 7 LCOL Suppose that a, b, c, d….represent an arithmetic sequence. For each of the sequences below, indicate if it is arithmetic, geometric, or neither. Explain your reasoning in each case.

1) a + 2, b + 2, c + 2, d + 2…..
2) 2a, 2b, 2c, 2d…..
3) a^2, b^2, c^2, d^2……

Task 8 LCHL

Find x to make the sequence 10, 30, 2x + 8

a) arithmetic

b) geometric

Task 9 LCOL

Suppose a litre of petrol cost €1.20 in January, and the price goes up by 3% every month throughout the year.

(a) Find the cost of a litre of petrol, rounded to the nearest cent, for each month of the year.

(b) Is this sequence arithmetic, geometric, or neither?

(c) If it keeps going at this rate, how many months will it take to reach €10 per litre?

(d) How about €1000 per litre?

Task 10 LCOL

In an arithmetic sequence, each term is the previous term plus a constant. In a geometric sequence, each term is the previous term times a constant. Is it possible to have a sequence which is both arithmetic and geometric? Can you give an example?

Task 11 LCOL

A ball is dropped from a height of 3.0 m vertically on a horizontal surface. After each bounce it rises to 75% of its previous height.

(a) What height does the ball reach *after 6 bounces*?

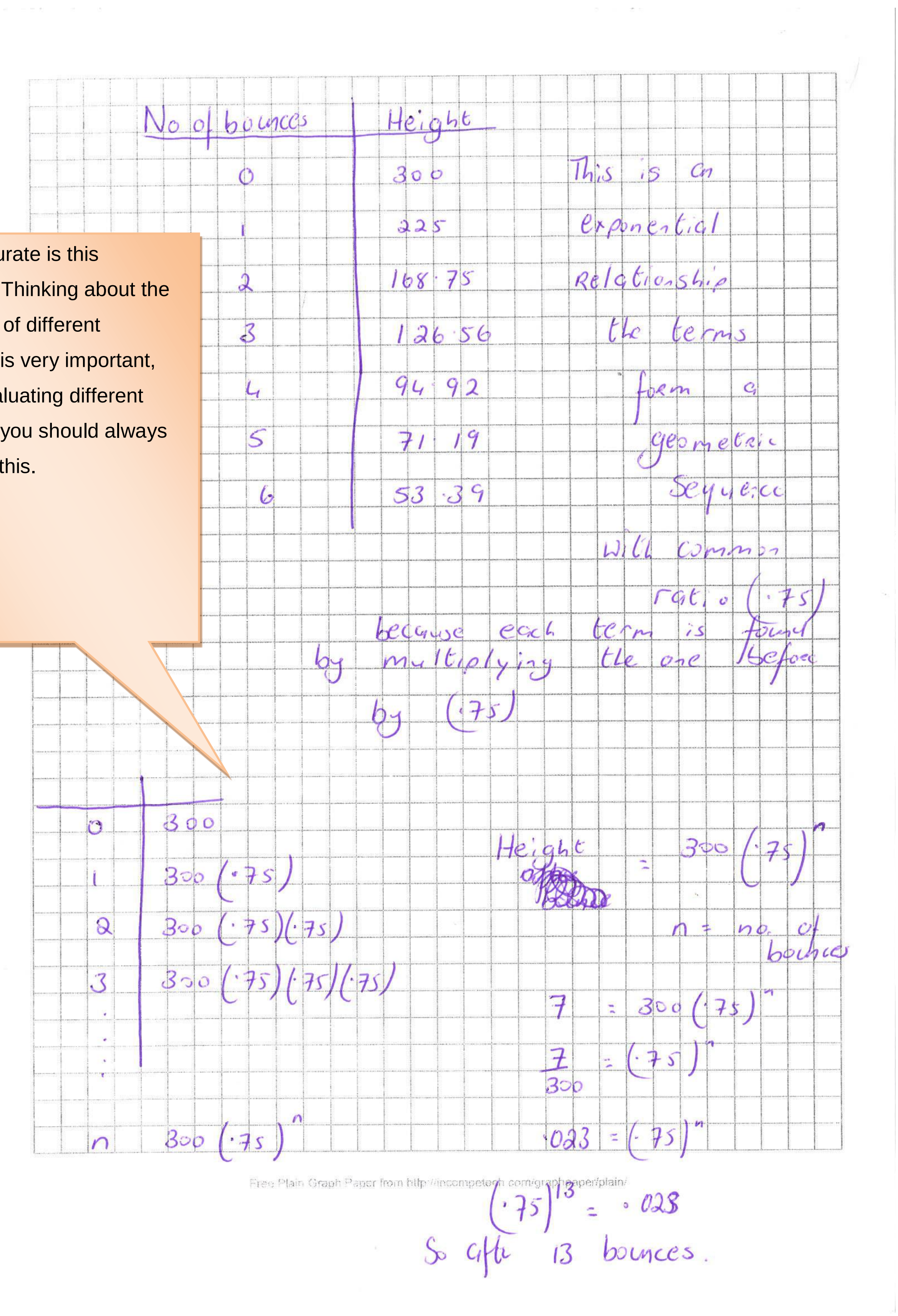
(b) The terms form a sequence; what type of sequence is it? Explain your reasoning.

(c) Write an expression showing how the height of the ball varies with the number of bounces.

(d) After how many bounces does the ball reach a height of only 7 cm?

Compare, Examine, Discuss and Evaluate

How accurate is this method? Thinking about the accuracy of different methods is very important, when evaluating different methods you should always consider this.



No of bounces	Height
0	300
1	225
2	168·75
3	126·56
4	94·92
5	71·19
6	53·39

This is an exponential Relationship the terms form a geometric Sequence will common ratio (·75) because each term is found by multiplying the one before by (·75)

0	300
1	300(·75)
2	300(·75)(·75)
3	300(·75)(·75)(·75)
⋮	
n	$300(\cdot 75)^n$

Height ~~of~~ = $300(\cdot 75)^n$

n = no. of bounces

$7 = 300(\cdot 75)^n$

$\frac{7}{300} = (\cdot 75)^n$

$\cdot 023 = (\cdot 75)^n$

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$(\cdot 75)^{13} = \cdot 028$

So afte 13 bounces.

Task 12: LCOL

Sarah was investigating savings options

Option 1: Invest €1000 at 11% per year for a number of years.

Option 2: Deposit €1000 and add €100 each year.

Investigate how the money begins to grow for each option by filling in the table.

Time in years (n)	0	1	2	3	4	5	6
Amount (A) Option 1	€1000						
Amount (A) Option 2	€1000						

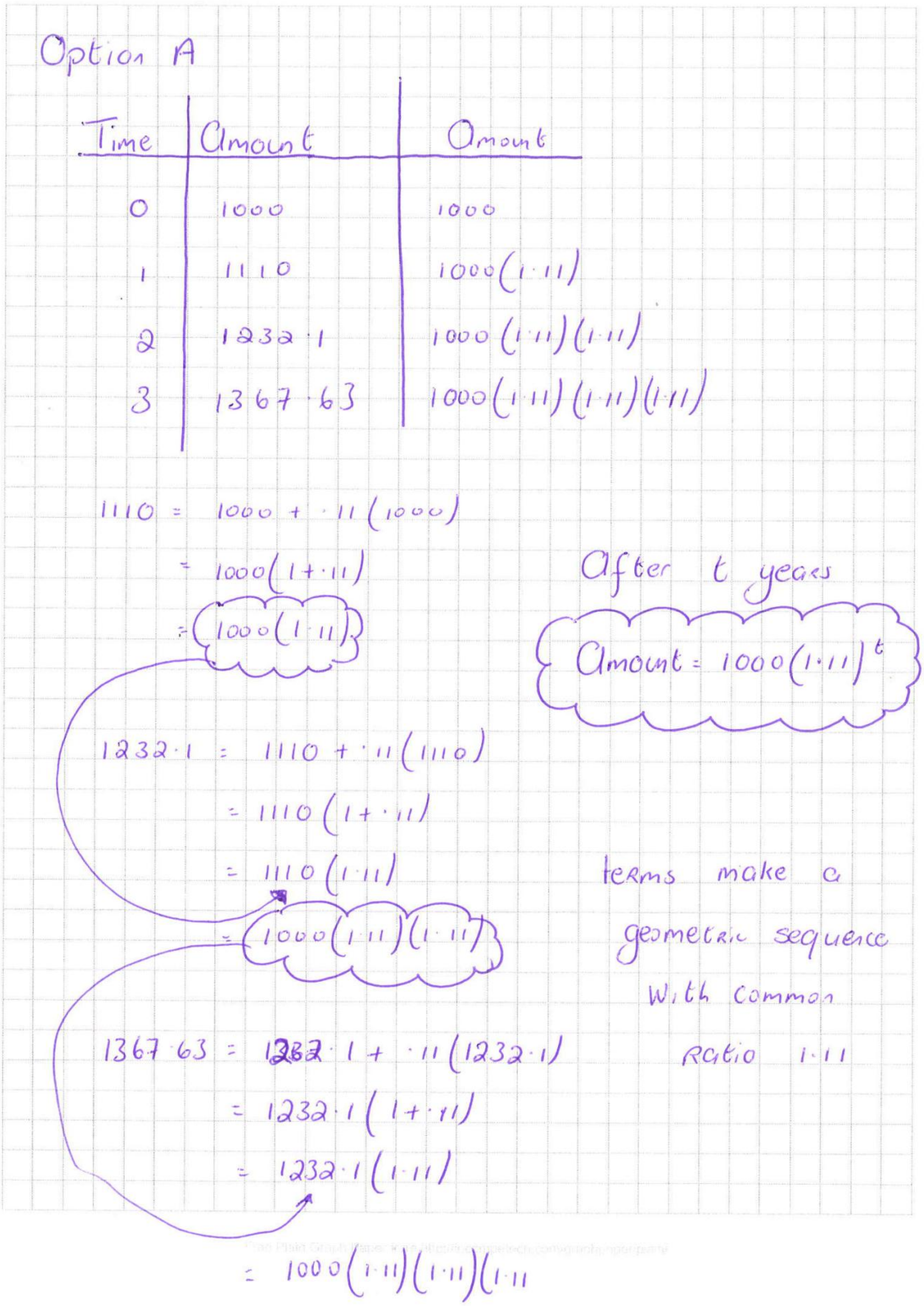
Sarah said "*the terms in option 1 produce a geometric sequence whilst the terms in option 2 produce an arithmetic sequence*"

(a) Examine these terms to see if Sarah is correct. Explain your reasoning

(b) Write an expression for the value of the *nth* term of each sequence (the general term)?

(c) Which investment option would you advise Sarah to go with? Explain your reasoning

Compare, Examine, Discuss and Evaluate



Option A

Time	Amount	Amount
0	1000	1000
1	1110	1000(1·11)
2	1232·1	1000(1·11)(1·11)
3	1367·63	1000(1·11)(1·11)(1·11)

$1110 = 1000 + ·11(1000)$

$= 1000(1 + ·11)$

$= \boxed{1000(1·11)}$

After t years

$\boxed{\text{Amount} = 1000(1·11)^t}$

$1232·1 = 1110 + ·11(1110)$

$= 1110(1 + ·11)$

$= 1110(1·11)$

$= \boxed{1000(1·11)(1·11)}$

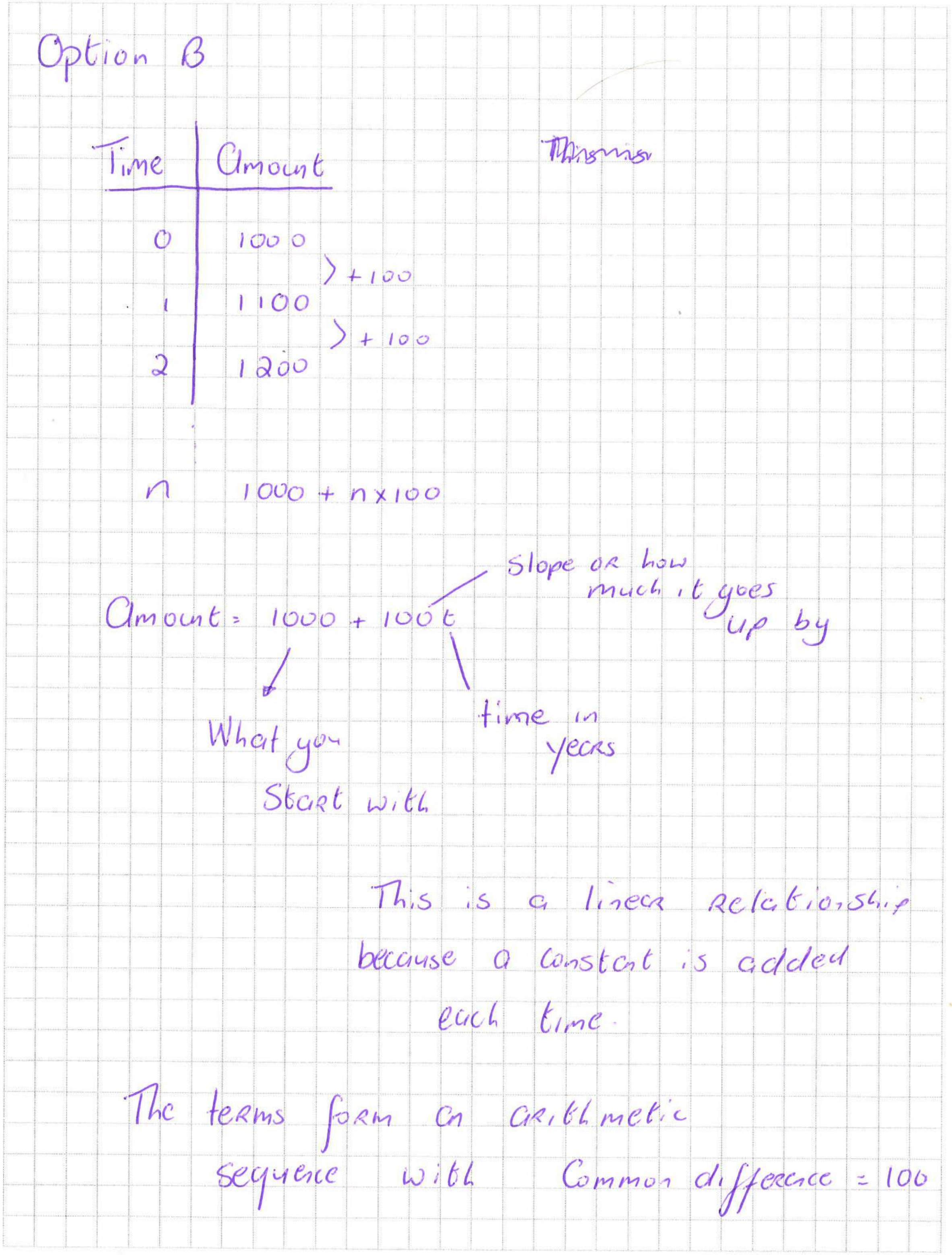
terms make a geometric sequence with common ratio 1·11

$1367·63 = 1232·1 + ·11(1232·1)$

$= 1232·1(1 + ·11)$

$= 1232·1(1·11)$

$= 1000(1·11)(1·11)(1·11$



Option B

Time	Amount	
0	1000	
		> +100
1	1100	
		> + 100
2	1200	
n	1000 + n×100	

Slope or how much it goes up by

$$\text{Amount} = 1000 + 100t$$

What you start with

time in years

This is a linear relationship because a constant is added each time.

The terms form an arithmetic sequence with Common difference = 100

Task 13: JCHL

(a) If a cent is dropped from a height of 45m, its height changes over time according to the formula: h=45 -4.9t^2, where t is measured in seconds. Use the mathematical tools (numerical analysis, tables and graphs) to determine how long it will take for the cent to hit the ground. What does the graph of height Vs time look like? What connections do you see between the graph and the table?

(b) Suppose you wanted the cent to land after exactly 4 seconds. From what height would you need to drop it? Explain how you figured this out.

(c) Suppose you dropped the cent from the top of the Eiffel Tower (300m). How long would it take to hit the ground? What does the graph of height Vs time look like? What connections do you see between the graph and the table?

(d) Suppose a machine tosses the coin vertically into the air so that the instant it leaves the machine it is travelling at 30m/s. The formula for height after t seconds is given by $h=30t -4.9 t^2 +4$.Why might that make sense? How long will it take to hit the ground? What does the graph of height Vs time look like? What connections do you see among the graph, the table, the situation, and the formula?

Note:

There is opportunity here to discuss the need for a more accurate approach to answering the question.

Would a graph have given a more accurate result? What about the equation?

Compare, Examine, Discuss and Evaluate

$h = 45 - 4{\cdot}9t^2$

t	h
0	45
1	40·1
2	20·99
3	·9
4	-33·4

It will hit the ground between 3 and 4 secs after it starts to fall

The table shows the height falling each time it doesn't fall by a constant amount so the graph won't be linear it will be curved downwards because it is losing height but not at a constant rate

If it is to hit the ground after 4 secs Then

The height must be 0

$0 = \cancel{45} - 4{\cdot}9(16)$

$X = 78{\cdot}4m$

Could you have worked this out from the table?

If you drop it from 300m

Then $h = 300 - 49(t^2)$

t	h
0	300
1	295
2	280
3	256
4	222
5	178
6	124
7	60
8	−13·6

It will hit the ground between 7 and 8 secs after it is dropped.

Compare, Examine, Discuss and Evaluate

$h = 30t - 4.9t^2 + 4$

When the coin is tossed it goes up and gravity pulls it down too.

How far it goes up

How far it comes down

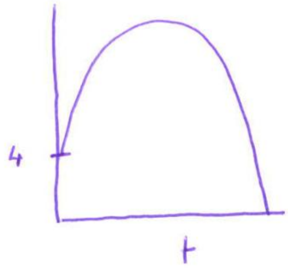
How high it is at the start

Altogether this is the height.

t	h	
0	4	increasing
1	29.1	
2	44.4	
3	49.9	
4	45.6	
5	31.5	decreasing
6	7.6	
7	-26.1	

The height increase then decreases and hits the ground between 6 and 7 secs after it is tossed

The graph will be like



Note What would a more accurate approach to answering the question involve?

Would a graph have given a more accurate result? What about the equation?

Task 14: LCOL

A radioactive substance has a ***half-life*** of 20 minutes. That means after 20 minutes only half the substance remains. If a substance starts with 1000 grams, complete the table to show the amount of substance remaining at each of the times shown.

Time (minutes)	Half-Lives	Substance remaining (grams)
0	0	1000
20	1	
40		
60		
80		
100		

(a) What type of sequence do the terms produce? How do you know?

(b) Write an equation showing how many grams there are remaining after n half-lives.

(c) After n half-lives, how many minutes have gone by?

(d) Write an equation showing after t minutes how many half-lives have gone by.

Now put it all together. After t minutes, how many grams are there?

(e) Test that equation to see if it gives you the same result you found above after 100 minutes.

Predict what the graph will look like. Explain your thinking

Check your prediction

(f) How much substance will be left after 70 minutes?

(g) How much substance will be left after two hours?

(h) How long will it be before only one gram of the original substance remains?

Compare, Examine, Discuss and Evaluate

LCOL A radioactive substance has a ***half-life*** of 20 minutes. That means after 20 minutes only half the substance remains. If a substance starts with 1000 grams, complete the table to show the amount of substance remaining at each of the times shown.

Time (minutes)	Half-Lives	Substance remaining (grams)
0	0	1000
20	1	$\frac{1}{2}(1000) = 500$
40	2	$\frac{1}{2}\frac{1}{2}(1000) = \frac{1}{4}(1000) = 250$
60	3	$\frac{1}{2}\frac{1}{2}\frac{1}{2}(1000) = \frac{1}{8}(1000) = 125$
80	4	$\frac{1}{2}\frac{1}{2}\frac{1}{2}\frac{1}{2}(1000) = \frac{1}{16}(1000) = 62.5$
100	5	$\frac{1}{32}(1000) = 31.25$

i) What type of sequence do the terms produce? How do you know?

Geometric because it doesn't go down by a constant amount

j) Write an equation to showing how many grams there are remaining after n half-lives.

$$\frac{1}{2^n}(1000)$$

k) After n half-lives, how many minutes have gone by?

$n X 20$

l) Write an equation showing after t minutes (for instance, after 60 minutes, or 80 minutes) how many half-lives have gone by.

after 60 minutes $\frac{60}{20} = 3$ half lives

" 80 minutes $\frac{80}{20} = 4$ half lives

" t minutes $\frac{t}{20}$ half lives

Now put it all together. After t minutes, how many grams are there?

$\frac{1}{2^{\frac{t}{20}}}(1000)$ grams

m) Test that equation to see if it gives you the same result you gave above after 100 minutes.

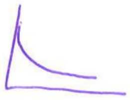
after 100 mins t= 100

$\frac{1}{2^{\frac{100}{20}}}(1000) = \frac{1}{2^5}(1000) = \frac{1}{32}(1000)$

$= 31{\cdot}25g$

Predict what the graph will look like. Explain your thinking

Curved downwards like this



Check your prediction

n) How much substance will be left after 70 minutes?

o) How much substance will be left after two hours?

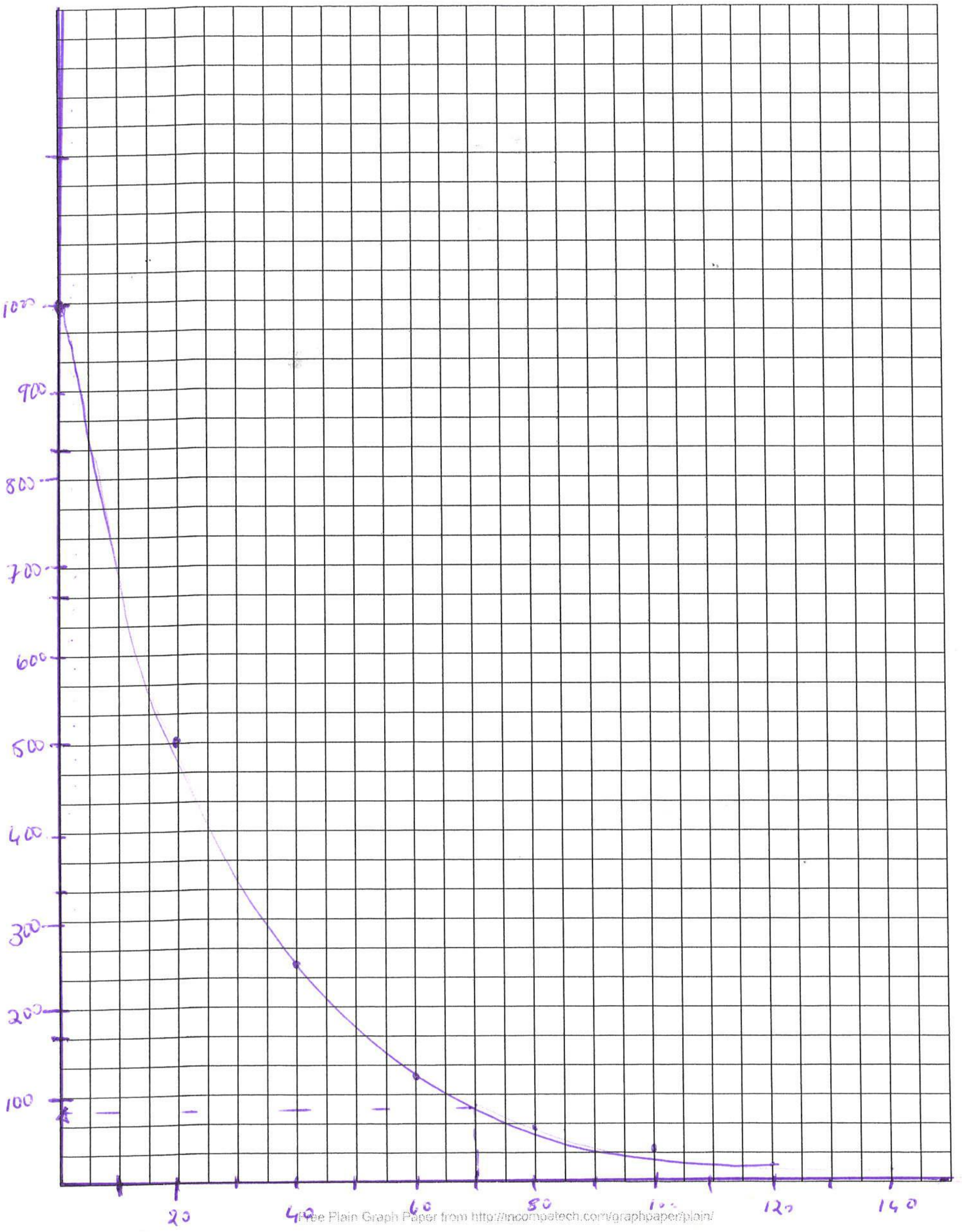
p) How long will it be before only one gram of the original substance remains?

after 70mins $t = 70$

$\frac{1}{2^{\frac{70}{20}}}(1000) =$

afte 2hrs $t = 120 = 6\frac{1}{2}$ lifes

$\frac{1}{2^{\frac{120}{20}}}(1000) = \frac{1}{2^6}(1000) = 15{\cdot}63g$


1000
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Free Plain Graph Paper from http://incompetech.com/graphpaper/plain/

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how they solved the radioactivity problem: Radioactivity 1

Well……when I started doing this question I just started filling in the table and I could see immediately that the relationship was not going to be linear because it was going down by different amounts each time ….first 500 then 250 …it was getting smaller each time. So I knew the terms couldn't form an arithmetic sequence,but they would form a geometric one.

Then I started looking at the grams remaining…. first one was half of a thousand …then half of half of a thousand and then half of half of half of a thousand and so on in a pattern that was a quarter of a thousand, one eight of a thousand and so on .It was easy really to see the relationship between the number of half lives and the grams remaining…..1 over 2 to the power of n times a thousand where n is the number of half lives

Then the question sort of made it easy for me to see the connection between the time and the grams remaining ....you just have to work out how many half lives have gone by in a given time….sometimes it's easy because it's a whole number of half lifes like in the table but other times it's not so easy ..like for 70 mins it's 3.5 half lives because that's 70 ÷ 20 = 3.5…..two hours was easy because that's just 120 ÷ 20 = 6 half lives…I had no problem predicting the graph because I knew it was an exponential relationship and that means the graph curves with the slope changing rapidly..I knew this time it would curve down because it was getting less and less. …If you draw the graph it's probably easier to find the grams remaining from that although it's probably not very accurate because it's hard to draw curves accurately on a graph….you just have to use a calculator ..once you get into exponential relationships the maths gets hard…it's all like powers and sometimes they're straightforward and sometimes they're not.

The last part is hard because it's like…you're given the grams and you have to find n. It's like we don't know how to do this we need to learn new maths. Because one over 2 to the n is equal to one over a thousand which means two to the n is equal to a thousand but like I can't do this because a thousand isn't two to the anything…..do you see what I mean? Like if you said two to the n is equal to 32 I would know that n is 5 because 32 is two to the 5..but a thousand isn't two to the anything…I can only do this by trying out numbers …so like two to the 8 is 256..(too small) two to the 9 is 512 (too small) ..two to the ten is one thousand and twenty four.(too big). So I know that n must be between 9 and 10; probably closer to 10. I could try two to the 9.6 that's 776.04 (too small); two to the 9.7 equals 831.75 (too small); two to the 9.9 equals 955.42 (too small); two to the 9.98 equals 1009.9 (too big); ..two to the 9.97 equals1.002 (too big); two to the 9.96 equals 995.99

(too small); so it must be between 9.96 and 9.97. This takes a long time and isn't really accurate We probably have to learn new maths to find a way to solve these types of questions.

The student who featured in the audio file has spoken about the need…." to find a way to solve these types of questions .." Logarithms were invented to facilitate difficult calculations; can logs help in this situation?

Well firstly, think about why the student thought the problem was difficult.

It was because, instead of going from time to amount, it asked her to go from amount to time. The question required her to invert the exponential function. This is called an ***inverse function***.

Exponential equations can be interpreted as questions.

√36 asks the question: What squared equals 36? The answer, of course, is 6.

Logarithmic functions are the inverse of exponential functions and they ask similar questions.

$\text{Log}_2 8 = x$ asks the question…. Two to what power equals 8 ?…The answer is 3.

The following question set in context gives you an indication of the usefulness of logs.

Task 15 LCHL

Sound is a wave in the air; the loudness of the sound is related to the intensity of the wave.

Type of sound	Intensity
Whisper	100
Background noise in a quiet rural area	1000
Normal conversation	1,000,000
Rock concert	1,000,000,000,000

Note: Try to place these points on a number line, and label them. Did you find this difficult? Why? It is because the range is huge, the function grows so quickly. Logarithmic scales are used when working with a function that, by itself, grows too quickly.

Sound volume is usually not measured in intensity, but in loudness, which is given by the formula:

$$L = 10 \log_{10} I$$

where L is the loudness (measured in decibels), and I is the intensity.

(a) What is the loudness, in decibels, of a whisper?

(b) What is the loudness, in decibels, of a rock concert?

(c) Now draw the number line again, labeling all the sounds; but this time, graph loudness instead of intensity.

(d) The quietest sound a human being can hear is intensity 1. What is the loudness of that sound?

(e) Sound intensity can never be negative, but it can be less than 1. What is the loudness of such inaudible sounds?

(f) If sound A is twenty decibels higher than sound B, how much more intense is it?

From Sequences to series

A series is essentially the sum of all the terms of a sequence. So we can have arithmetic series and geometric ones.

Note: A difficulty in dealing with series is getting to grips with the notation ($T_{n,}$, S_n, a, r, d); spend some time finding out about using notation. You can get help in textbooks, on the internet or by asking your teacher.

Now use the notation to describe each finite series below:

(a) 6 + 7 + 8 + 9 + 10
(b) -6 - 7 - 8 - 9 -10
(c) 6 + 8 + 10 + 12 + 14
(d) 6 + 12 + 24 + 48
(e) 6 - 7 + 8 - 9 + 10
(f) All the even numbers between 50 and 100.

Sequences can be finite or infinite. So too can series.

Applications of series

In economics, **geometric series** are used to represent the present value of an annuity (a sum of money to be paid in regular intervals).

For example, suppose that you expect to receive a payment of €100 once per year for an indefinitely long time. Receiving €100 a year from now is worth less to you than an immediate €100, because you cannot invest the money until you receive it. In particular, the present value of a €100 one year in the future is €100 / (1 +yearly interest rate)

Similarly, a payment of €100 two years in the future has a present value of €100 / (1 + yearly interest rate)^2 – squared because it would have received the yearly interest twice. Therefore, the present value of receiving €100 per year for an indefinitely long time can be expressed as an infinite series because the payment is being made for an indefinite length of time.

$$\frac{100}{1+I} + \frac{100}{(1+I)^2} + \frac{100}{(1+I)^3} + \frac{100}{(1+I)^4} + \cdots$$

Investigate this.
Have a look at the transcript from the audio file “Investment 1”

This is a geometric series with common ratio 1 / (1 + *I*). The sum is

$$\frac{a}{1-r} = \frac{100/(1+I)}{1-1/(1+I)} = \frac{100}{I}.$$

For example, if the yearly interest rate is 10% (***I*** = 0.10), then the entire annuity has a present value of €1000.

This sort of calculation is used to compute the **APR** of a loan (such as a **mortgage** loan). It can also be used to estimate the present value of expected **stock dividends**, or the **terminal value** of a **security**. You may find financial terminology intimidating use Google to help you make sense of it.

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how they solved the investment problem: Investment 1

Well…what we did was to look at €100 and what it would become 3, 4 or 5 years from now if we invested it at, say, 5% interest.

So we made a table…it's always useful to do this because you can see patterns easily.

Time	Amount
0	100
1	100+5/100(100)= 100(1+.05)
2	100(1+.05) +.05(100(1+.05) =100(1+.05)(1+.05)
3	100(1+.05)(1+.05)(1+.05)

So I see the pattern it is an exponential relationship and the terms form a geometric sequence with common ratio (1+.05). That means to find the amount for each year you multiply the previous year by (1+.05).

So then we started to think about the €100 we are going to get each year.

The present value of €100 one year in the future is 100/ (1+.05)

Then the present value of €100 two years in the future is $100/(1+.05)^2$

This payment goes on indefinitely so the present value forms an infinite series with common ratio

1/(1+.05)… we can get the sum of this series by using the formula from the tables.

a/1-r = 100/(1+.05) divided by 1 -1/(1+.05)…2000 is what the whole annuity is worth now….. that is its **present value.**

We can generalise this for any value and any interest rate.

LCHL

You have won a lottery that pays €1,000 per month for the next 20 years. But, you prefer to have the entire amount now. If the interest rate is 8%, how much will you accept?

Note: This annuity is different from the last example because there is a finite (20 years) sequence of payments which form a **finite** series. The reasoning will be the same as the above except the sum of the finite series is found using the formula

$$\frac{a(r^n - 1)}{r - 1}$$

The "Formulae and tables" book states this formula in a different way. Are they both the same? Why?

Look on this questions as a situation where two people have won the same lottery; John and Susan.

John, a young student is happy with his €1000 monthly payment, but Susan, a slightly older lady wants to have the entire amount now. Your job is to determine how much Susan should get.

Work out how much Susan would get if she invested this lump sum at 8% per annum compounded monthly. You should find it interesting that in 20 years her lump sum should be worth €568, 999.07. Have a look through the next transcript from the audio file.

It's a good idea to look for other contexts; look in the papers, go online to banking websites and create your own questions.

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how she solved the investment problem: Investment 2 (John and Susan)

Well; let's say Susan accepts x euro, then that x euro deposited at 8% for 20 years should yield the same amount as the €1,000 monthly payments for 20 years. So what we are doing really is comparing the future values for both Susan and John, and these future values have to be equal.

Now since Susan is getting a lump sum of x euro, its future value is given by

$$x(1+(\sqrt[12]{1.08}-1))^{240}$$

$$x(\sqrt[12]{1.08})^{240}$$

and John is getting a sequence of payments, or an annuity, of €1,000 per month; its future value is given by

$1000 + 1000(1+\sqrt[12]{1.08}-1) + 1000(1+\sqrt[12]{1.08}-1)^2 + \ldots\ldots..$

So you can see that $\boldsymbol{a} = 1000$, $\boldsymbol{n} = 240$ $\boldsymbol{r} = \sqrt[12]{1.08}$

The sum of 240 of these payments is

$$\frac{1000[(\sqrt[12]{1.08})^{240}-1]}{\sqrt[12]{1.08}-1}$$

Susan will only agree to the amount she gets if these two future values are equal. So then, all we do is make them equal and solve the equation for x

$$\frac{1000[(\sqrt[12]{1.08})^{240}-1]}{\sqrt[12]{1.08}-1} = x(\sqrt[12]{1.08})^{240}$$

x = €122,077.73

See how much this amount would be worth after 20 years at the given rate.

Applications of sequences and series tasks

Task 15 LCHL

Jack won a lottery; he has been given a choice of two options:

> **Option A:** Receive an annuity of €1500, each month for 25 years. An annuity is a sum of money to be paid in regular intervals.
>
> **Option B:** Take the present value of the annuity (based on an annual growth rate of 10%)

Jack decides to take Option **B** and invests it himself in an account that pays 9% compounded monthly for 20 years.

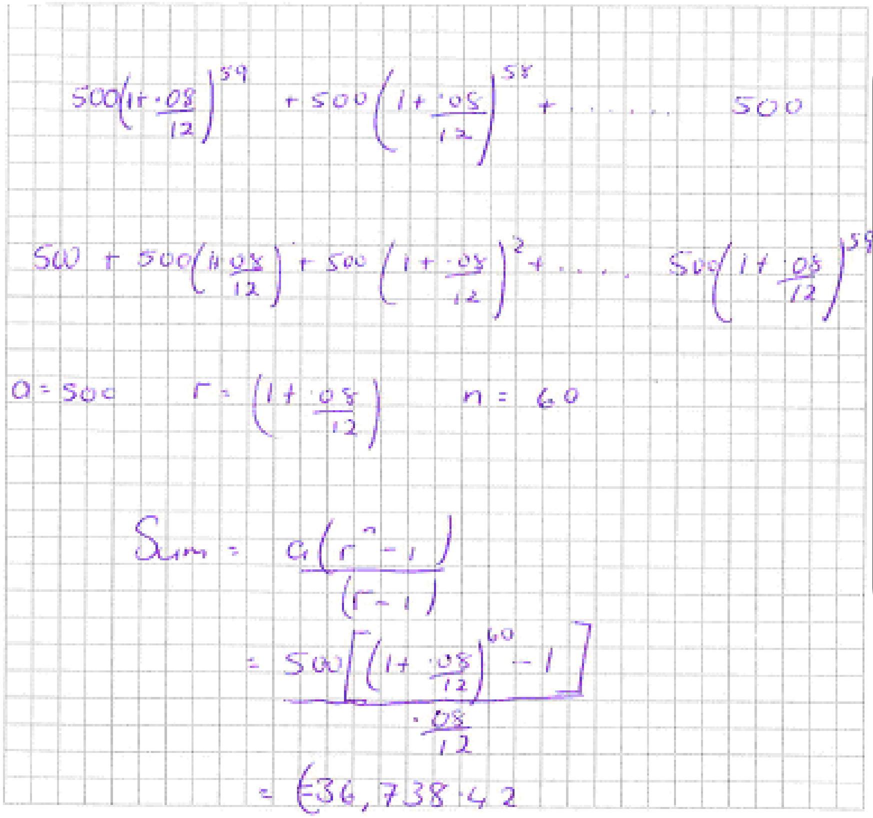
What is the present value of the annuity?

How much will Jack's investment have amounted to after the 20 years?

Task 16 LCHL

At the end of each month a deposit of €500 is made in an account that pays 8% per annum compounded monthly. What will the final amount be after 5 years?

A student calculated this as follows:



$$500\left(1+\frac{.08}{12}\right)^{59} + 500\left(1+\frac{.08}{12}\right)^{58} + \ldots\ldots 500$$

$$500 + 500\left(1+\frac{.08}{12}\right) + 500\left(1+\frac{.08}{12}\right)^{2} + \ldots\ldots 500\left(1+\frac{.08}{12}\right)^{59}$$

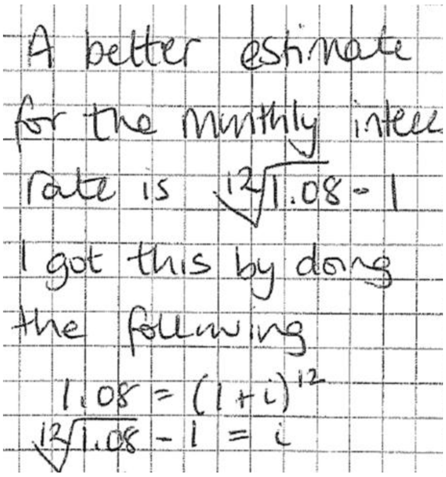
$$a = 500 \qquad r = \left(1+\frac{.08}{12}\right) \qquad n = 60$$

$$\text{Sum} = \frac{a\left(r^{n}-1\right)}{(r-1)}$$

$$= \frac{500\left[\left(1+\frac{.08}{12}\right)^{60}-1\right]}{\frac{.08}{12}}$$

$$= €36{,}738{\cdot}42$$

Another student argues:



A better estimate for the monthly intere rate is $\sqrt[12]{1.08}-1$

I got this by doing the following

$$1.08 = (1+i)^{12}$$

$$\sqrt[12]{1.08} - 1 = i$$

What do you think?

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how he solved the problem: Investment 3 (Task 16)

Well, you see***....***There are 60 deposits made in this account…because 5 times 12 is 60. The first payment stays in the account for 59 months, the second payment for 58 months, the third for 57 months, and so on.

I thought OK… how much will the first €500 accumulate to at the end of the 59 months…… I worked this out by saying it will be €500 + the interest but this will be 8% x 500 / 12 because the interest is compounded monthly. The first payment of €500 will accumulate to an amount of €500$(1 + .08/12)^{59}$. But then another student said that because the interest is compounded monthly you can't just divide by twelve; they calculated the interest rate to be $\sqrt[12]{1.08} - 1$. So then I used this rate.

The second payment of €500 will accumulate to an amount of €500$(1 + (\sqrt[12]{1.08} - 1))^{58}$.

The third payment will accumulate to €500$(1 + (\sqrt[12]{1.08} - 1))^{57}$.

And so on. It's just a geometric sequence.

The last payment is taken out the same time it is made, and will not earn any interest.

To find the total amount in five years, we need to add the accumulated value of these sixty payments.

So in other words, I need to find the sum of the following series.

€500$(1 +(\sqrt[12]{1.08} - 1))^{59}$ + €500$(1 +(\sqrt[12]{1.08} - 1))^{58}$ + €500$(1 +(\sqrt[12]{1.08} - 1))^{57}$ + . . . + €500

I write that backwards because it's easier to see

€500 + €500$(1 +(\sqrt[12]{1.08} - 1))$+ €500$(1 +(\sqrt[12]{1.08} - 1))^2$ + . . . + €500$(1 +(\sqrt[12]{1.08} - 1))^{59}$

This is a geometric series with ***a*** = €500, ***r*** = $(1 +(\sqrt[12]{1.08} - 1))$, and ***n*** = 60 Therefore the sum is

$$\frac{a(r^n - 1)}{r - 1}$$

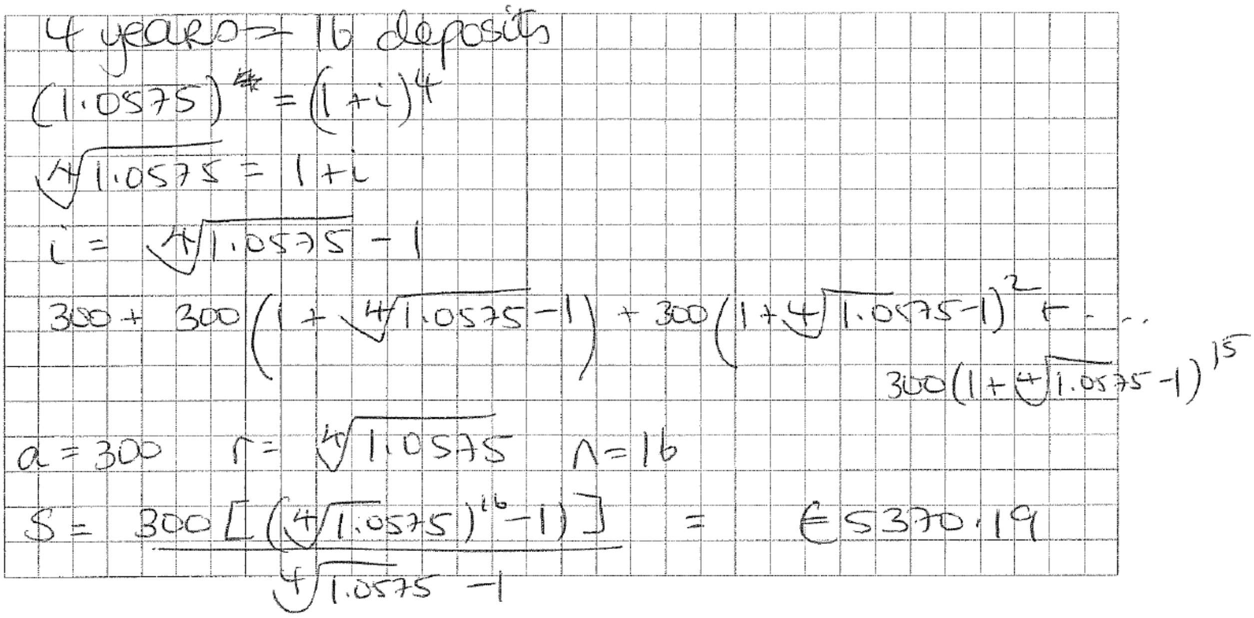
$$\frac{500((1 + \sqrt[12]{1.08} - 1)^{60} - 1)}{\sqrt[12]{1.08} - 1}$$

Answer = €36, 471.70

Task 17 LCHL

Sonya deposits €300 at the end of each quarter in her savings account. If the account earns 5.75% per annum compound interest how much will she have in 4 years?

Compare, Examine, Discuss and Evaluate



4 years → 16 deposits

$$(1.0575)^{\cancel{4}} = (1+i)^4$$

$$\sqrt[4]{1.0575} = 1+i$$

$$i = \sqrt[4]{1.0575} - 1$$

$$300 + 300\left(1+\sqrt[4]{1.0575}-1\right) + 300\left(1+\sqrt[4]{1.0575}-1\right)^2 + \ldots$$

$$300\left(1+\sqrt[4]{1.0575}-1\right)^{15}$$

$a = 300 \quad r = \sqrt[4]{1.0575} \quad n = 16$

$$S = \frac{300\left[\left(\sqrt[4]{1.0575}\right)^{16}-1\right]}{\sqrt[4]{1.0575}-1} = €5370.19$$

Task 18 LCHL

Robert needs €5000 in three years. How much should he deposit at the end of each month in an account that pays 8% per annum in order to achieve his goal?

LCHL

Q. The New Horizons computer company needs to raise money to expand. It issues a 10-year €1,000 bond that pays €30 every six months. If the current market interest rate is 7%, what is the fair market value of the bond?

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how he solved the problem.

I wasn't sure what a bond certificate was so I Googled it and basically a bond certificate promises two things – an amount of €1,000 to be paid in 10 years, and a semi-annual payment of €30 for ten years. So, to find the fair market value of the bond, I need to find the present value of the lump sum of €1,000 that you'd receive in 10 years, as well as the present value of the €30 semi-annual payments for the 10 years.

I calculated the present value of the lump sum €1,000 as follows:

$$x\ (1+(\sqrt{1.07}-1))^{20} = \text{€}1{,}000$$

$$x(1.96715) = \text{€}1{,}000$$

$$x = \text{€}508.35$$

The present value of the €30 semi-annual payments is $30\left[\dfrac{(1+(\sqrt{1.07}-1))^{20}-1}{\sqrt{1.07}-1}\right]$

Hence, $$x\ (1+(\sqrt{1.07}-1))^{20} = 30\left[\frac{(1+(\sqrt{1.07}-1))^{20}-1}{\sqrt{1.07}-1}\right]$$

$$x = \text{€}428.67$$

So

The present value of the lump sum €1,000 = €508.35

The present value of the €30 semi-annual payments = €428.67

Therefore, the fair market value of the bond is €508.35+ €428.67= €937.02

Task 19 LCHL

Mr. Mooney bought his house in 1975, and financed the loan for 30 years at an annual interest rate of 9.8% compounded monthly. His monthly payment was €1260. In 1995, Mr. Mooney decided to pay off the loan. Find the balance of the loan he still owed at that time.

Compare, Examine, Discuss and Evaluate

This is a transcript of a student reflecting on how she solved the investment problem: Investment 4

When I looked at this problem first I thought…I can't answer this because I don't know how much he borrowed to start with…so I even asked my teacher was this information missing from the question…she said no you don't need to know this to work it out so I thought… over the course of the loan Mr Mooney committed to make (30 x12 = 360) 360 payments of €1260. By deciding to pay off the loan in 1995 he still has 120 more payments to make so I thought the present value of these instalments is what the bank should charge him.

So I set about finding the present value of these payments, by putting the two amounts equal.

I used this formula

$$X\,(1+(\sqrt[12]{1.098}-1))^{120} = \frac{1260[(1+\sqrt[12]{1.098}-1)^{120}-1]}{\sqrt[12]{1.098}-1}$$

The left hand side shows the future amount he would owe if he paid the lump sum X, which is its present value. The right hand side shows how much the bank will get if he continues to make the monthly payments.

They must be equal. Thus X = €97,863.68