

Section A

Concepts and Skills

Question

A computer is going to choose a letter at random from the text of an English novel. The table shows the probabilities of the computer choosing the various vowels.

Vowel	A	E	I	O	U
Probability	0.06	0.13	0.07	0.08	0.03

(a) What is the probability it will **not** choose a vowel?

$$\begin{array}{l}
 A = 0.06 \\
 E = 0.13 \\
 I = 0.07 \\
 O = 0.08 \\
 U = 0.03 \\
 \hline
 = 0.37
 \end{array}$$

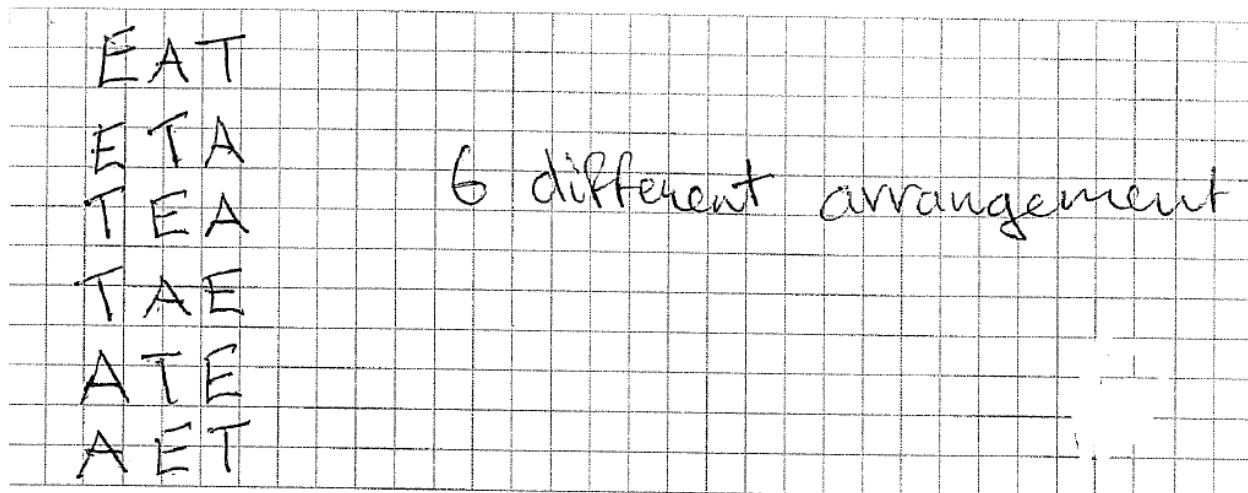
$$\begin{array}{l}
 0.37 \times 100 = 37\% \\
 100\% - 37\% = 63\% \\
 \text{Ans} = 63\%
 \end{array}$$

(b) The probability that the computer will choose the letter **T** is **0.09**.

The computer chooses a letter at random, and then a second, and then a third letter. What is the probability that these letters will be **E, A and T** (in that order)?

$$\begin{array}{l}
 E \quad \times \quad A \quad \times \quad T \\
 0.13 \quad \times \quad 0.06 \quad \times \quad 0.09 \quad = \quad \frac{351}{500,000} \\
 \\
 \frac{351}{500,000} \quad \times \quad 100 \quad = \quad 0.0702\%
 \end{array}$$

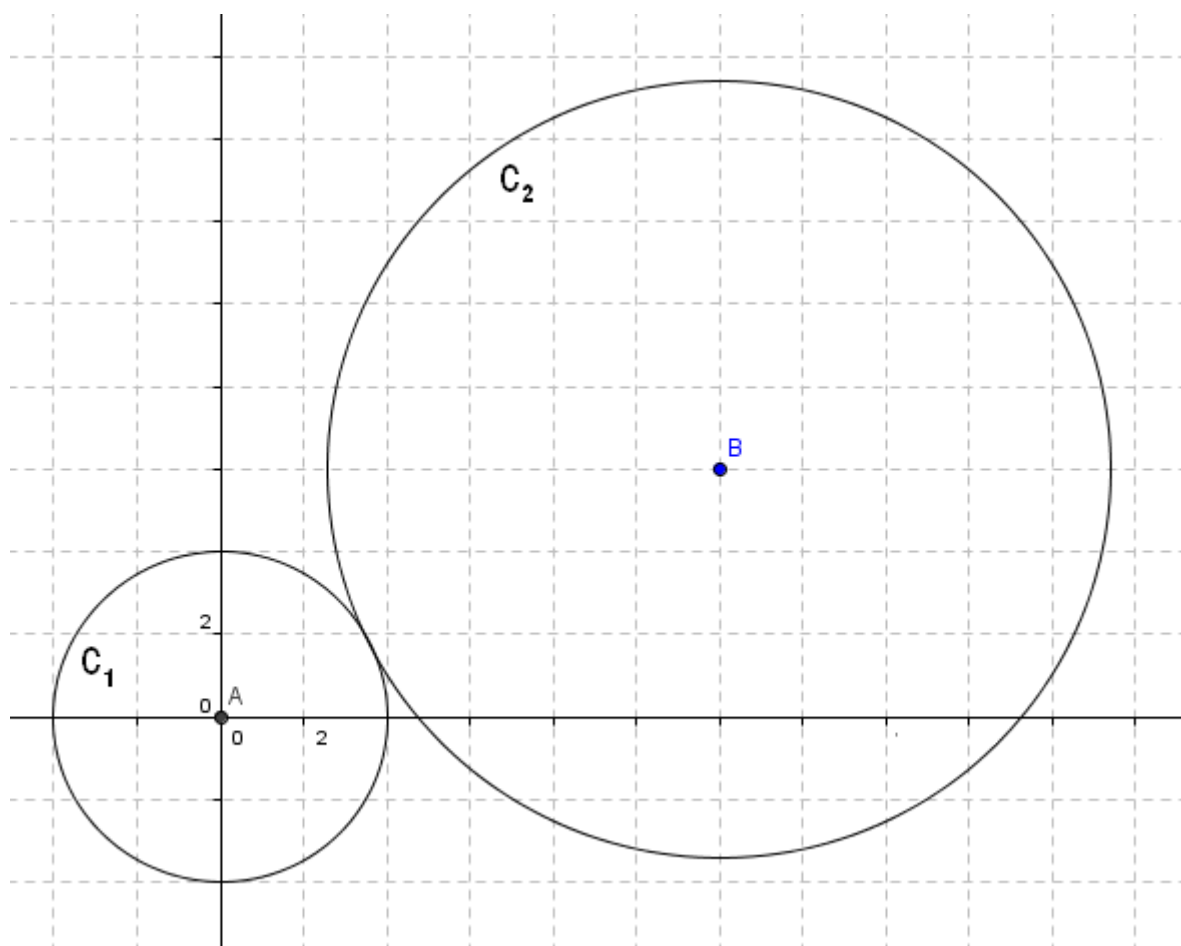
(c) How many ways can these three letters be arranged? Show each arrangement.



6 different arrangements.

Question

(a) The diagram shows two touching circles; c_1 and c_2 . Using the diagram to estimate the centres and radii as accurately as you can, find the equations of the two circles.



$$C_1 - R = 4, C = (0, 0)$$

$$C_2 - R = 9.5, C = (12, 6)$$

$$C_1 \quad (x-h)^2 + (y-k)^2 = r^2$$

$$x^2 - 2xh + h^2 + y^2 - 2yh + k^2 = r^2$$

$$x^2 - 2x(0) + (0)^2 + y^2 - 2y(0) + (0)^2 = 16$$

$$x^2 + y^2 = 16$$

$$C_2 \quad x^2 - 2xh + h^2 + y^2 - 2yh + k^2 = r^2$$

$$x^2 - 2x(12) + (12)^2 + y^2 - 2y(6) + (6)^2 = (9.5)^2$$

$$x^2 - 24x + 144 + y^2 - 12y + 36 = 90.25$$

$$x^2 + y^2 - 24x - 12y - 84.25 = 0$$

- (b) It is claimed that the line with equation $x - y + 6 = 0$ is a tangent to both circles. By performing suitable calculations, decide whether this claim is true or false. Explain your answer.

$$x = y - 6 \quad (y-6)^2 + y^2 = 16$$

$$y^2 - 12y + 36 + y^2 = 16$$

$$2y^2 - 12y + 20 = 0$$

$$y^2 - 6y + 10 = 0$$

More than one point of contact \rightarrow NOT a tangent

$$(x-12)^2 + (y-6)^2 = 90.25$$

$$(y-6-12)^2 + (y-6)^2 = 90.25$$

$$(y-18)^2 + (y-6)^2 = 90.25$$

$$y^2 - 36y + 324 + y^2 - 12y + 36 = 90.25$$

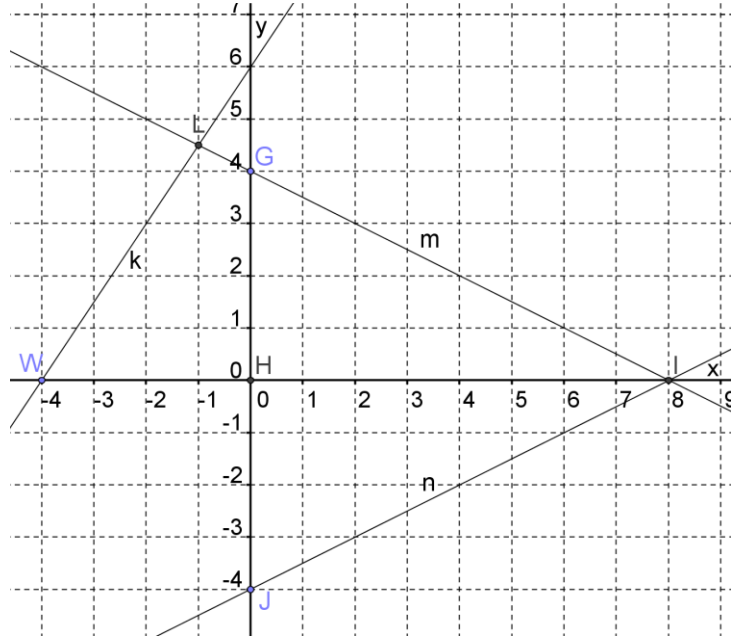
$$2y^2 - 48y + 269.75 = 0$$

$$y^2 - 24y + 134.875 = 0$$

More than one point of contact
NOT a tangent

Question

In this diagram state whether each of the following statements is true or false (by placing a ✓ in the appropriate box) and in each case give a reason for your answer.



a) $k \perp m$

True	False
	x

$\text{Slope } k = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{4 - 0}{-1 - (-4)}$ $= \frac{4}{3}$	$\text{Slope } m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{0 - 4}{8 - 0}$ $= -\frac{1}{2}$
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b) area $\Delta GIJ = 32$ sq. units

True	False
x	

$\frac{1}{2} \times \text{base} \times \text{perpendicular height}$
 $\frac{1}{2} \times 8 \times 8 = 32 \text{ sq. units}$

c) the equation of k is $y = -\frac{2}{3}x + 6$

True	False
	x

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{2}{3}(x - (-4))$$

$$y = -\frac{2}{3}x - \frac{8}{3}$$

d) $m \perp n$

True	False
	x

Slope of $m = -\frac{1}{2}$
 Slope of $n = \frac{1}{2}$
 $\therefore m \not\perp n$

e) the line $y = -2x + 1$ is perpendicular to n

True	False
x	

$m = -2$
 Slope $n = \frac{1}{2}$ $-2 \times \frac{1}{2} = -1$

f) the line $y = 2x$ is parallel to m

True	False
	x

$m = 2$
 Slope $m = -\frac{1}{2}$

g) $\triangle GIJ$ is an isosceles triangle

True	False
x	

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\sqrt{80} = 8\sqrt{4}$ $|6-5| = 8$
 $\sqrt{(0-8)^2 + (4-0)^2}$ $\sqrt{(0-8)^2 + (4-0)^2}$
 $\sqrt{64+16} = \sqrt{80}$ $= \sqrt{80}$

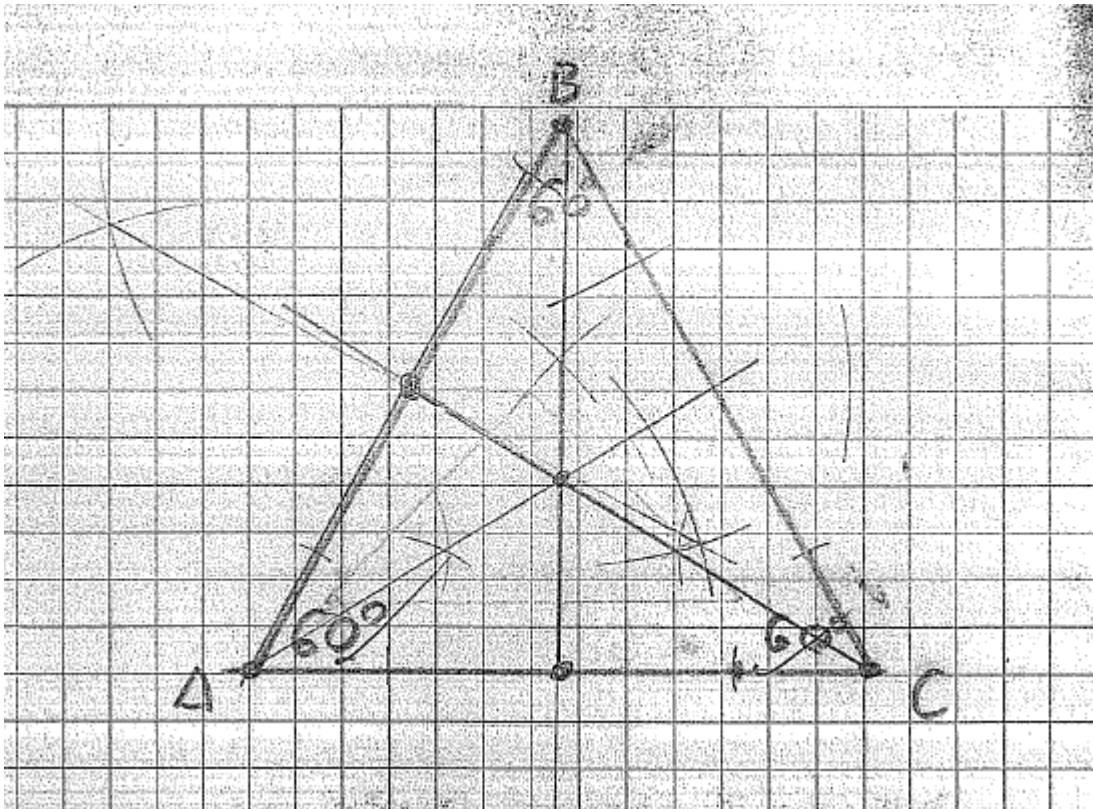
h) the x -axis is the bisector of $\angle GIJ$

True	False

$\tan A = \frac{4}{8}$ $\tan B = \frac{4}{8}$
 $A = B$
 $\therefore x\text{-axis bisects } \angle GIJ$

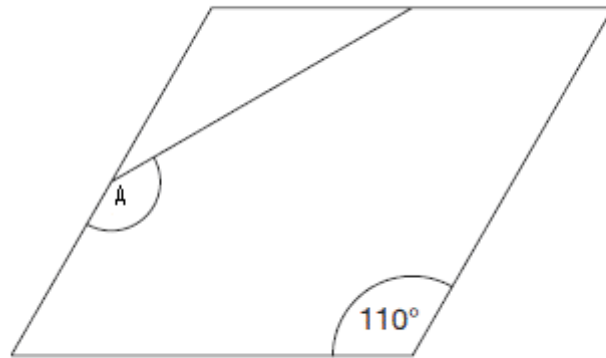
Question

Construct an equilateral triangle. Prove that the inscribed circle and the circumcircle have the same centre.



Question

- (a) The diagram shows a rhombus (that is, a parallelogram with four sides of equal length). The midpoints of two of its sides are joined with a straight line segment.

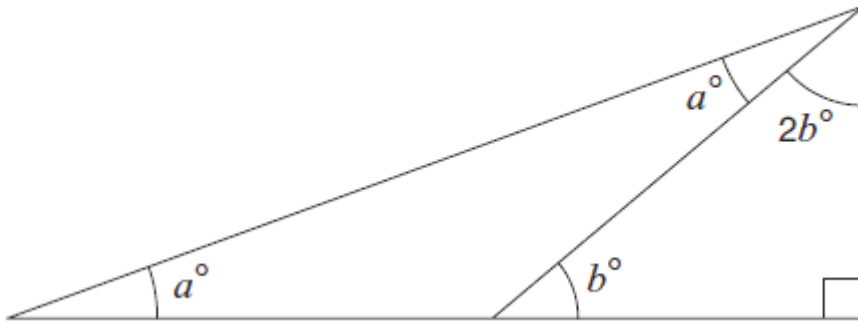


Not drawn
to scale.

Calculate the size of angle A. Show how you found your answer.

Opposite angles are equal, Angles in a Δ add up to 180°
 $180 - 110 = 70$
Isosceles triangle so - $70/2 = 35$
 $35 + A = 180 \Rightarrow$ Straight line = 180°
 $A = 145^\circ$

(b)



Not drawn
to scale.

Find the value of a . Show how you found your answer.

$$1. \quad = 180^\circ - 90^\circ = 90^\circ$$

$$3b = 90^\circ$$

$$b = \frac{90^\circ}{3}$$

$$b = 30^\circ$$

$$\begin{aligned} \text{Straight line} &= 180^\circ - 30^\circ \\ &= 150^\circ \end{aligned}$$

$$3 \text{ angle in triangle} = 180^\circ$$

$$\therefore 180^\circ = 150^\circ + 2a$$

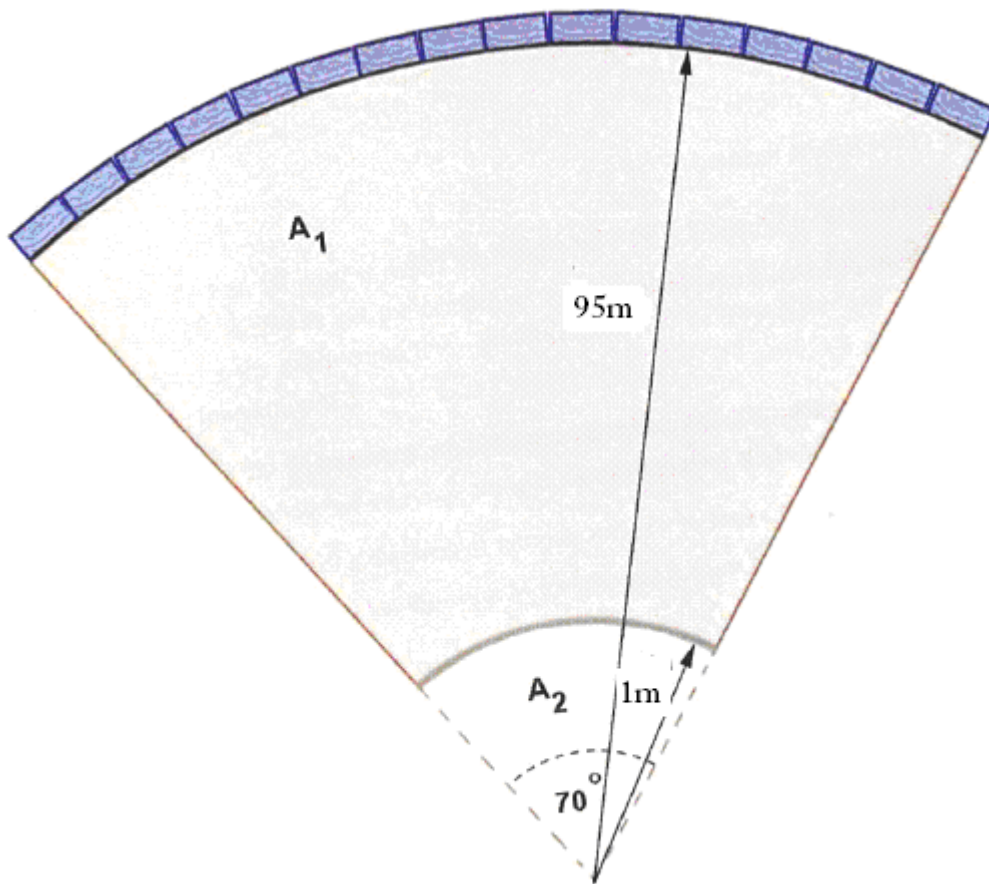
$$180^\circ - 150^\circ = 2a$$

$$30^\circ = 2a$$

$$15^\circ = a$$

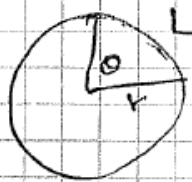
Question

(a) The modern or Olympic *hammer throw* is an athletic throwing event where the object is to throw a heavy metal ball attached to a wire and handle. In the diagram below A_2 represents a portion of the *throwing circle* and A_1 represents the area in which the hammer should land. The diagram is not drawn to scale.



- (i) A net is to be erected at the end of the landing area. The foundation consists of a single row of bricks; each brick is 41cm long. How many bricks will be needed to lay the foundations?
- (ii) The area A_1 will be planted with grass. A 10kg bag of lawn seed covers approximately $220m^2$. How many bags of grass seed must be bought?

Show all your work and state any assumptions you make.



$$L = 2\pi r \left[\frac{\theta}{360^\circ} \right]$$

$$L = (2)(\pi)(95) \left[\frac{70^\circ}{360^\circ} \right]$$

$$\text{cm } 11606.4$$

$$L = 116.064 \text{ m}^2$$

$$11606.4 \div 41$$

$$\underline{L \geq 1}$$

$$= 283.08$$

284 bags needed

$$\text{Area of } A_1 = A = \pi r^2 \left[\frac{\theta}{360^\circ} \right]$$

$$A = \pi (95)^2 \left[\frac{70^\circ}{360^\circ} \right]$$

$$A = 5513.058$$

$$\text{Area of } A_2 = A = \pi r^2 \left[\frac{\theta}{360^\circ} \right]$$

$$A = \pi (1)^2 \left[\frac{70^\circ}{360^\circ} \right]$$

$$A = .06108$$

$$A_1 - A_2 = 5513.058 - .06108 = 5512.99 \div 220$$

$$= 25.059$$

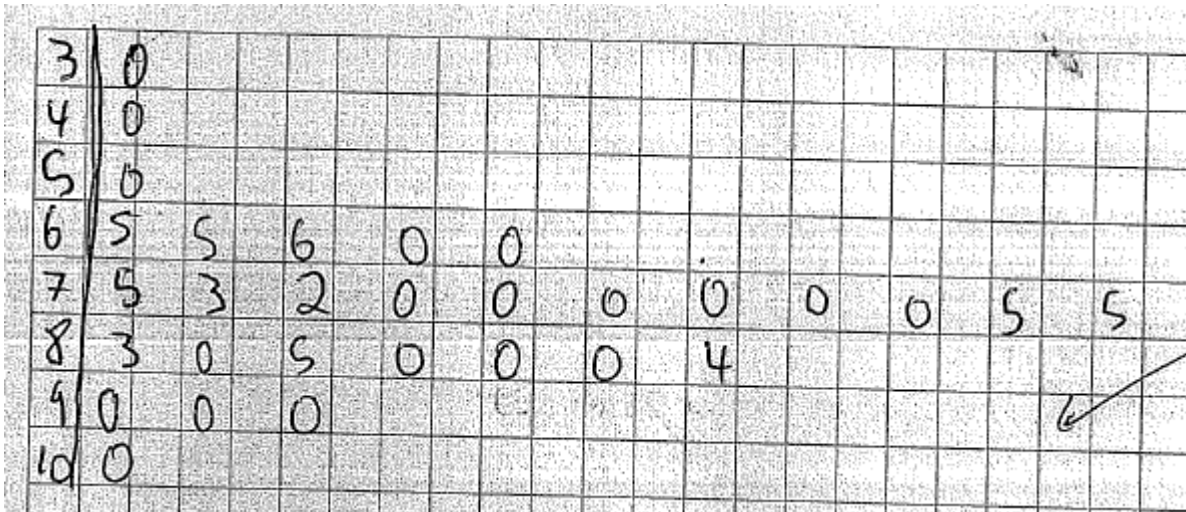
Ans = 26 bags

Question

The lengths of the ring fingers of 30 Irish students chosen randomly from amongst those who completed the *censusatschool* phase 9 questionnaire are displayed below. The measurements are in cm.

7.5	8	7	6	7.5
8.3	6.5	8	5	9
7.3	8.5	7	7	9
7.2	6.5	7	10	9
3	4	6.6	6	8
7	8	7	7.5	8.4

(a) Use the data to investigate whether ring finger lengths are normally distributed. Explain your answer.



- (b) Sharon measured the length of her ring finger and found it to be 11.3cm. Her boyfriend says her finger length is most unusual; Sharon disagrees. By calculating the mean and standard deviation of the distribution above, present evidence to support either Sharon's argument, or that of her boyfriend.

$$\text{Mean} = 7.28$$

$$\text{Standard deviation} = 1.46$$

Sharon's ring size is most unusual as it lies between the second and third standard deviation.

Mathematics (Project Maths – Phase 2)

Pre-Leaving Certificate – Ordinary Level Paper 2

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