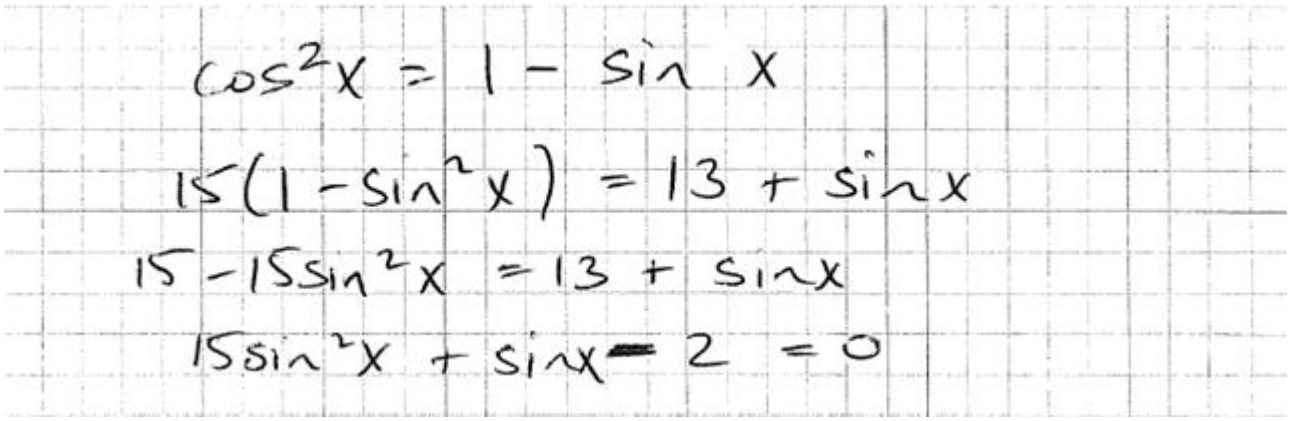
Section A Concepts and Skills

Question

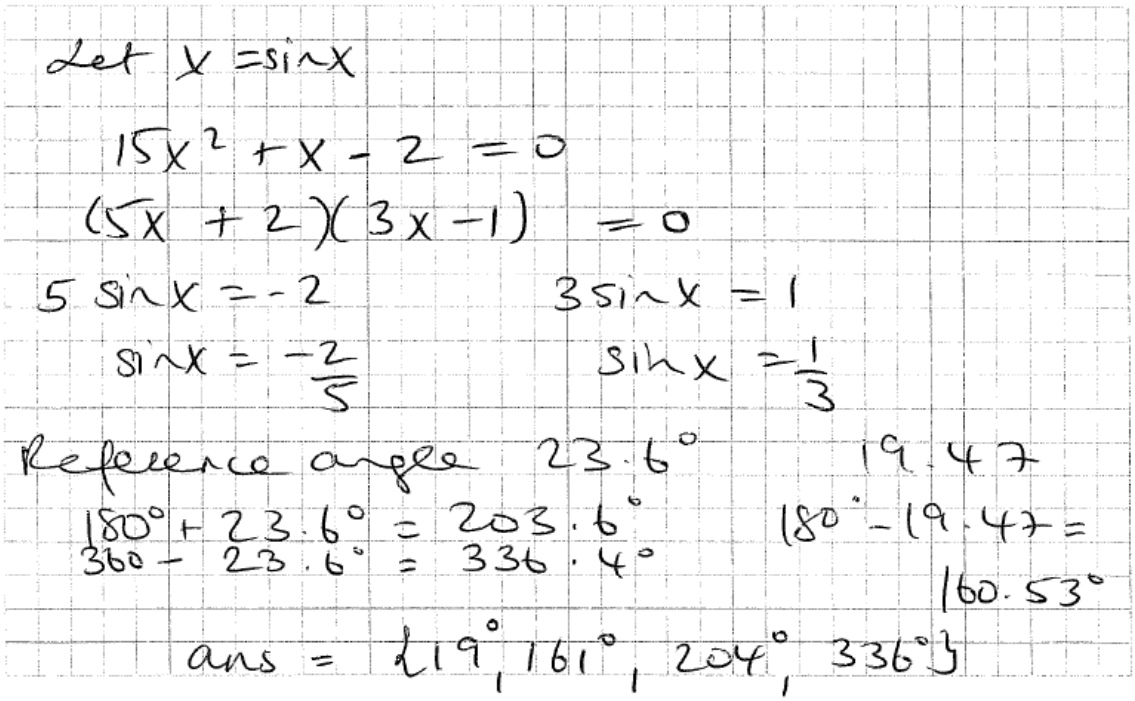
(a) Show that the equation

$$15\cos^2 x = 13 + \sin x$$

may be written as a quadratic equation in sin x.

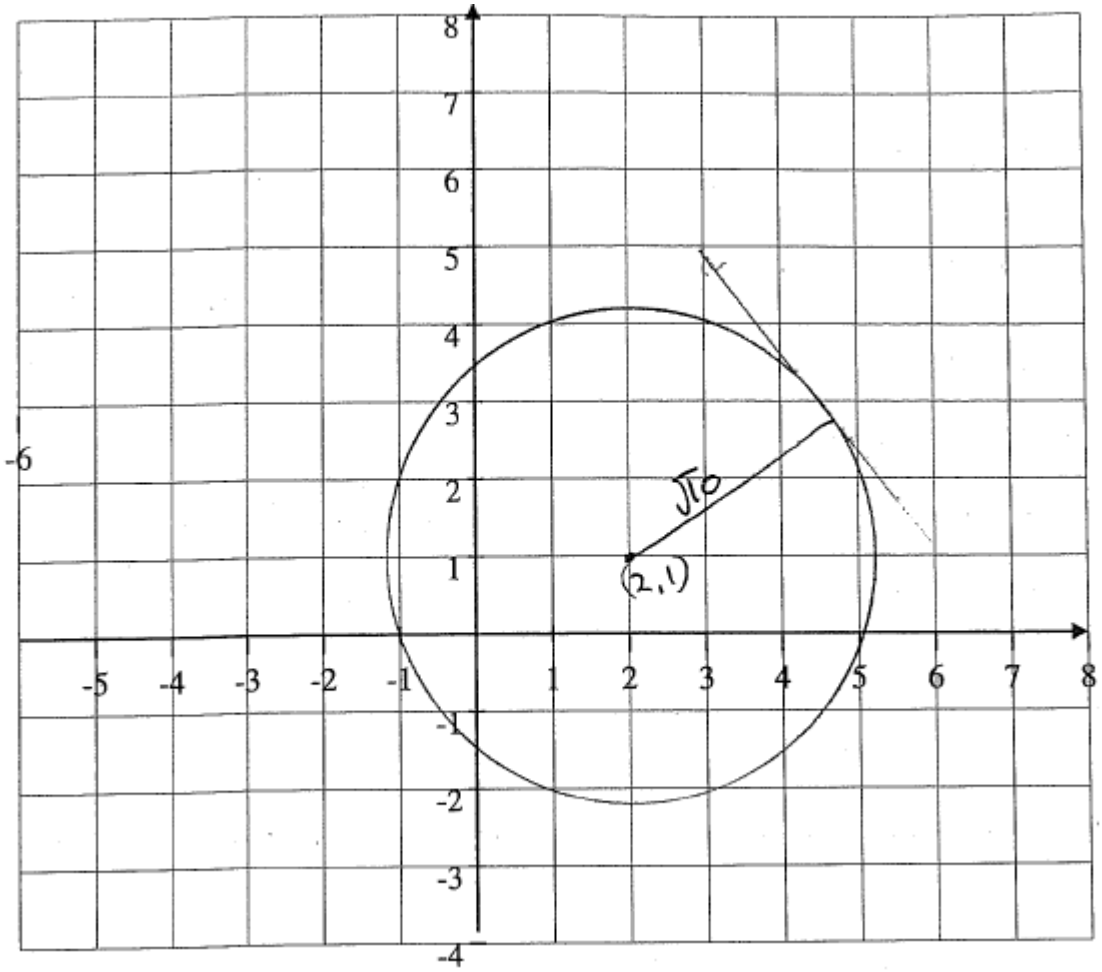


(b) Solve the quadratic equation for sin x, and hence solve for all values of x where $0° \leq x \leq 360°$. Give your answer(s) correct to the nearest degree.

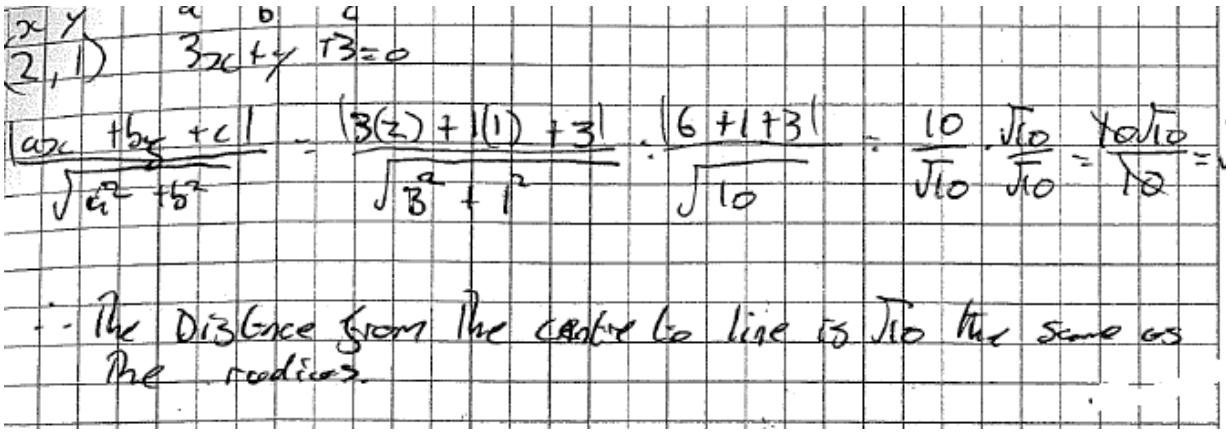


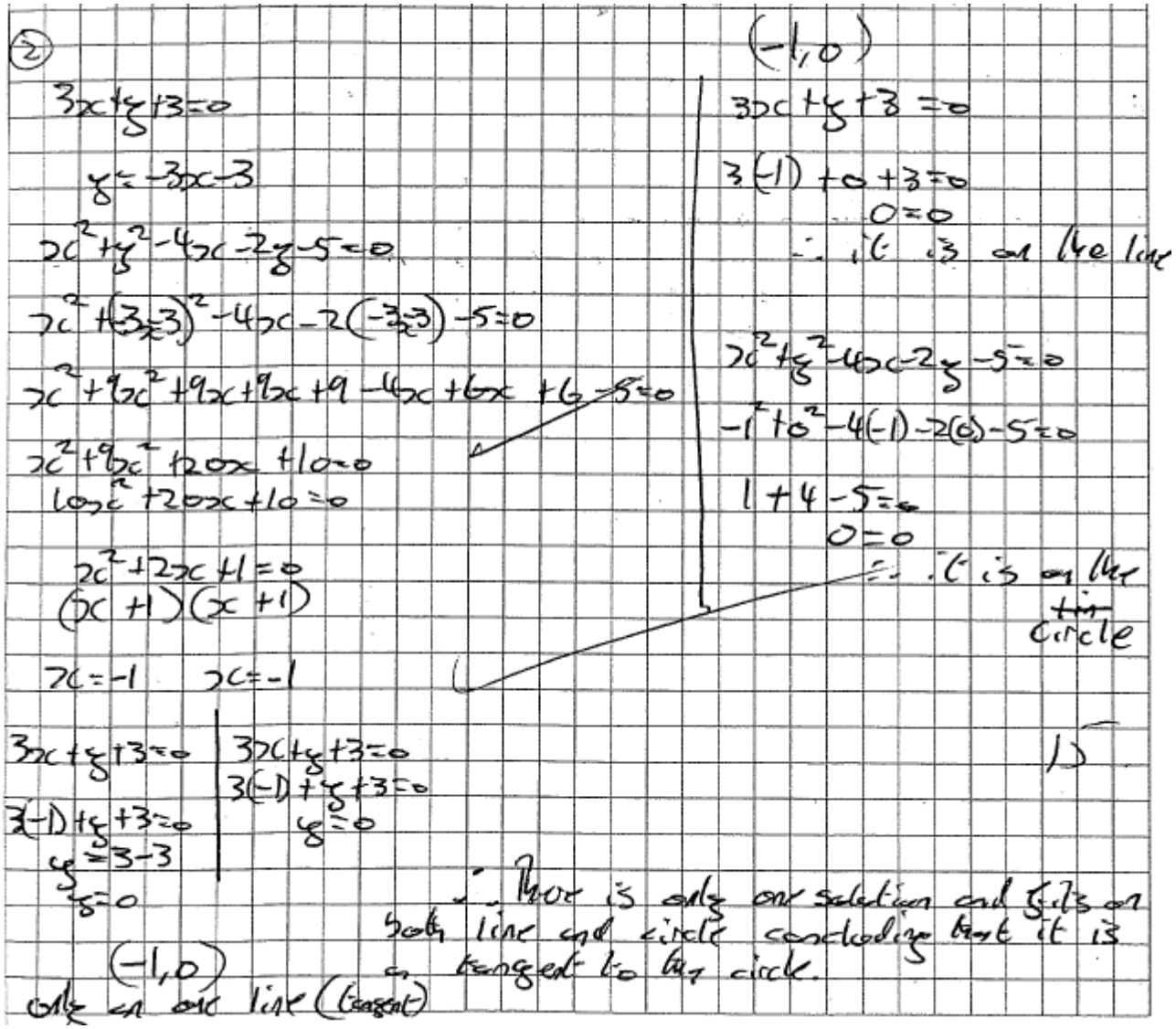
Question

(a) On the grid provided draw circle p whose equation is $x^2 + y^2 - 4x - 2y - 5 = 0$.



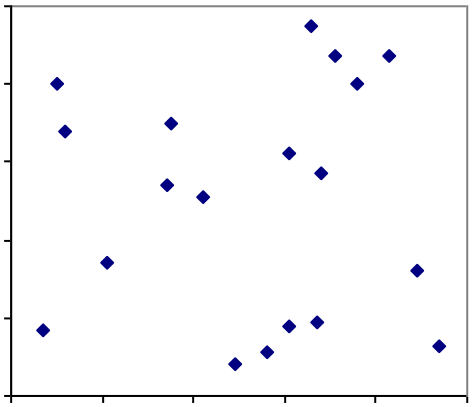
(b) Use **two different** methods to determine whether the line $l: 3x + y + 3 = 0$ is tangent to this circle p.



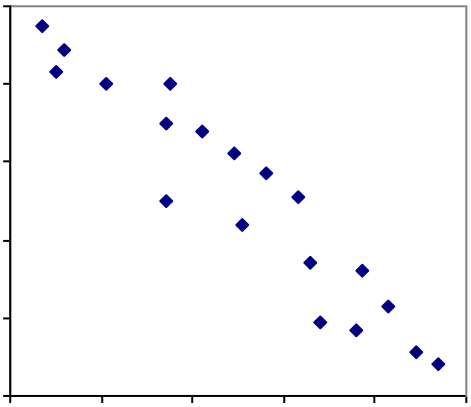


Question

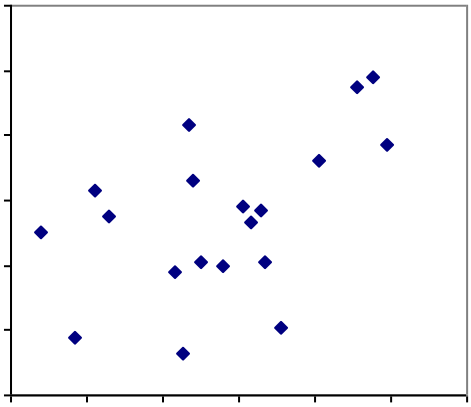
(a) For each of the four scatter plots below, estimate the correlation coefficient.



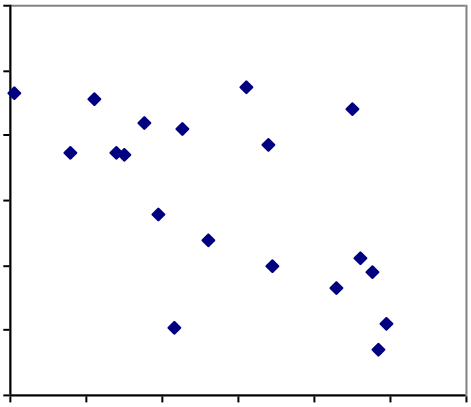
Correlation ≈ 0



Correlation ≈ -0.9



Correlation ≈ 0.5



Correlation ≈ 0.6

(b) Using your calculator, or otherwise, find the correlation coefficient for the data given in the table.

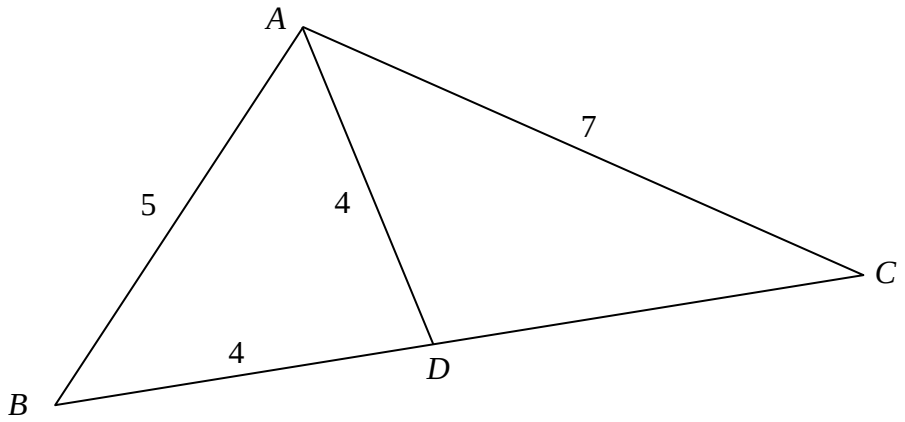
Give your answer correct to two decimal places.

Answer: 0.76

x	*y*
0.0	0.5
5.0	1.3
5.2	3.3
6.1	6.7
9.3	4.5
9.5	4.6
9.9	6.5

Question

(a) *ABC* is a triangle, and *D* is a point on [*BC*].
The lengths |*AB*|, |*AD*|, |*AC*| and |*BD*| are as shown in the diagram.



Find |*DC*|, correct to one decimal place.

$\triangle ABD \quad 5^2 = 16 + 16 - 2(4)(4)\cos D$

$\cos D = 7/32 \quad D = 77^\circ$

$\angle CDA = 103^\circ$

$7^2 = 4^2 + x^2 - 2(4)(x)\cos 103^\circ$

$49 = 16 + x^2 - 8x\cos 103^\circ$

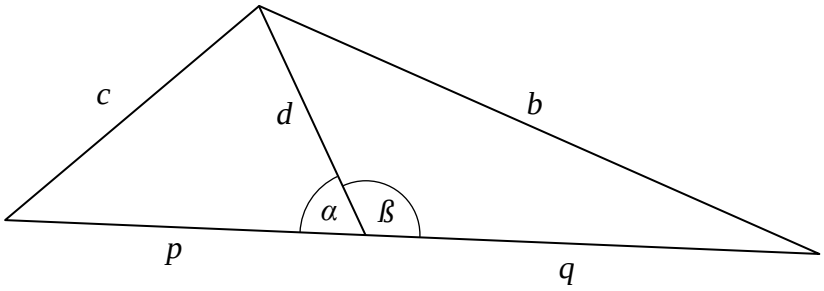
$x^2 + 1.8x - 33 = 0 \quad a = 1 \; b = 1.8 \; c = -33$

$\frac{-1.8 \pm \sqrt{3.24 + 132}}{2} = \frac{-1.8 \pm \sqrt{135.24}}{2}$

$x = 4.9 \text{ or } -6.7$

$\therefore \quad x = 4.9 \qquad |DC| = 4.9$

(b) Consider the diagram below.



Express $\cos\alpha$ **and** $\cos\beta$ **in terms of the labelled lengths**

$c^2 = d^2 + p^2 - 2(d)(p)\cos\alpha$

$c^2 - d^2 - p^2 = -2dp\cos\alpha$

$\frac{c^2 - d^2 - p^2}{-2dp} = \cos\alpha$

$b^2 = q^2 + d^2 - 2(q)(d)\cos\beta$

$b^2 - q^2 - d^2 = -2qd\cos\beta$

$\frac{b^2 - q^2 - d^2}{-2qd} = \cos\beta$

(ii) Show that $pb^2 + qc^2 = (p+q)(pq+d^2)$

$\cos\alpha = -\cos\beta$

$\frac{c^2 - p^2 - d^2}{-2pd} = \frac{b^2 - q^2 - q^2}{-2dq}$

$-2dqc^2 + 2dqp^2 + 2d^2q = 2pdb^2 - 2pd^2 - 2pdq^2$

$qp^2 + qd^2 + pd^2 + pq^2 = pb^2 + qc^2$

$p(pq+d^2) + q(pq+d^2) = pb^2 + qc^2$

$(p+q)(pq+d^2) = pb^2 + qc^2$

Question

(a) The following formula relates to the *binomial distribution*.

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

(i) State what each of the letters *p*, *q*, *n*, and *r* represents in the formula above.

(i) State what each of the letters *p*, *q*, *n*, and *r* represents in the formula above.

p is The probability of success

q is The probability of failure

n is Number of trials

r is Number of successes.

(ii) Describe the type of experiment that results in a random variable that has a binomial distribution.

Repeated experiments that are independent of each other where there are two possible outcomes, success or failure and where the probability of success is the same for each trial.

(b) In a certain type of archery competition, Laura hits the target with an average of two out of every three shots. The shots are independent of each other. During one such competition, she has ten shots at the target.

(i) Find the probability that Laura hits the target exactly nine times.

Give your answer correct to three decimal places.

$p = 2/3 \quad q = 1/3 \quad n = 10 \quad r = 9$

$P(9 \text{ hits}) = \binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1 = \frac{10 \times 2^9}{3^{10}} = 0.0867$

$P(9 \text{ hits}) = 0.087$

(ii) Find the probability that Laura hits the target fewer than nine times.

Give your answer correct to three decimal places.

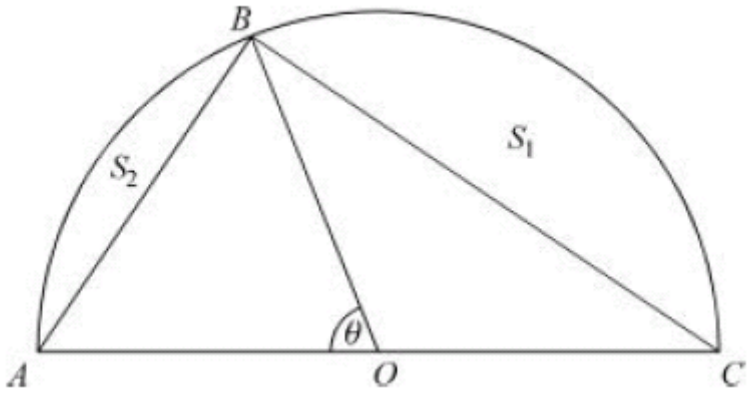
$P(10 \text{ hits}) = \left(\frac{2}{3}\right)^{10} = 0.0173$

$P(9 \text{ hits}) = 0.0867$

$P(<9) = 1 - (0.0867 + 0.0173) = 0.8960$

Thus fewer than 9 times $\Rightarrow 0.896$

Question



The diagram shows a semicircle *ABC* on [*AC*] as diameter. The mid-point of [*AC*] is *O*, and angle *AOB* = θ radians, where $0 < \theta < \frac{\pi}{2}$. The area of the segment S_1 cut off by the chord *BC* is twice the area of the segment S_2 bounded by the chord *AB*.

Show that $3\theta = \pi + \sin\theta$

Area S_1 = 2 Area S_2

Area S_1 = Area of a sector − area of ΔBOC

$\frac{1}{2}|OC|^2(180-\theta) - \frac{1}{2}|OC||OB|\sin(180-\theta)$

$\frac{1}{2}|OC|^2(180-\theta) - \frac{1}{2}|OC|^2\sin(180-\theta)$

Area of S_2 = Area of sector − area of ΔBOA

$\frac{1}{2}|OA|^2(\theta) - \frac{1}{2}|OA||OB|\sin\theta$

$\frac{1}{2}|OA|^2(\theta) - \frac{1}{2}|OA|^2\sin\theta$

$\frac{1}{2}|OC|^2[(\pi-\theta) - \sin(\pi-\theta)] = 2[\frac{1}{2}|OA|^2(\theta-\sin\theta)]$

$[\pi-\theta-\sin(\pi-\theta)] = 2(\theta-\sin\theta)$

$[\pi-\theta-\sin\pi\cos\theta+\cos\pi\sin\theta] = 2\theta-2\sin\theta$

$\pi-\theta-0-\sin\theta = 2\theta-2\sin\theta$

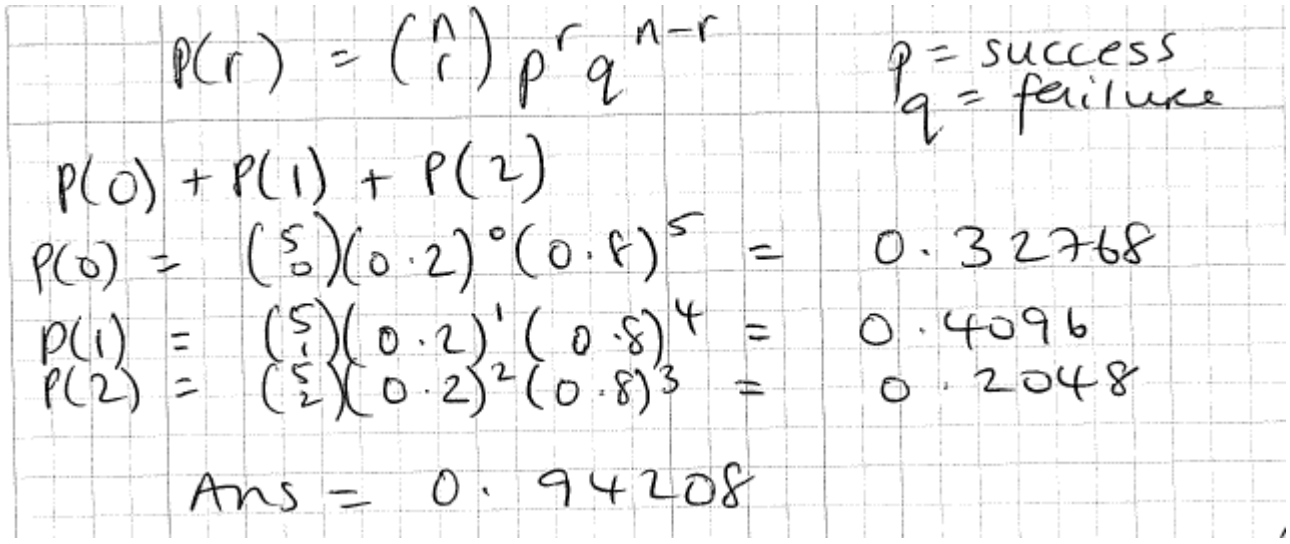
$-\theta-2\theta = -\pi-\sin\theta$

$3\theta = \pi+\sin\theta$

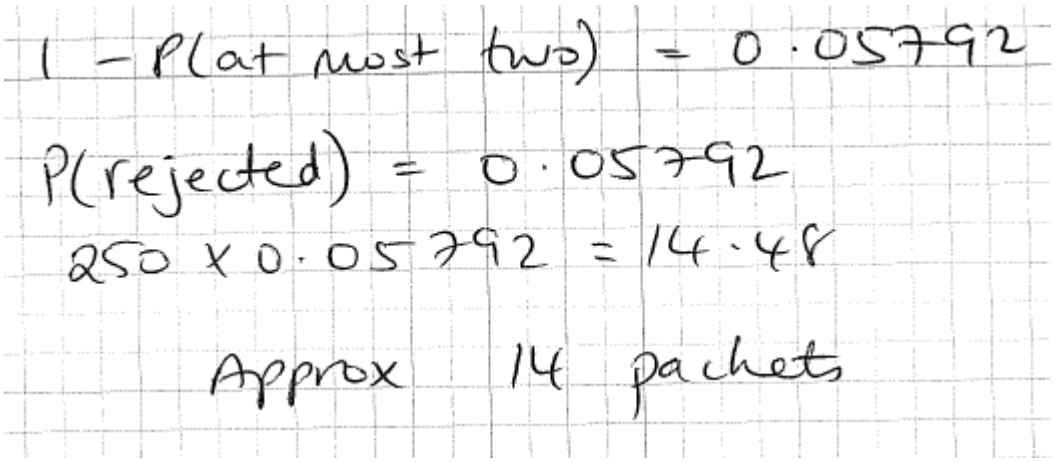
Question

20% of the bolts produced by a machine are defective.

(a) Find the probability that, in a group of five bolts randomly selected from a batch produced by the machine, at most two are defective.



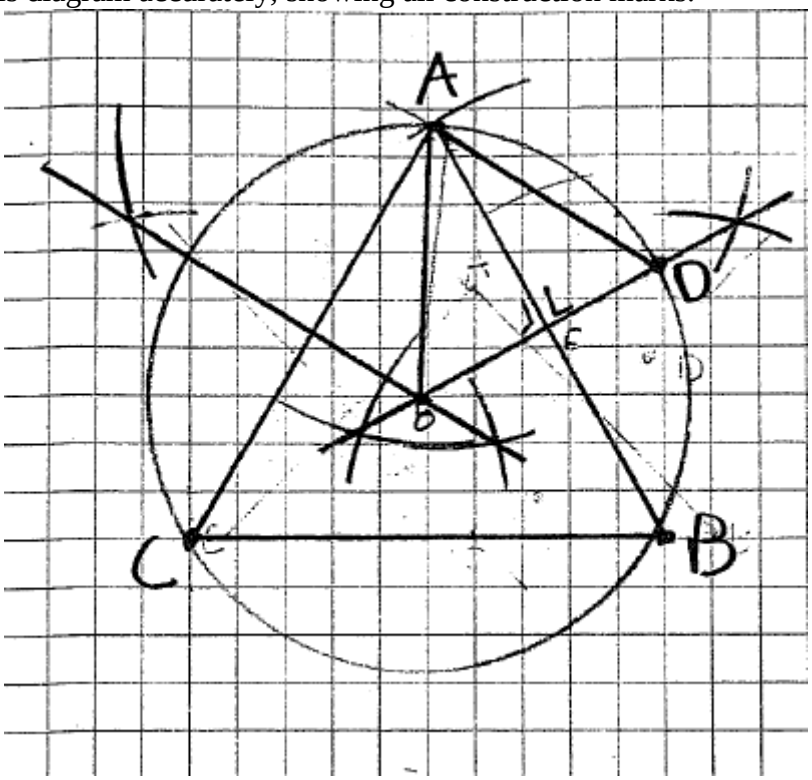
(b) A shipment of 250 packets of 5 bolts produced by this machine is inspected. A packet is rejected if it has more than two defective bolts. Show that approximately 14 packets are expected to be rejected.



Question

ABC is an equilateral triangle inscribed in a circle centre O. A radius is drawn from *O* through the midpoint of *AB* to meet the circumference of the circle at *D*.

(a) Construct this diagram accurately, showing all construction marks.



(b) Prove that *ODA* is equilateral.

Let E be the midpoint of AB

$\Delta AOE = \Delta BOE$

$|OA| = |OB|$ radii

$|AE| = |EB|$ midpoint E

$|OE| = |OE|$ common

$|\angle AOB| = 120^\circ$ on same arc as $|\angle ACB| = 60^\circ$

But $\angle AOE = \angle BOE \Rightarrow \angle AOE = 60^\circ$

$|OA| = |OD|$ both radii ΔOAD isosceles

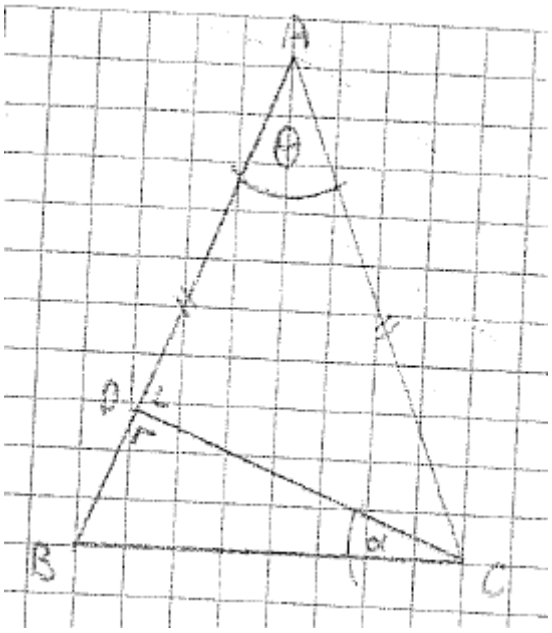
$|\angle OAD| = |\angle ODA| = \frac{1}{2}(180^\circ - 60^\circ) = 60^\circ$

$\therefore$ ODA is equilateral

Question

ABC is an isosceles triangle such that $|AB| = |AC|$ and *D* is a point on *AB* such that $CD \perp AB$. Represent this on a diagram.

Show that $|\angle BCD| = \frac{1}{2}|\angle BAC|$



$\angle B = \angle C$

$\angle B = \frac{1}{2}(180^\circ - \angle BAC)$

$\angle B = 90^\circ - \frac{1}{2}|\angle BAC|$

$\angle B = 180^\circ - 90^\circ - |\angle BCD|$

$= 90^\circ - |\angle BCD|$

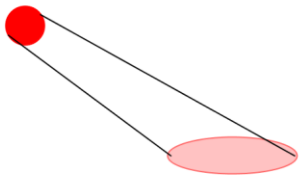
$90^\circ - |\angle BCD| = 90^\circ - \frac{1}{2}|\angle BAC|$

$|\angle BCD| = \frac{1}{2}|\angle BAC|$

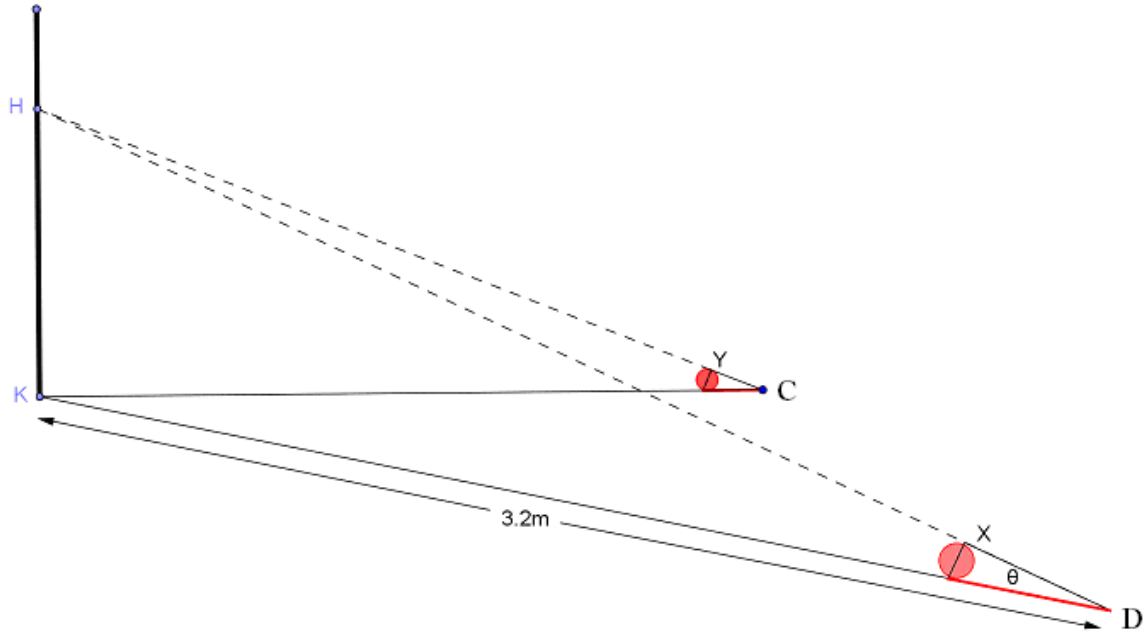
Section B Contexts and Applications

Question

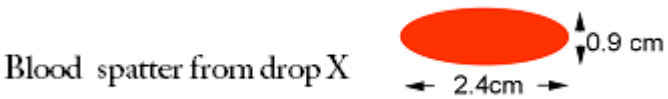
(a) Forensic investigators encounter crime scenes containing traces of blood. A spherical drop of blood makes an elliptical spatter when it hits the ground at an angle**.**



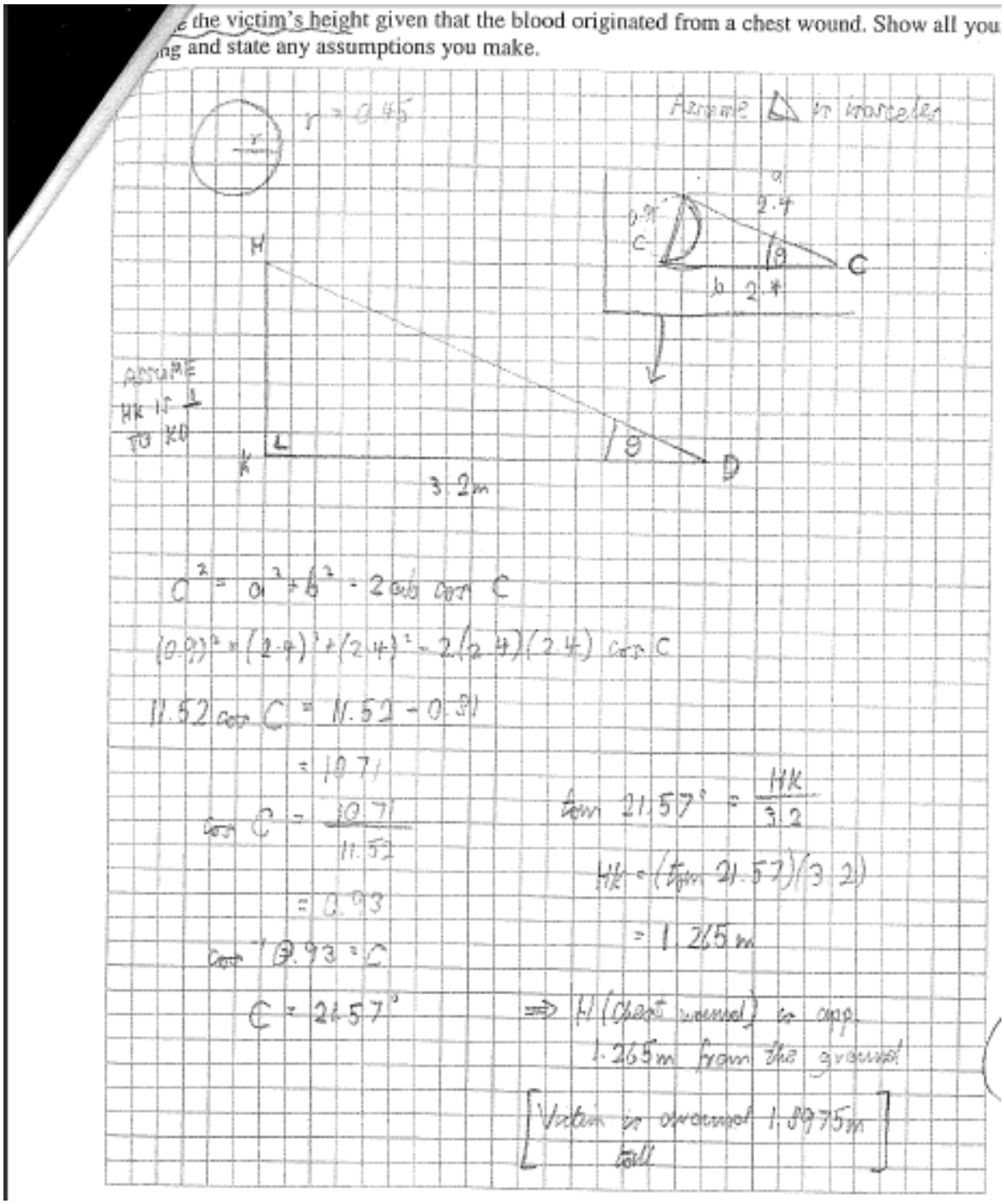
Investigators have mapped out the crime scene below showing two blood spatters on the floor at C and D. They measure a spatter in order to calculate the angle of impact and trace this back to an approximate starting place (assuming the blood drops travel in a straight line).



Since they know the drop started as a sphere, the width of the spatter drop will be the same as its diameter. They record the measurements of the blood spatter X:



Estimate the victim's height given that the blood originated from a chest wound. Show all your working and state any assumptions you make.



Question

(a) In a component factory, machine A produces 30% of the output, machine B 25% and machine C the remainder.

Over a period of time, 1% of the output from machine A is found to be defective, 1.2% from machine B and 2% from machine C.

(i) On a given day, the three machines produce a total of 10,000 components. How many components are likely to be defective?

A => 0.3 B => 0.25 C = 0.45

= 3000 units 2500 units 4500 units

3000 × 1% = 30 2500 × 1.2% = 30 4500 × 2% = 90

30 + 30 + 90 = 150 units are likely to be defective.

(ii) A quality controller selects a component at random from that day's output and finds that it is defective. What is the probability that this component was produced by machine B?

Probability it's from B given its defective.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

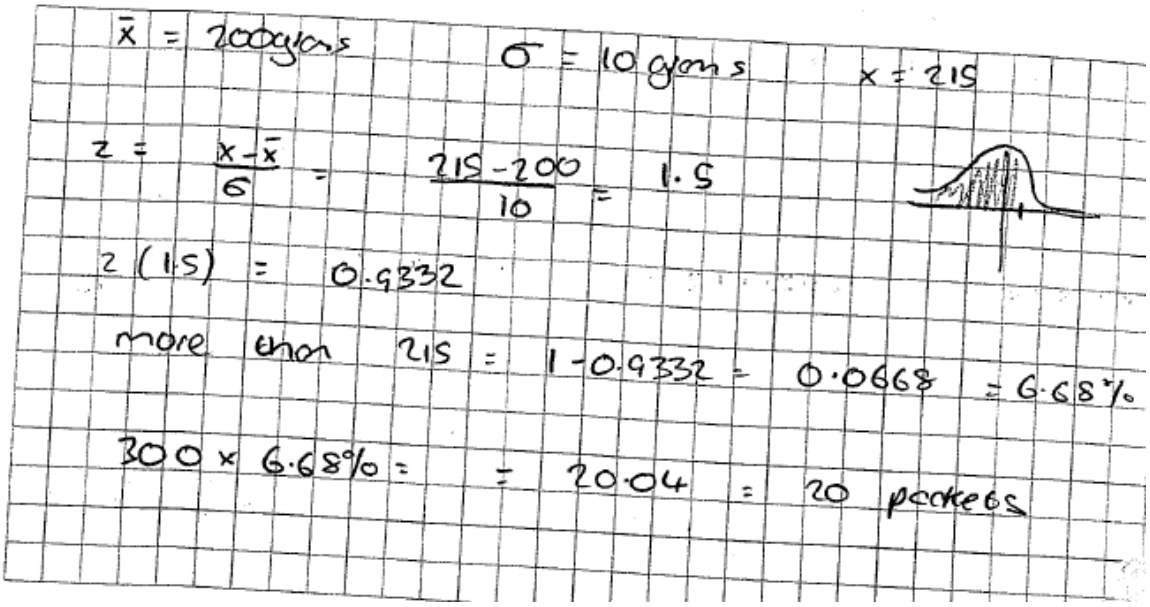
Defective overall = 150

Defective from B = 30

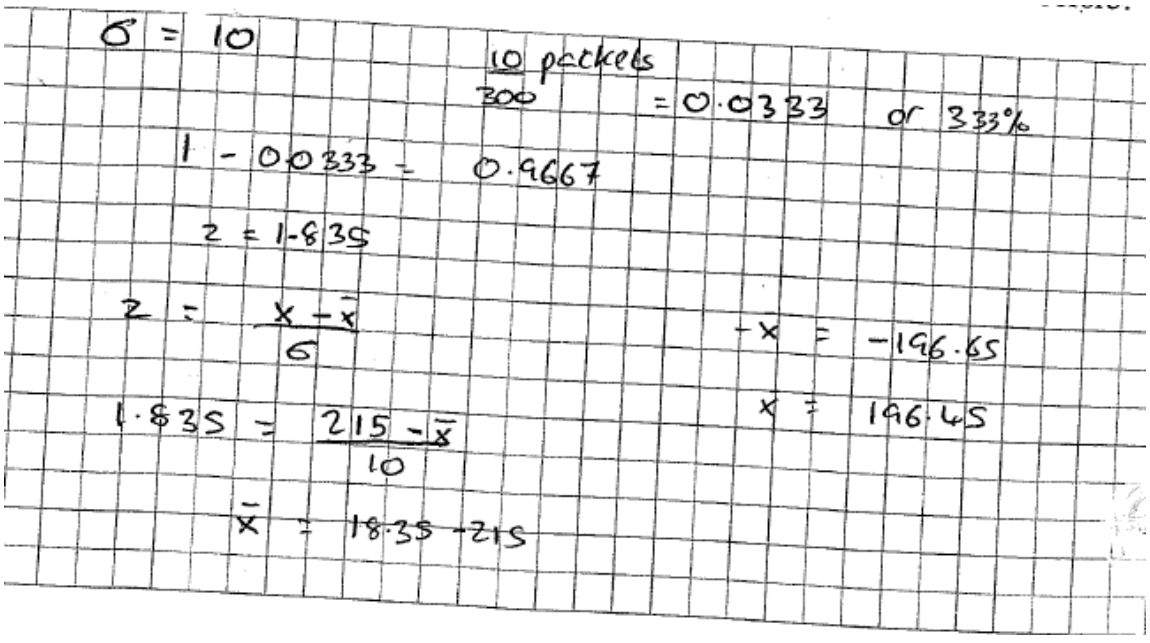
P(Produced from B) = $\frac{30}{150}$ = 20%

(b) Cereal packets filled by a machine have a mean of 200 grams with a standard deviation of 10 grams.

(i) In a sample of 300 packets, how many can be expected to weigh more than 215 grams? Show all your work.

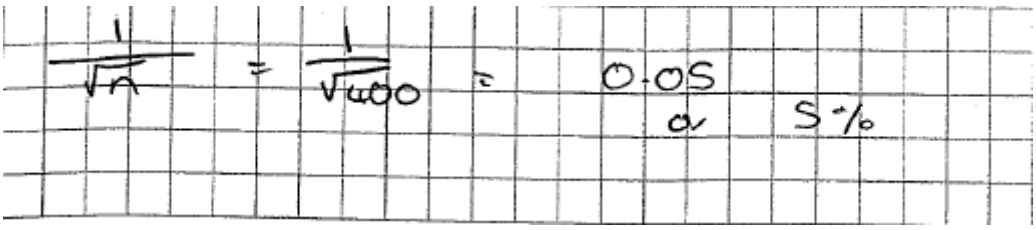


(ii) The machine is adjusted and another sample of 300 is checked. If the expected value of the number of packets that weigh more than 215 grams is now 10, calculate the new mean, assuming that the standard deviation after adjustment is the same as before?

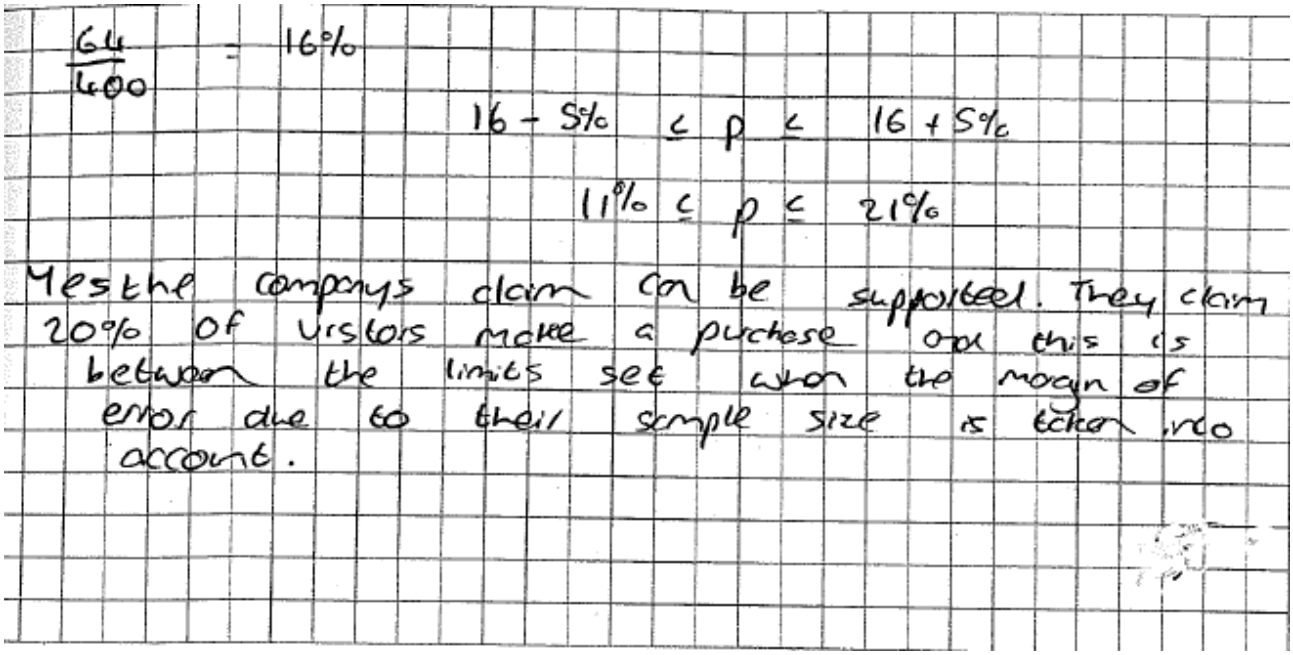


(c) A company states that 20% of the visitors to its website purchase at least one of their products. A sample of 400 site visitors is checked and the number who purchased a product is found to be 64.

(i) Calculate the margin of error in this case.



(ii) Based on this sample, should the company's claim be accepted? Explain your reasoning.



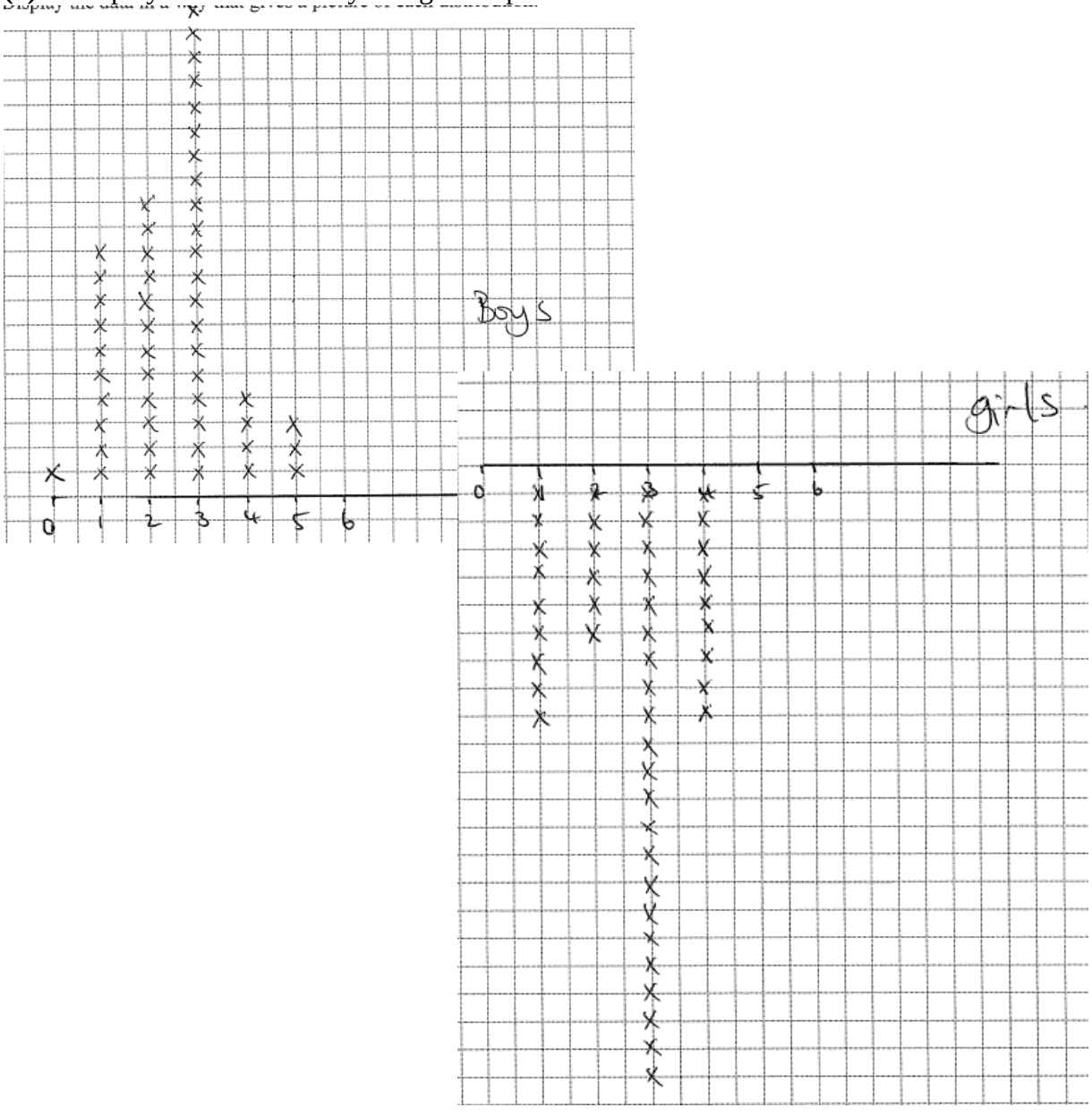
Question

Some research was carried out into the participation of girls and boys in sport. The researchers selected a simple random sample of fifty male and fifty female teenagers enrolled in GAA clubs in the greater Cork area. They asked the teenagers the question: *How many sports do you play?*

The data collected were as follows:

Boys	Girls
0, 4, 5, 1, 4, 1, 3, 3, 3, 1,	3, 3, 3, 1, 1, 3, 3, 1, 3, 3,
1, 2, 2, 2, 5, 3, 3, 4, 1, 2,	2, 2, 4, 4, 4, 5, 5, 2, 2, 3,
2, 2, 2, 3, 3, 3, 4, 5, 1, 1,	3, 3, 4, 1, 6, 2, 3, 3, 3, 4,
1, 1, 1, 2, 2, 2, 2, 2, 3, 3,	4, 5, 3, 4, 3, 3, 3, 4, 4, 3,
3, 3, 3, 3, 3, 3, 3, 3, 3, 3	1, 1, 3, 2, 1, 3, 1, 3, 1, 3

(a) Display the data in a way that gives a picture of each distribution.



(b) State **one difference** and **one similarity** between the distributions of the two samples.

Difference: 12 out of 50 girls play more than 3 sports whereas only 7 out of 50 boys play more than 3 sports

Similarity:

the mode is the same for both.

(c) Do you think that there is evidence that there are differences between the two populations? Explain your answer.

Note: you are not required to conduct a formal hypothesis test.

Answer: No

Justification:

I think the samples are so similiar that it is unlikely to be due to chance.

(d) The researchers are planning to repeat this research on a larger scale. List **two** improvements they could make to the design of the research in order to reduce the possibility of *bias* in the samples. Explain why each improvement you suggest will reduce the likelihood of bias.

They could sample people other than those in GAA clubs as this could result in potential bias.

They could include urban + rural areas in a wider area than just Cork

Question

The *Wonder Building* is an arched building that does not need any support inside, due partly to the fact that its shape is an arc of a circle.

The photograph shows a *Wonder Building* being used in Antarctica.

The arc for a *Wonder Building* can be a full semicircle or less than a semicircle. It cannot be more than a semi-circle. The "span" of the building is the total width from one side of the arch to the other.

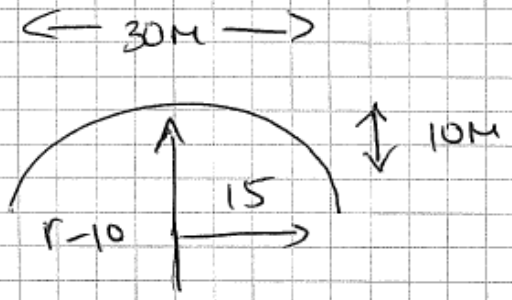
(a) A particular *Wonder Building* has a span of 30 metres and a height of 10 metres. Find the radius of the arc.

$r^2 = 15^2 + (r-10)^2$

$r^2 = 225 + r^2 - 20r + 100$

$20r = 325$

$r = 16.25$ M



(b) A customer wants a building with a span of 18 metres and a height of 10 metres.

(i) What arc radius would be required to give such a building?

$r^2 = 9^2 + (r-10)^2$

$r^2 = 81 + r^2 - 20r + 100$

$20r = 181$

$r = 9.05$M

(ii) Explain why the *Wonder Building* that the customer wants is not possible.

r − 10 can't be negative

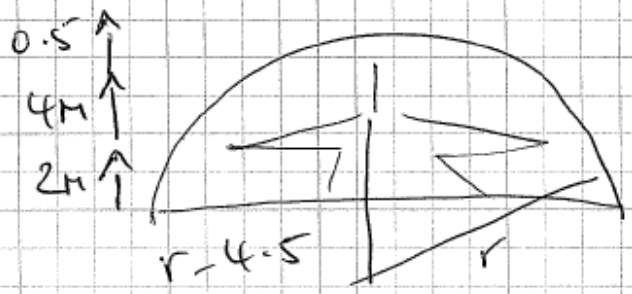
(c) An air force needs a *Wonder Building* to house a *Tornado* military jet.

The dimensions of the aircraft are as follows:

- Wingspan: 14 metres
- Height: 6 metres
- Height of wingtips above ground: 2 metres.

The shelter must be at least 0·5 metres above the top of the tail, and at least 1 metre clear horizontally of the wingtips.

For the shelter to have the exact clearance required, find the radius of the arc.



$r^2 = 8^2 + (r - 4.5)^2$

$= 64 + r^2 - 9r + 20.25$

$9r = 84.25$

$r = 9.361$ metres.