Supporting the shift from a 'groups of' way of thinking about multiplication to an array-based representation.

A group of First Year students were given the following task. The mathematical purpose of the task was to see how students were thinking about multiplication. There are several *ideas* that support the development of multiplicative thinking, and the ability to think multiplicatively is very important if students are to engage meaningfully with the *Strand 3 Number* in subsequent years. Consequently the development of multiplicative thinking is a major goal of the bridging period.

Task: Solve the following problem using a diagram.

4 people go to a party and they each bring 3 balloons. How many balloons in total do they bring?

The majority of students represented the situation as in the diagram below.



This is a *groups of* model of multiplication. The students are "*accumulating groups of equal size*" to represent the situation. It is a valid representation and learners can easily see the 4 "lots of" or "sets of" 3 balloons and can represent the situation with the arithmetic sentence

4 x 3 = 12

It is however, an additive model of multiplication and students need to move beyond this idea if they are to develop multiplicative thinking. A more useful model for making sense of the operation of multiplication is the *array model*

Why is it important that students make this shift?

Working with array representations enables students to

- simultaneously co-ordinate the number of groups, the number in each group and the total
- recognise commutativity
- relate the two ideas for division partition (or sharing) and quotition (or how many groups in), to multiplication

It also provides a basis for moving from a count of equal groups (eg, 1 three, 2 threes, 3 threes, 4 threes,...) to a constant number of groups (eg, 4 ones, 4 twos, 4 threes, 4 fours, 4 fives ...) which supports more efficient mental strategies (eg, 6 groups of anything is double 3 groups or 5 groups and 1 more group).



These learners have arranged the balloons in an array and, as with the above model, they can easily see the 4 "lots of" or "sets of" 3 balloons and can represent the situation with the arithmetic sentence

4 x 3 = 12

The array model will only be useful to learners if they fully understand how it can represent the story context and the arithmetic sentence. Learners need time to discuss this model and to reason and make sense of it; hence the initial simple task. Discuss each group's answer to the task and encourage learners to see how, of all the representations given, the array model is the most useful.

Useful questions to ask

- What does the "x" symbol represent in the story context? In the array?
- What does the "4" represent in the story context?
- What does the "3" represent in the story context?

Once learners have established the array model as a useful way to represent multiplication you can set further tasks that will allow them to use the model and reason and make sense of the operation of multiplication.