## Reasoning about Comparison

This task, set in a real-world context is particularly useful in helping students to construct an understanding of the difference between absolute comparison and relative comparison and to become aware of the relevance of proportion.

Task: John and Mark were making squash for the school sports. It is important that they get the right strength, they decided to choose a strength by mixing the orange concentrate and clear water together until they get just the right taste then they recorded the shade of orange and this shade is the desirable strength.

The boys have several jugs of liquid, some with orange concentrate and some with clear water. They plan to mix these together in big bowls. Before they mix the liquids, they guess the shade of orange the mixture will be.

In the diagrams below, there are two sets ( $A$ and $B$ ) of orange-clear combinations to mix. Predict which set will be darker orange, and explain your reasoning.
a) A

B

b)

c)
A

B

B

d) $A$

e)
B

f)


Challenge: The boys decided to see what would happen if they took two different mixtures and mixed them together. They called this the "union" of the two mixtures. For example, in the first example above, they took the two mixtures $A$ and $B$ and formed the union of the mixtures. Standard mathematical notation for the union of two things (usually sets) is $U$, so they named their new mixture $A \cup B$.
A

B


## A U B


a) In the above example which will be darker orange $A, B$ or $A U B$ ?
b) Can $A$ u $B$ ever be more orange than either $A$ or $B$ ? Explain

## Thoughts for teachers

- How would you use these tasks with your students?
- What mathematics do you want your students to learn from engaging with these tasks?
- What prior knowledge should your students bring to the task?
- What do you think your students might find difficult about this task?
- What questions might you ask as your students as they are working on the task
- How might an additive thinker answer which is darker orange?
- How might they justify their reasoning?
- How might a multiplicative (relative) thinker respond to this task?
- How might they justify their reasoning? How could you extend those who reason multiplicatively about more?
- How could you help those who reason additively begin to think multiplicatively about comparison?

Problem solving reminder: If you are going to use these tasks remember, answers are important but what is more important is the mathematics students can learn from engaging with the tasks.

